TEXT BOOK OF

BASIC ELECTRICAL ENGINEERING

For I / II Semester
Visveswaraiah Technological University

As per Latest Syllabus
As per Choice Based credit System (CBCS) Scheme

MV Rao [redacted]

M.V. Rao
Formerly Senior Faculty Member
A. F. Technical College, Bangalore
Ex-Principal, Hindustan Electronics
Academy Polytechnic, Bangalore.

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PUBLISHER'S NOTE

The author, Mr. M. V. Rao, has wide teaching experience in the areas of Electrical, Electronics and Communication Engineering. He was a Senior Lecturer at the Air Force Technical College, Bangalore, where he imparted instructions to Engineering Graduates undergoing post-graduate technical training. Later on, he was instrumental in setting up the Hindustan Electronics Academy Polytechnic, Bangalore, of which he was the first Principal.

Apart, from his wide teaching experience, the author has more than 25 years of practical experience in the aforementioned fields.

The author has already written books having the following titles -'ELECTRONIC DEVICES', 'DIGITAL ELECTRONICS', 'DIGITAL COMPONENTS & CIRCUITS', 'ELECTRICAL ENGINEERING', 'ELECTRICAL & ELECTRONIC ENGINEERING, several editions of BASIC ELECTRONICS and ELECTRICAL SCIENCES (including those which have been revised and enlarged), three editions each of BASIC ELECTRONICS and **BASIC ELECTRICAL ENGINEERING.** This Edition has been modified to cater to the latest Blow-up Syllabus.

Bangalore
August, 2014

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for **SUBHAS STORES**
A. PRAKASH
A. P. PRADEEP

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MODULE 1

(a) D.C. Circuits

1.1 Introduction

An electric circuit is an interconnection of the various elements such as, a voltage source, a current source, resistors, inductors and capacitors.

Two types of current may flow in an electric circuit (i) direct current (ii) alternating current and accordingly we have D.C. circuits & A.C. circuits. We shall take up the discussion of only D.C. circuits.

A direct current always remains constant and does not vary with time. The flow of direct current characterises the flow of electric charge in one particular direction. Fig. 1.1 depicts the direct current which does not vary with time.

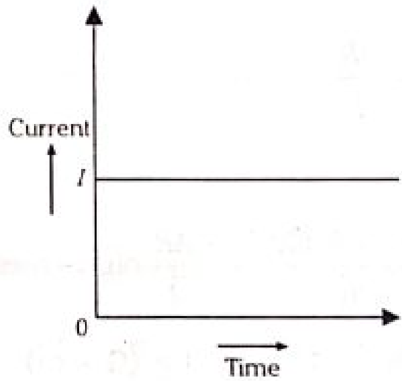


Fig. 1.1

A D.C. circuit consists of constant voltage sources, constant current sources and their interconnections with resistances only.

1.2 Basic Definitions

1 Independent Voltage Source

If the voltage of a source is constant and will not depend on any parameters of the circuit, it is called an *independent voltage source*. An independent d.c.voltage source is symbolically represented in Fig. 1.2.

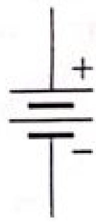


Fig. 1.2

2 Resistance & Resistivity

The property of a material due to which it opposes (or restricts the flow of current through it) is called **resistance**. The unit of resistance is *ohm* and its symbol is Ω.

The resistance of a conductor is directly proportional to its length and inversely proportional to its area of cross-section, *i.e.*,

$$R \propto \frac{l}{A}$$

or $$R = \rho \frac{l}{A} \qquad \text{---(i)}$$

where ρ (Greek letter "Rho") is a constant and is called *resistivity* or *specific resistance of the conductor material.*

If, in eqn (i) we put $l = 1$ metre and $A = 1$ metre^2, then $R = \rho$.

Hence, resistivity or specific resistance of a material may be defined as the resistance offered by one metre length of the material, having an area of cross-section of one square metre.

From eqn (i), we have $\rho = \dfrac{AR}{l}$

In the S.I system of units,

$$\rho = \frac{A \text{ metre}^2 \times R \text{ ohms}}{l \text{ metres}} = \frac{AR}{l} \text{ ohm} - \text{metre}$$

Hence the unit of resistivity is ohm - metre (Ω – m).

3 The Electric Current

In the outermost orbit of the atoms of conductors, there are free electrons which can be dislodged from the parent atom by the application of an external force. The continuous drift of electrons in a conductor on the application of an external force in a particular direction, constitutes the flow of current. *The rate at which the electric charge is transferred across a point in a conductor is known as the current flowing through the conductor, i.e.,*

$$I = \frac{dq}{dt} = \frac{q}{t} \qquad \text{---(ii)}$$

The unit of current is *ampere*.

4 The Ampere

One ampere of current is defined as that current which, when flowing through a resistance of one ohm, causes a potential difference of one volt across it.

From eqn (ii), one ampere of current may also be defined as the current flowing through a conductor, when a charge of one coulomb crosses a point in the conductor in one second.

One coulomb of charge is equal to the charge of 6.242×10^{18} electrons. Hence one ampere of current is said to be flowing through a conductor when 6.242×10^{18} electrons cross a point in the conductor in one second.

5 The Electric Potential

The electric potential always refers to a point in a charged conductor. The electric potential at any point in a charged conductor is defined as the work done to bring a unit positive charge from infinity to that point. The unit of electric potential is *volt*.

6 The Potential Difference

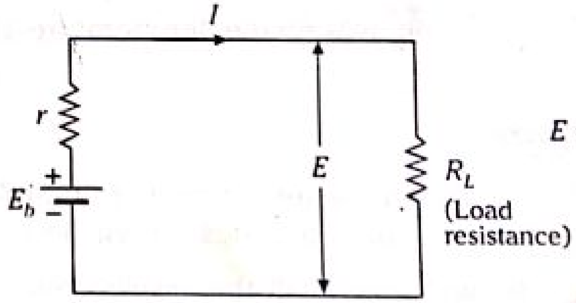
The potential difference between any two points of a charged conductor is the amount of work that has to be done to bring a unit positive charge from the point of lower potential to the point of higher potential. The unit of potential difference is *volt*. The potential difference is also referred to as the voltage between the two points of a conductor.

7 Volt (V)

One volt is defined as the potential difference across a resistance of one ohm, through which a current of one ampere is flowing.

8 E.M.F (Electromotive Force) of a Source (*E*)

The E.M.F of a source is the voltage available across its terminals. The voltage available across the terminals of a voltage source is slightly less than the internal voltage E_b because of the small voltage drop across its internal resistance r as shown in Fig. 1.3.



$$E = E_b - Ir$$

Fig. 1.3

The unit of E.M.F is also *volt*.

1.3 Ohm's Law

1.3.1 Definition :

The Ohm's Law states that, temperature remaining constant, the current through a passive element is directly proportional to the voltage across the element.

The passive element may be an individual element or an equivalent of a number of passive elements connected in series or parallel or a combination of some in series and some in parallel. Ohm's Law can be applied to any particular part of the circuit or to a complete circuit.

Taking a simple d.c.circuit of Fig. 1.4.

$$I \propto V$$

$$i.e., \frac{V}{I} = constant = R$$

where R is known as the *resistance of the element*

$$\text{or} \quad I = \frac{V}{R} \quad \text{or} \quad V = IR$$

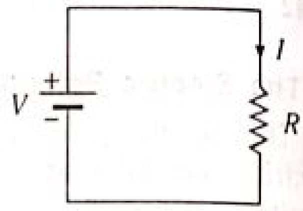


Fig. 1.4

1.3.2 Illustration of Ohm's Law

The application of Ohm's Law to Series, Parallel and Serial - Parallel circuit are illustrated in the problems below.

1.3.3 Limitations of Ohm's Law

(a) It does not hold true for non-linear devices such as semiconductors and zener diodes.

(b) It is not applicable to non-metallic conductors, such as silicon carbide, where the following relation is applicable :

$$V = KI^m$$

where K and m are constants

(c) Ohm's Law cannot be applied to arc-lamps.

(d) It does not hold good where the temperature rise is rapid in some metals.

1.4 Series Circuit

The circuit in which resistances are connected end-to-end, so that there is only one path for current flow, is called a series circuit. In a series circuit,

i) The same current flows through all the resistances.

ii) There will be a voltage drop across each resistance, according to Ohm's Law

iii) The sum of the voltage drops is equal to the applied voltage.

Fig. 1.5 show a series circuit, where resistors R_1, R_2 and R_3 are connected in series, and a voltage of V volts is applied at the extreme ends A and B, to cause a current of I amperes to flow through all these resistors.

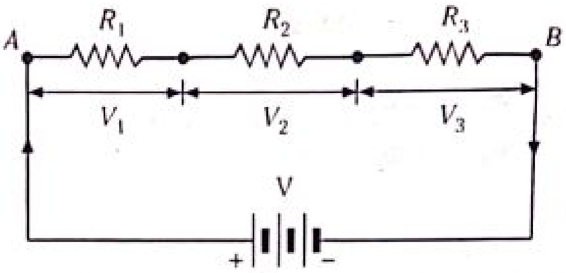


Fig. 1.5

Let V_1, V_2 and V_3 be the voltage drops across resistors R_1, R_2 and R_3 respectively.

Now, $V = V_1 + V_2 + V_3$

$$= IR_1 + IR_2 + IR_3 = I(R_1 + R_2 + R_3)$$

$$\therefore \quad \frac{V}{I} = R_1 + R_2 + R_3$$

According to Ohm's Law, V/I is the total circuit resistance R.

$$\therefore \quad R = R_1 + R_2 + R_3$$

i.e., Total resistance = Sum of individual resistances

Thus, when a number of resistors are connected in series, the equivalent resistance (total circuit resistance) is given by the arithmetic sum of their individual resistances.

Problem 1.1

Calculate for each of the circuits shown in Fig. 1.6 the current flowing in the circuit given that each resistor is of 2 kΩ.

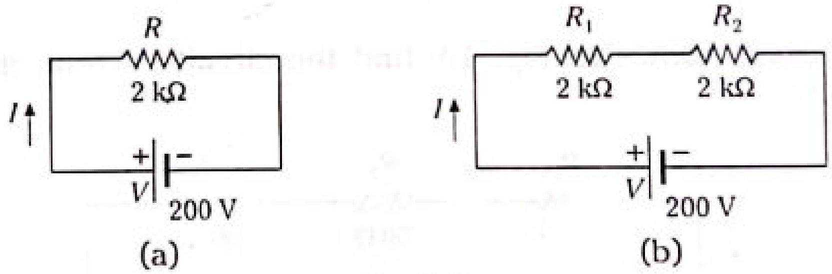


Fig. 1.6

Circuit (a) : $I = \frac{V}{R} = \frac{200}{2 \times 10^3} = 0.1 \text{ A} = \mathbf{100 \text{ mA}}$

Circuit (b) : The circuit resistance is

$$R_t = R_1 + R_2 = \left(2 \times 10^3\right) + \left(2 \times 10^3\right)$$

$$= 4000\ \Omega$$

$$\therefore\ I = \frac{V}{R_t} = \frac{200}{4000} = 0.05\,\text{A} = \mathbf{50\ mA}$$

Problem 1.2

Calculate the voltage across each of the resistors shown in Fig. 1.7 then calculate the supply voltage V.

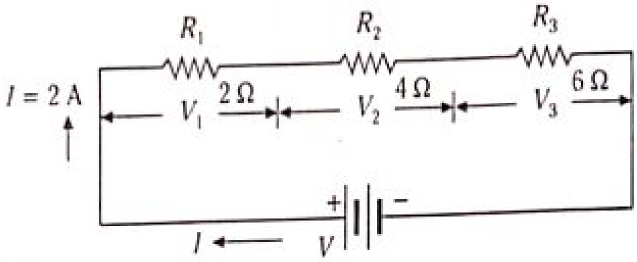


Fig. 1.7

$$V_1 = IR_1 = 2 \times 2 = 4.0\ \text{V}$$

$$V_2 = IR_2 = 2 \times 4 = 8.0\ \text{V}$$

$$V_3 = IR_3 = 2 \times 6 = 12.0\ \text{V}$$

$$V = V_1 + V_2 + V_3$$
$$= 4.0 + 8.0 + 12.0$$
$$= \mathbf{24\ V}$$

Problem 1.3

For the circuit shown in Fig. 1.8 find the circuit current, given that supply is 200 V.

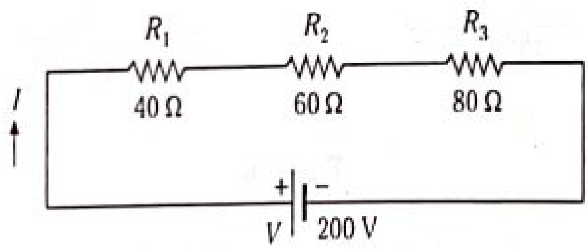


Fig. 1.8

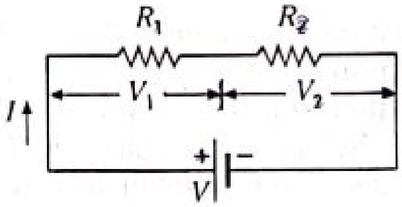
Total resistance

$$R = R_1 + R_2 + R_3$$
$$= 40 + 60 + 80 = 180\ \Omega$$

$$\therefore \quad \text{Circuit current } I = \frac{V}{R} = \frac{200}{180} = \mathbf{1.11\ A}$$

.4.1 Voltage Division between Two Resistors

Let us consider the circuit of Fig. 1.9 which deals with the *division of voltage etween only two resistors connected in series*. Given the supply voltage V, we eed to find the voltage drop across the resistor R_1.



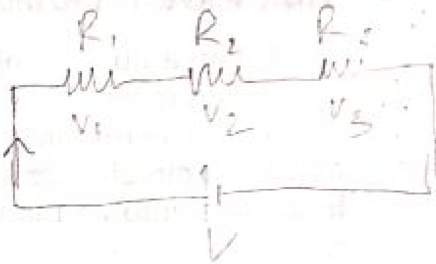


Fig. 1.9

The total resistance of the circuit is

$$R = R_1 + R_2$$

$R = R_1 + R_2 + R_3$

Hence, the current in the circuit is

$$I = \frac{V}{R_1 + R_2}$$

$I = \frac{V}{R_1+R_2+R_3}$

The voltage drop across the resistor R_1 is given by

$$IR_1 = \frac{V}{R_1 + R_2} \times R_1 = V_1$$

$IR_1 = \frac{V}{R_1+R_2+R_3} \times R_1 = V_1$

$$\therefore \quad \frac{\mathbf{V_1}}{\mathbf{V}} = \frac{\mathbf{R_1}}{\mathbf{R_1 + R_2}}$$

$\frac{V_1}{V} = \frac{R_1}{R_1+R_2+R_3}$

Thus, Fig. 1.9 is a typical example of a *voltage divider*, where the ratio of the voltages depends upon the ratio of the resistances in the case of a simple series circuit.

Problem 1.4

In the voltage divider of Fig. 1.10, given that V = 50 V, and the voltage across R_2 (100 Ω) is 20 V, calculate the value of R_1.

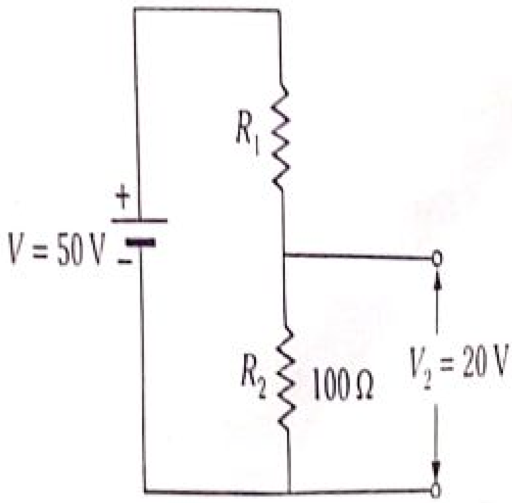


Fig. 1.10

$$\frac{V_2}{V} = \frac{R_2}{R_1 + R_2}$$

$$\frac{20}{50} = \frac{100}{R_1 + 100}$$

$$\therefore\ R_1 + 100 = 2.5 \times 100$$

$$= 250$$

$$\therefore\ R_1 = 150\ \Omega$$

1.5 Parallel Circuits

When a number of resistors are connected in such a way that one end of them is joined to a common point, and the other end of each of them is to another common point, then the resistors are said to be connected in and such circuits are known as parallel circuits (Fig. 1.11). In these circuits, is divided into as many paths as the number of resistances.

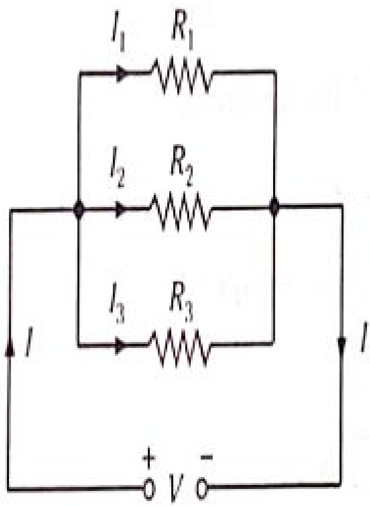


Fig. 1.11

As per Ohm's Law, $\frac{V}{I}$ is the total circuit resistance R, so that $\frac{I}{V} = \frac{1}{R}$

$$\therefore \quad \frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

Hence, when a number of resistors are connected in parallel, the reciprocal of the total resistance is equal to the sum of the reciprocals of the individual resistances.

1.5.1 Current Distribution in Parallel Circuits

Let two resistors R_1 and R_2 be connected in parallel (Fig. 1.12) across a potential difference of V volts. As per Ohm's Law, the current flowing through resistor R_1,

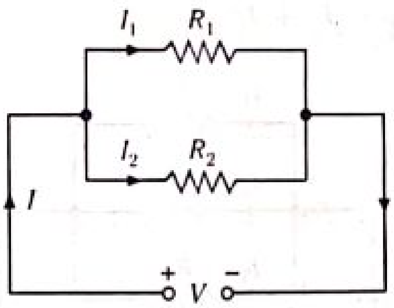


Fig. 1.12

$$I_1 = \frac{V}{R_1} \qquad \text{---(i)}$$

Current through resistor R_2.

$$I_2 = \frac{V}{R_2} \qquad \text{---(ii)}$$

Dividing expression (i) by expression (ii), we have

$$\frac{I_1}{I_2} = \frac{\frac{V}{R_1}}{\frac{V}{R_2}} = \frac{R_2}{R_1} \qquad \text{---(iii)}$$

The above expression shows that the *current through each resistor, when connected in parallel, is inversely proportional to their respective resistances.*

We could also express the branch currents in terms of the total circuit current in the following manner :

$$I = I_1 + I_2 \qquad \therefore \quad I_2 = I - I_1$$

We have seen from eqn.(iii) that $\frac{I_1}{I_2} = \frac{R_2}{R_1}$

$$\therefore \quad \frac{I_1}{I - I_1} = \frac{R_2}{R_1}$$

or $$I_1 R_1 = R_2 (I - I_1)$$

$$\therefore \quad I_1 = I \cdot \frac{R_2}{R_1 + R_2}$$

and $$I_2 = I \cdot \frac{R_1}{R_1 + R_2}$$

Let us now consider three resistors in parallel, connected across a v (Fig. 1.13).

The total current $I = I_1 + I_2 + I_3$

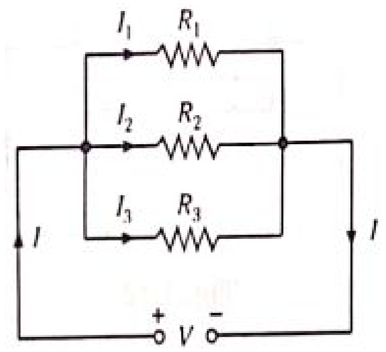


Fig. 1.13

Let the resistance of the parallel combination, or the equivalent resi be = R.

$$V = IR$$

Also $$V = I_1 R_1$$

$$\therefore \quad IR = I_1 R_1$$

or $$\frac{I}{I_1} = \frac{R_1}{R}$$

or $$I_1 = \frac{IR}{R_1}$$

Now, $$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

or $$R = \frac{R_1 R_2 R_3}{R_2 R_3 + R_3 R_1 + R_1 R_2}$$

From (iv) above, $$I_1 = I\left(\frac{R_2 R_3}{R_1 R_2 + R_2 R_3 + R_3 R_1}\right)$$

Similarly, $$I_2 = I\left(\frac{R_1 R_3}{R_1 R_2 + R_2 R_3 + R_3 R_1}\right)$$

$$I_3 = I\left(\frac{R_1 R_2}{R_1 R_2 + R_2 R_3 + R_3 R_1}\right)$$

Problem 1.5

Find the supply current to the current to the circuit shown in Fig 1.14.

The supply current I and the branch currents are shown in Fig.1.14.

$$I_1 = \frac{V}{R_1} = \frac{100}{20} = 5\text{ A}$$

$$I_2 = \frac{V}{R_2} = \frac{100}{40} = 2.5\text{ A}$$

$$\therefore \quad I = I_1 + I_2 = 5 + 2.5 = \mathbf{7.5\ A}$$

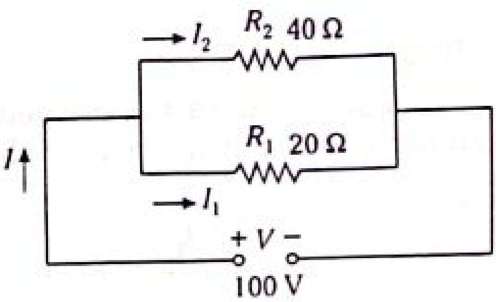


Fig. 1.14

Problem 1.6

Find the effective resistance and the supply current to the circuit of Fig. 1.15.

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

$$= \frac{1}{8} + \frac{1}{6} + \frac{1}{4}$$

$$= 0.125 + 0.166 + 0.250$$

$$= 0.541$$

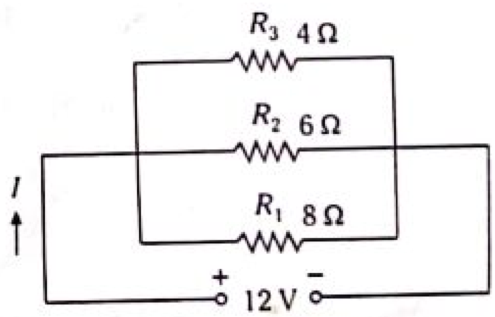


Fig. 1.15

$$\therefore \text{ Effective resistance } R = \frac{1}{0.541}$$

$$= \mathbf{1.85\ \Omega}$$

$$I = \frac{V}{R} = \frac{12}{1.85} = \mathbf{6.49\ A}$$

Problem 1.7

A current of 10 A is divided between two resistors in the circuit of Fig. 1.16. Find the current in each of the two resistors.

$$I_1 = I.\frac{R_2}{R_1 + R_2}$$

$$= 10 \times \frac{2}{2+2}$$

$$= \mathbf{5\ A}$$

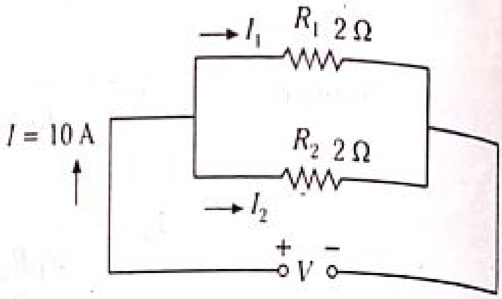


Fig. 1.16

$$\therefore\ I_2 = I - I_1 = 10 - 5 = \mathbf{5\ A}$$

Thus, we see that equal resistances share the supply current equally.

Problem 1.8

A current of 10 A is divided between two resistors in the circuit of Fig. 1.17. Find the current in each of the two resistors.

$$I_1 = I.\frac{R_2}{R_1 + R_2}$$

$$= 10 \times \frac{4}{2+4}$$

$$= \mathbf{6.66\ A}$$

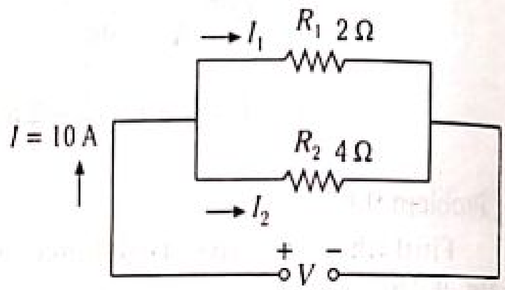


Fig. 1.17

$$I_2 = I - I_1 = 10 - 6.66 = \mathbf{3.34}$$

Alternately, $I_2 = I.\dfrac{R_1}{R_1 + R_2}$

$$= 10 \times \frac{2}{2+4} = \mathbf{3.34}$$

Thus it is seen that the lesser the resistance the greater its share of the supply current. The converse too is applicable.

1.6 Series-Parallel Circuit

It may be often necessary to use combinations of series and parallel arrangement of resistors. Such circuits can be solved by combining first the parallel groups into an equivalent resistance and then adding this equivalent resistance to other series resistances to determine the total resistance. The current produced by the voltage applied can then be obtained as usual. The following problems illustrate this.

Problem 1.9

A coil of 12 Ohms resistance is in parallel with a coil of 20 Ohms resistance. This combination is connected in series with a third coil of 8 Ohms resistance, The whole circuit is connected across a battery having an e.m.f. of 30 V and internal resistance of 2 Ohms. Calculate (a) the terminal voltage of the battery and (b) the power in the 12 Ohm coil.

Solution :

The circuit is shown in Fig. 1.18.

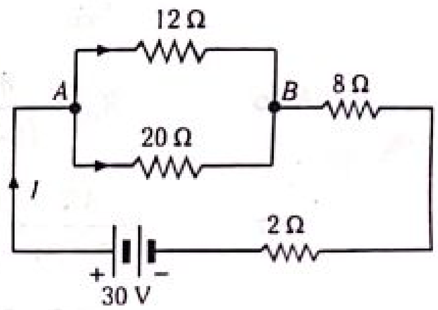


Fig. 1.18

i) The resistance of the parallel combination between A and B, R_{AB}, is given by

$$\frac{1}{R_{AB}}=\frac{1}{12}+\frac{1}{20}=\frac{8}{60}$$

$$\therefore \quad R_{AB}=\frac{60}{8}=7.5 \text{ ohms}$$

ii) The resistance of 7.5 ohms is in series with the 8 ohms and 2 ohms resistances

$\therefore$ Total circuit Resistance, R_t = 7.5 + 8 + 2 + = 17.5 ohms.

$\therefore$ Total current drawn from the battery,

$$I=\frac{V}{R_t}=\frac{30}{17.5}=1.714 \text{ A}$$

The internal resistance of the battery = 2 ohms

$\therefore$ Internal voltage drop in battery = 2 × 1.714 = 3.428 V

$\therefore$ Terminal voltage of the battery = 30 − 3.428 = **26.572 V**

Voltage across $AB = I \times R_{AB}$ = 1.714 × 7.5 = 12. 855 V

$\therefore$ Power in the 12 ohm coil $=\frac{12.855^2}{12}=$ **13.77 Watts**

(Because Power = VI or I^2R or V^2/R).

Problem 1.10

If the total power dissipated in the circuit shown is 18 Watts, find the value of R and its current.

(April, 1997, B.U.)

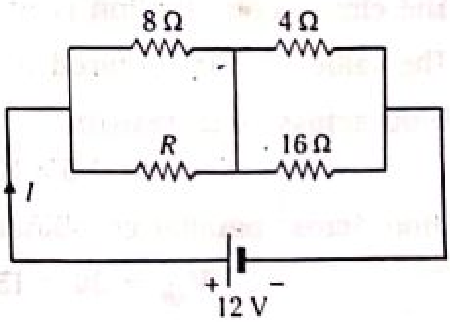


Fig. 1.19

Solution :

Total power dissipated, P = 18 Watts

$$P = \frac{V^2}{R_{eq}}$$

where R_{eq} Equivalent resistance of the circuit

$$\therefore \quad 18 = \frac{12^2}{R_{eq}} \quad \text{or } R_{eq} = 8\,\Omega$$

Also $$R_{eq} = \frac{8R}{(8+R)} + \frac{(4\times16)}{(4+16)}$$

or $$8 = \frac{8R}{(8+R)} + \frac{64}{20}$$

$$= \frac{160R + 64(8+R)}{20(8+R)}$$

$$160\,(8+R) = 160\,R + 512 + 64\,R$$

$$1280 + 160R = 160R + 512 + 64R$$

$$768 = 64\,R$$

$$\therefore \quad R = 12\,\Omega$$

Total current $$I = \frac{V}{R_{eq}} = \frac{12}{8} = 1.5\text{ A}$$

Problem 1.11

A resistance of 10 Ω is connected in series with two resistances eacl 15 Ω arranged in parallel. What resistance must be shunted across this para combination so that the total current taken shall be 1.5 A with 20 V applie

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Solution :

The circuit combination is as shown in Fig. 1.20.

The value of R is required to be found.

Drop across 10 Ω resistor

$$= 1.5 \times 10 = 15\text{ V}$$

Drop across parallel combination,

$$V_{AB} = 20 - 15 = 5\text{ V}$$

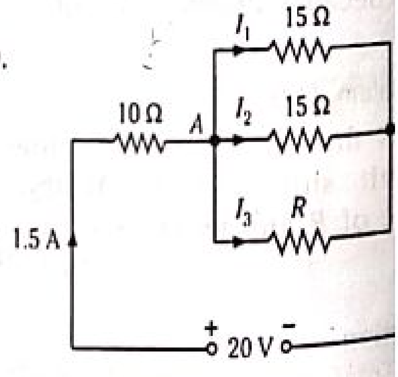

Fig. 1.20

So, voltage across each parallel resistance is 5 V.

$$I_1 = \frac{5}{15} = \frac{1}{3}\text{A}; \quad I_2 = \frac{5}{15} = \frac{1}{3}\text{ A}$$

$$I_3 = 1.5 - \left(\frac{1}{3} + \frac{1}{3}\right) = \frac{5}{6}\text{ A}$$

$\therefore \quad I_3 R = 5$

or $\quad \left(\frac{5}{6}\right) R = 5$

or $\quad \boldsymbol{R = 6\ \Omega}$

Problem 1.12

Determine the value of R if the power dissipated in 10 ohm resistor is 40 W for the circuit Fig. 1.21. (Jan 93, B.U.)

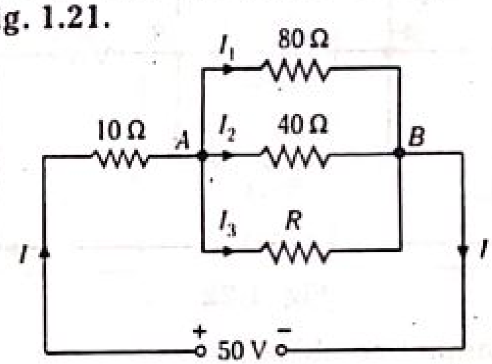

Fig. 1.21

Solution :

Power dissipated in 10 Ω resistor is 40 W

$\therefore$ The circuit current flowing through this resistor is given by the expression

$$I^2 = \frac{P}{R} = \frac{40}{10} = 4$$

$\therefore \quad I = 2$ Amps

$\therefore$ Voltage drop across 10 Ω resistor

$= 10 \times 2 = 20\ V$

The voltage across the entire circuit (given)

$= 50\ V$

Voltage across the parallel combination

$$V_{AB} = 50 - 20 = 30\ V$$

$\therefore$ Current through 80 Ω resistor, $I_1 = \frac{30}{80} = 0.375\ A$

and Current through 40 W resistor, $I_2 = \frac{30}{40} = 0.750\ A$

$\therefore$ Current through resistor R,

$$I_3 = I - (I_1 + I_2)$$

$$= 2 - (0.375 + 0.750) = 0.875\ A$$

$$\therefore \quad R = \frac{V_{AB}}{I_3} = \frac{30}{0.875} = \mathbf{34.3\ \Omega}$$

Problem 1.13

In the circuit shown, find the value of the resistance R, when the consumed by the 12 Ω resistor is 36 W. (June 89,

Solution :

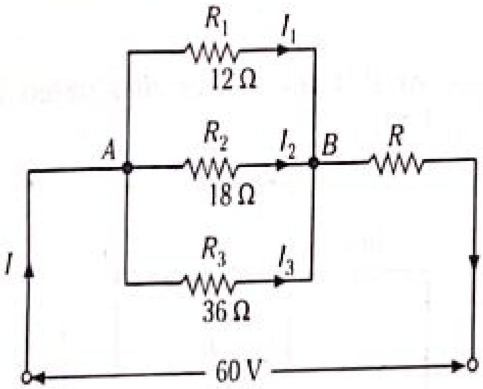

Fig. 1.22

Equivalent resistance of parallel combination,

$$R_{AB} = \frac{12 \times 18 \times 36}{(12 \times 18) + (18 \times 36) + (36 \times 12)} = \frac{7776}{1296} = 6\Omega$$

Power consumed by 12 Ω resistor, P_1 = 36 watts

$$\frac{V_{AB}^2}{R_1} = 36 \quad \text{or} \quad \frac{V_{AB}^2}{12} = 36 \quad \text{or} \quad V_{AB} = 20.78\ \text{V}$$

$$I_1 = \frac{V_{AB}}{R_1} = \frac{20.78}{12} = 1.73\ \text{A}$$

$$I_2 = \frac{V_{AB}}{R_2} = \frac{20.78}{18} = 1.15\ \text{A}$$

$$I_3 = \frac{V_{AB}}{R_3} = \frac{20.78}{36} = 0.57\ \text{A}$$

$$\therefore \quad I = I_1 + I_2 + I_3 = 3.45\ \text{A}$$

Now, $I = \dfrac{V}{6+R}$

or $3.45 = \dfrac{60}{6+R}$

$3.45\,(6+R) = 60$

$20.7 + 3.45\,R = 60$

or $3.45\,R = 39.3$

or $R = \mathbf{11.4\ \Omega}$

Problem 1.14

In the circuit shown in Fig. 1.23, determine
(i) the current supplied by the 100 V source
(ii) the voltage across the 6 Ω resistor, (May 90, KUD)

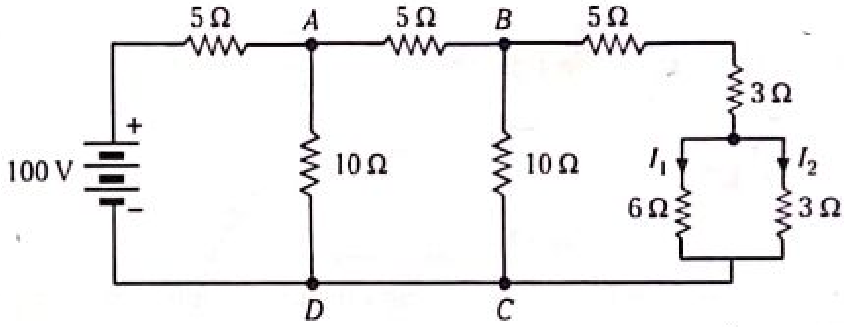

Fig. 1.23

Solution :

a) As 6 Ω and 3 Ω are in parallel, their equivalent resistance is $\dfrac{(6\times 3)}{6+3} = 2\Omega$

b) This is in series with the 3 Ω and 5 Ω resistors,
∴ Total resistance, R_t = 2+3+5 = 10 Ω

c) R_t comes in parallel with the 10 Ω resistor between B and C
∴ Their equivalent resistance, $R_e = \dfrac{10\times 10}{10+10} = 5\Omega$

d) R_e comes in series with the 5 Ω resistor between A and B,
∴ Their combined resistance, R_c = 5+5 = 10 Ω

e) R_c comes in parallel with the 10Ω resistor between A and D,
∴ Their equivalent resistance, $R_{eq} = \dfrac{10\times 10}{10+10} = 5\Omega$

f) R_{eq} is in series with the other 5Ω resistor

$\therefore$ Total Circuit Resistance $R_{tcr} = 5 + 5 = 10\Omega$

$\therefore$ Current supplied by battery $= \dfrac{100}{R_{tcr}} = \dfrac{100}{10} = 10\,A$

At A, the current divides equally between AB and AD.

$\therefore$ Current in $AB = 5$ A

Again, at B, the current is equally divided.

$\therefore$ Current in $BE = \dfrac{5}{2} = 2.5\,A$

Current again divides at E. The current of 2.5 A divides in the ratio 1 : 2 in the $6\,\Omega$ and $3\,\Omega$ branch resistors.

$\therefore$ Current in the $6\,\Omega$ resistor $= \dfrac{2.5 \times 1}{3} = \dfrac{2.5}{3}\,A$

$\therefore$ Voltage drop across the $6\,\Omega$ resistor $= \dfrac{2.5}{3} \times 6 = 5V$

1.7 Kirchoff's Laws

1.7.1 Definitions, illustration

Kirchoff's Laws are very useful in solving circuits which cannot be easily by using Ohm's Law.

Kirchoff's First Law (Current Law)

In any network of wires carrying currents, the algebraic sum of all currents meeting at a point is zero. Or, the sum of incoming currents towards any point is equal to the sum of outgoing currents away from that point.

An algebraic sum is one in which the sign of the quantity is taken into consideration. For example 4 wires meet at a point O, carrying currents I_1, I_2, I_3 and I_4 as shown in Fig. 1.24.

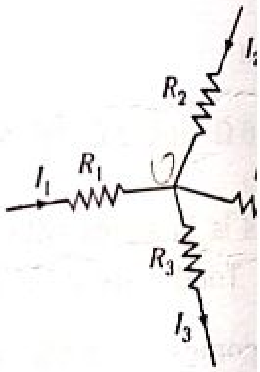

Fig. 1.24

If we take the sign of currents flowing towards point O as positive, th sign of currents flowing away from point O will be treated as negative.

Applying Kirchoff's First Law to point O in Fig. 1.24, the algebraic s currents at that point, will be zero.

$$i.e., \quad (I_1) + (I_2) + (-I_3) + (-I_4) = 0$$

$$\text{or} \quad I_1 + I_2 = I_3 + I_4$$

Kirchoff's Second Law (Voltage Law)

In any closed circuit or mesh, the algebraic sum of products of currents and resistances (voltage drops) plus the algebraic sum of all the e.m.f.s in that circuit is zero, i.e.,

Algebraic sum of e.m.f's + Algebraic sum of voltage drops = 0

Kirchoff's Second Law is valid because, when we start from any point in a closed circuit and travel round the circuit and return to the same point, there is no increase or decrease in potential.

This means that all the sources of e.m.f's coming along the path *plus* the voltage drops in the resistances must together amount to zero.

Signs of e.m.f's and voltage drops : While applying Kirchoff's Second Law to a closed circuit or mesh, algebraic sums are considered, and so e.m.f's and voltage drops must be assigned proper signs.

a) **Signs of e.m.f's :** *A rise in potential* should be taken as *positive* and a *fall in potential* should be taken as *negative*. So, if we start from the negative terminal of a battery or source towards the positive terminal, there is a rise in potential and it must be considered positive. On the other hand, if we go from the positive terminal of a battery or voltage source to the negative terminal, there is a fall in potential which should be taken as negative. It should be borne in mind that the sign of e.m.f. is independent of the direction of current through that branch.

b) **Signs of voltage drops :** When current passes through a resistance there is a voltage drop in it. If we go with the current, the voltage drop should be considered to be *negative* as the current flows from higher potential to lower potential (*fall in potential*). On the other hand, if we go against the current flow, the voltage drop should be considered to be *positive* as it is a *rise in potential*. We should observe that the sign of voltage drop depends upon the direction of current and is independent of the polarity of e.m.f. in the concerned circuit.

Illustration

We shall now illustrate Kirchoff's Second Law, with the help of the circuit given as Fig. 1.25. We shall mark the directions of currents as shown in the circuit. If, on calculation, we find that the value of current has a positive sign, then the direction we have assumed is correct.

If, on the other hand, it has a negative value, then the actual direction is opposite to the assumed one.

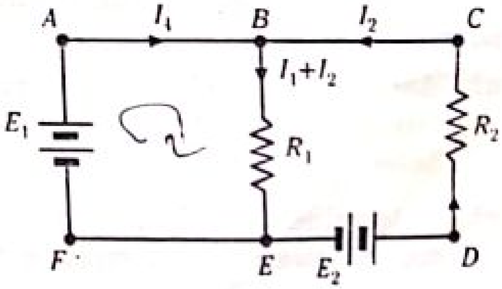

Fig. 1.25

f) R_{eq} is in series with the other 5Ω resistor

∴ Total Circuit Resistance $R_{tcr} = 5 + 5 = 10\Omega$

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Again, at B, the current is equally divided.

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Current again divides at E. The current of 2.5 A divides in the ratio 1 : 2 in the $6\,\Omega$ and $3\,\Omega$ branch resistors.

∴ Current in the $6\,\Omega$ resistor $= \dfrac{2.5\times1}{3} = \dfrac{2.5}{3}\,A$

∴ Voltage drop across the $6\,\Omega$ resistor $= \dfrac{2.5}{3}\times6 = 5V$

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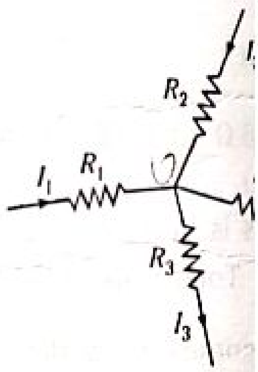

Fig. 1.24

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Kirchoff's Second Law is valid because, when we start from any point in a closed circuit and travel round the circuit and return to the same point, there is no increase or decrease in potential.

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Illustration

We shall now illustrate Kirchoff's Second Law, with the help of the circuit given as Fig. 1.25. We shall mark the directions of currents as shown in the circuit. If, on calculation, we find that the value of current has a positive sign, then the direction we have assumed is correct.

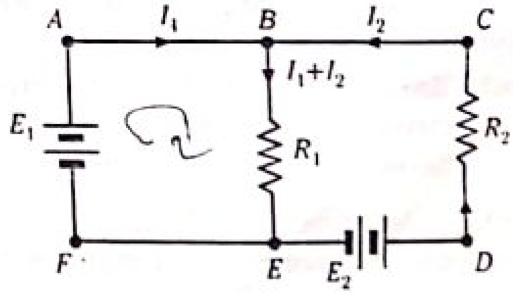

Fig. 1.25

If, on the other hand, it has a negative value, then the actual direction is opposite to the assumed one.

We shall now consider two loops - *FABEF* and *BCDEB*, and deal with them as follows :

a) Loop *FABEF*

The voltage drop in R_1 is $(I_1 + I_2)\,R_1$ and is negative, *i.e.*,

Voltage drop in $R_1 = -(I_1 + I_2)\,R_1$

This is so because, as the voltage drop is along *FABEF*, we proceed with the current in the branch *BE*, and so there will be a fall in potential. Thus, the voltage drop in R_1 is negative.

The e.m.f. E_1 is positive; this is so because, as we consider the loop *FABEF*, in that order, we go from the *–ve* terminal to the *+ve* terminal of the battery in the branch *FA* and so there is a rise in potential. Therefore e.m.f. E_1 is *+ve*.

Applying Kirchoff's Second Law to this closed loop,

$$-(I_1+I_2)\,R_1 + E_1 = 0$$

$$E_1 = (I_1+I_2)R_1$$

b) Loop *BCDEB*

The voltage drop in R_2 is $I_2 R_2$ and is positive, so

Voltage drop in $R_2 = I_2 R_2$

As we are considering the loop in the order *BCDEB*, we are proceeding against the current in the branch *CD*, and so there is a rise in potential. In a similar manner, the voltage drop in R_1 is positive.

As we are considering loop *BCDEB*, in that order, we go from the positive terminal of the battery to the negative terminal in the branch *DE*, and so there is a fall in potential.

By applying Kirchoff's Second Law to this closed loop,

$$I_2 R_2 + (I_1+I_2)R_1 + (-E_2) = 0$$

or $$E_2 = I_2 R_2 + (I_1+I_2)R_1$$

To summarise, Kirchoff's Laws can be utilised to solve circuits in accordance with the following rules :

a) **Mark the direction of currents in the different arms (branches) of the circuit, as per the First Law.**

b) **Select any closed circuit/circuits.**

c) **Determine the algebraic sum of the e.m.f.'s and the voltage drops in each closed circuit, and equate them to zero.**

1.7.2 Application of Kirchoff's Laws

Kirchoff's Laws are applied to two-loop circuit analysis by Branch Current Method and Loop Current (or Mesh Current) Method, explained in the following sections.

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.8 The Branch Current Method

Take a two-loop circuit of Fig 1.26 and go through the following steps :

Mark all the simple nodes, *a, b, f* & *e* and principal nodes *c* and *d*.

Choose the current direction arbitrarily, and apply *Kirchoff's Current Law* at each principal node, keeping in mind that the number of branch currents chosen is kept minimum.

Choose any closed path, *i.e.*, the loop covering all the elements of the circuit, and apply *Kirchoff's Voltage Law* and form equations equal to the number of unknowns.

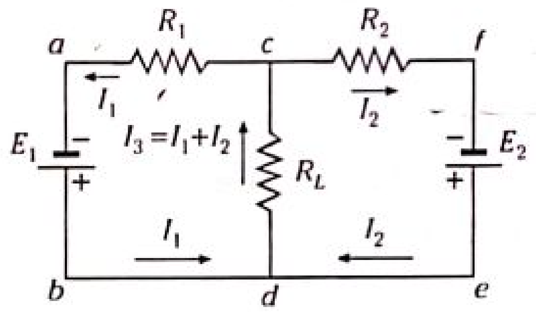

Fig. 1.26

Solve the simultaneous equations. In a case like this where only two loops are involved, it will suffice to use the usual method of solving them. However, if three or more loops are to be considered it will be more convenient to apply Cramer's Rule.

If, on calculation, we find that the value of current has a positive sign, then the direction we have assumed is correct. If, on the other hand, it has a negative sign, then the actual direction is opposite to the assumed one.

The following solved problems will illustrate the above.

Problem 1.15 1

In the circuit given below (Fig. 1.27), find the value of D.C. current through
L **by using Kirchoff's Laws.** **(Mar 89, M.U.)**

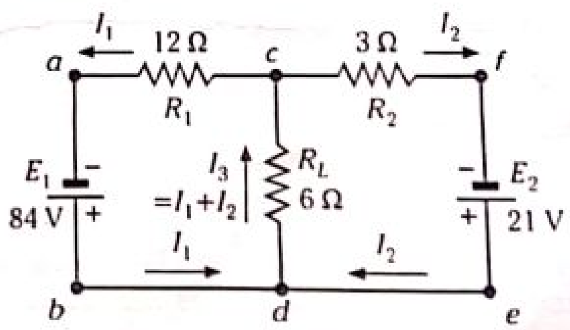

Fig. 1.27

Solution :

(a) Loop *bdcab*

Applying Kirchoffs Second Law to this closed loop,

$$I_1 R_1 + (I_1 + I_2) R_L - E_1 = 0$$

or $$12 I_1 + 6(I_1 + I_2) - 84 = 0$$

or $$18 I_1 + 6 I_2 - 84 = 0 \quad \text{—(i)}$$

(b) Loop *edcfe*

$$-(I_1 + I_2) R_L - I_2 R_2 + E_2 = 0$$

$$-6(I_1 + I_2) - 3 I_2 + 21 = 0$$

$$-6 I_1 - 9 I_2 + 21 = 0 \quad \text{—(ii)}$$

Multiplying the above (ii) by 3, we have

$$-18 I_1 - 27 I_2 + 63 = 0$$

$$18 I_1 + 6 I_2 - 84 = 0 \quad \text{—(i) reproduced}$$

$$-21 I_2 - 21 = 0$$

or $$I_2 = -1$$

The negative sign for the current I_2 means that it is flowing in the direction opposite to that assumed, *i.e.*, it actually flows from d to e.

Substituting $I_2 = -1$ in eqn.(ii), we have

$$-6 I_1 - 9(-1) + 21 = 0$$

or $$-6 I_1 = -30$$

or $$I_1 = 5 \text{ Amps}$$

The current flowing through R_L

$$I_3 = I_1 + I_2$$

$$= 5 + (-1)$$

$$= 4 \text{ Amps}$$

I_3 flows in the direction assumed, *i.e.*, from d to c.

Problem 1.16

Two 12 V batteries with internal resistances 0.2 ohm and 0.25 ohm respectively are joined in parallel and a resistance of 1 Ohm is placed across the terminals. Find the current supplied by each battery.

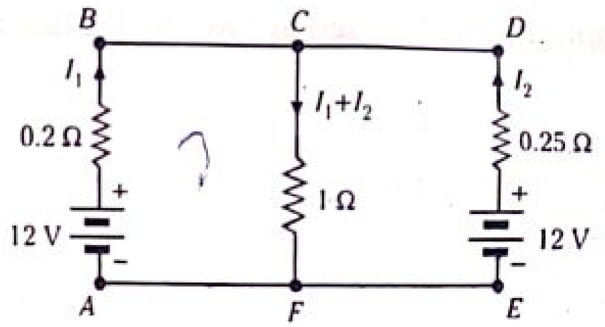

Fig. 1.28

olution :

The circuit is shown in Fig. 1.28.

Let I_1 and I_2 be the currents supplied by the batteries.

Current through the 1 ohm resistor = $I_1 + I_2$

As there are two unknown quantities, we shall require two loop equations.

Applying Kirchoff's Voltage Law to loop *BCFAB*,

$$-1(I_1+I_2)+12-0.2I_1=0$$

or $\quad 1.2I_1 + I_2 = 12$ —(i)

Similarly, for loop *CDEFC*,

$$+0.25I_2-12+1(I_1+I_2)=0$$

or $\quad I_1+1.25I_2 = 12$ —(ii)

Eqn.(i) is rewritten : $1.2I_1 + I_2 = 12$

Eqn.(ii) × 1.2 : $1.2I_1 + 1.5I_2 = 14.4$

$$-0.5I_2 = -2.4$$

$\therefore \quad I_2 = \mathbf{4.8\ Amps}$

Substituting the value of I_2 in Eqn.(ii),

$$I_1 = 12 - (1.25 \times 4.8)$$
$$= 12 - 6 = \mathbf{6\ Amps}$$

Problem 1.17

In the circuit shown find E_1, E_2 and I when the power dissipated in the 5 Ω resistor is 125 W.

(June/July 90, B.U.)

Solution :

Applying Kirchoff's First Law, the current through the 5 Ω resistor is = I + 2.5.

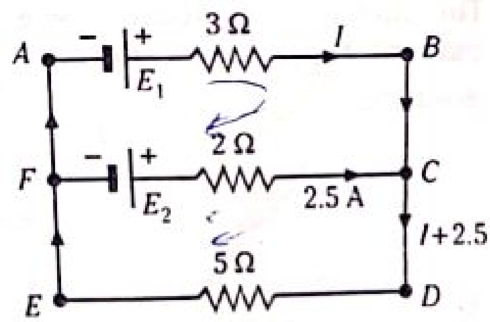

Fig. 1.29

Next, we shall apply Kirchoff's Second Law to the loops *ABCFA* and *FC*
(Fig. 1.29).

Loop *ABCFA*

$$E_1 - 3I + (2 \times 2.5) - E_2 = 0$$

or $$E_1 - 3I + 5 - E_2 = 0$$

or $$E_1 - E_2 = 3I - 5 \qquad \text{—(i)}$$

Loop *FCDEF*

$$E_2 - (2 \times 2.5) - 5(I + 2.5) = 0$$

or $$E_2 - 5 - 5I - 12.5 = 0$$

or $$E_2 - 5I = 17.5 \qquad \text{—(ii)}$$

Given that power dissipated in $5\,\Omega$ resistor,

$P = 125$ W

Current flowing through $5\,\Omega$ resistor $= I + 2.5$

$$\therefore \quad (I + 2.5)^2 \times 5 = 125$$

or $$(I + 2.5)^2 = 25 \quad \text{or} \quad I + 2.5 = 5$$

$$\therefore \quad I = \mathbf{2.5\ A}$$

Substituting this value of I in eqn.(ii),

$$E_2 - (5 \times 2.5) = 17.5$$

or $$E_2 = \mathbf{30\ V}$$

Substituting the values of I and E_2 in eqn.(i),

$$E_1 - 30 = (3 \times 2.5) - 5$$

$$E_1 = \mathbf{32.5\ V}$$

Problem 1.18

Two storage batteries *A* and *B* are connected in parallel to supply a l of 0.30 ohm. The open circuit e.m.f. of *A* is 11.7 V and that of *B* is 12. The internal resistances are 0.06 ohm and 0.05 ohm respectively. Find current supplied to the load.

Solution :

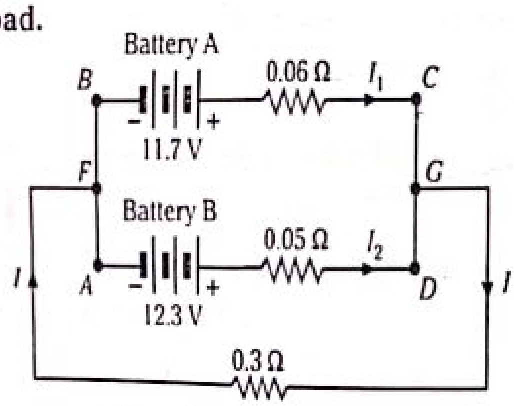

Fig. 1.30

Let the currents supplied by the batteries be I_1 and I_2 respectively.

Current through load, $I = I_1 + I_2$

Applying Kirchoff's Second Law to loops $ABCDA$ and $FBCGF$ we have

$$-0.06I_1 + 0.05I_2 + 11.7 - 12.3 = 0$$

or $$0.06I_1 - 0.05I_2 = 11.7 - 12.3$$

or $$0.06I_1 - 0.05I_2 = -0.6 \qquad \text{—(i)}$$

$$-0.06I_1 - 0.3(I_1 + I_2) + 11.7 = 0$$

or $$0.36I_1 + 0.3I_2 = 11.7 \qquad \text{—(ii)}$$

Multiplying expression (i) by 6, we get

$$0.36I_1 - 0.3I_2 = -3.6 \qquad \text{—(iii)}$$

Subtracting expression (iii) from expression (ii), we get

$$0.6I_2 = 15.3 \quad \text{or} \quad I_2 = 25.5 \text{ Amps}$$

Substituting the value of I_2 in eqn.(i), we get

$$0.06I_1 - (0.05 \times 25.5) = -0.6$$

or $$0.06I_1 = 1.275 - 0.6 = 0.675$$

$$\therefore \quad I_1 = 11.25 \text{ Amps}$$

$\therefore$ Current through the load of $0.3\,\Omega$

$$I = I_1 + I_2 = 11.25 + 25.50 = \mathbf{36.75\ Amps}$$

Problem 1.19

Find (i) Current in 15 Ω resistor (ii) Voltage across 18 Ω resistor and (iii) Power dissipated in 7 Ω resistor. **(Jun/July 90, B.U.)**

Solution :

Referring to Fig. 1.31 the currents are assumed to be in the directions indicated, keeping in mind Kirchoff's First Law.

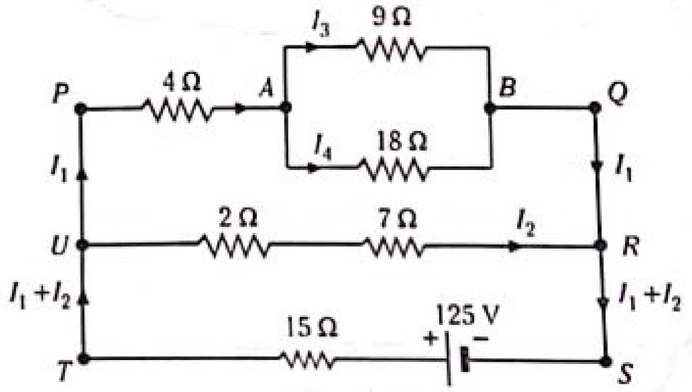

Fig. 1.31

The equivalent resistance of the parallel combination, $R_{AB} = \frac{9\times18}{9+18} = 16\Omega$

We shall now apply Kirchoff's Second Law to the meshes *PQRUP* and *UR*

Mesh *PQRUP*

$$-4I_1 - R_{AB} \cdot I_1 + 7I_2 + 2I_2 = 0$$

$$-4I_1 - 6I_1 + 7I_2 + 2I_2 = 0$$

or $$-10I_1 + 9I_2 = 0 \qquad \text{—(i)}$$

Mesh *URSTU*

$$-2I_2 - 7I_2 - 15(I_1 + I_2) + 125 = 0$$

or $$15I_1 + 24I_2 = 125 \qquad \text{—(ii)}$$

Multiplying (i) by 3 and (ii) by 2, and add :

$$-30I_1 + 27I_2 = 0$$

$$+30I_1 + 48I_2 = 250$$

$$75\, I_2 = 250$$

$$\therefore \qquad I_2 = \frac{10}{3}\text{A}$$

Substituting this value of I_2 in eqn.(i),

$$-10I_1 + 9\left(\frac{10}{3}\right) = 0$$

$$-10I_1 + 30 = 0$$

or $$I_1 = 3\text{A}$$

(i) Current in 15Ω resistor, $I = I_1 + I_2 = 3 + 3.33 =$ **6.33 A**

Current in 9Ω resistor of parallel combination,

$$I_3 = I_1\left(\frac{18}{9+18}\right) = 3\left(\frac{18}{27}\right) = 2\text{A}$$

$$I_4 = I_1 - I_3 = 3 - 2 = 1\text{ A}$$

(ii) Voltage across 18Ω resistor,

$V_{AB} = 18 \times I_4 = 18 \times 1 =$ **18 volts**

(iii) Power dissipated in 7Ω resistor,

$P = I_2^2 \times 7 = (3.33)^2 \times 7 =$ **77.62 watts**

Problem 1.20

Determine the magnitude and direction of the current in the 2 V battery in the circuit shown in Fig. 1.32. **(Nov/Dec 84, B.U.)**

Solution :

The direction of the currents have been taken as shown in Fig. 1.32 in accordance with Kirchoff's First Law.

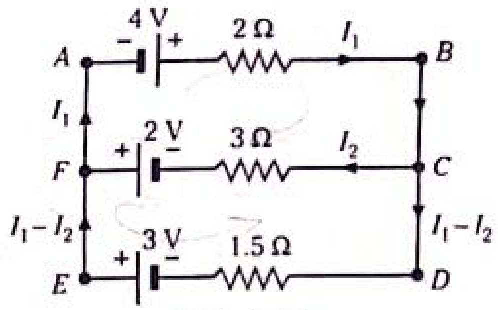

Fig. 1.32

We will solve the circuit, using Kirchoff's Second Law, for the loops *ABCFA* and *FCDEF*, as given below :

Loop *ABCFA*

$$+4 - 2I_1 - 3I_2 + 2 = 0$$

or $\quad 2I_1 + 3I_2 = 6 \qquad$ —(i)

Loop *FCDEF*

$$-2 + 3I_2 - 1.5(I_1 - I_2) + 3 = 0$$

or $\quad 1.5I_1 - 4.5I_2 = 1 \qquad$ —(ii)

Multiplying eqn.(i) by 3, we have : $\quad 6I_1 + 9I_2 = 18$

Multiplying eqn.(ii) by 4, we have : $\quad 6I_1 - 18I_2 = 4$

$$27I_2 = 14 \quad \text{or} \quad I_2 = 0.52\text{ A}$$

Thus, current in the 2 V battery is I_2 = **0.52 A** and its direction is as shown in the figure.

Problem 1.21

For the network shown below, determine all the branch currents. **(Aug 94, B.U.)**

Solution :

Let the current through branches *CB* and *BO* be I_1 and I_2 respectively.

Applying Kirchoff's First Law to junctions *B*, *A*, *O* and *C*, we have

Current in $BA = I_1 - I_2$

Let current in AO be I_3

∴ Current in $AC = I_1 - I_2 - I_3$

Current in $OC = I_2 + I_3$

Next, applying Kirchoffs Second Law to loops *BAO*, *CAO* and *OCB*, we obtain the following equations :

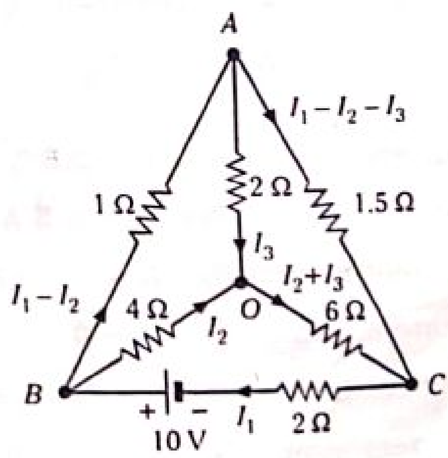

Fig. 1.33

$$-(I_1-I_2)-2I_3+4I_2=0$$

or $\quad -I_1+5I_2-2I_3=0 \qquad \text{—(i)}$

$$-1.5(I_1-I_2-I_3)+6(I_2+I_3)+2I_3=0$$

or $\quad -1.5I_1+7.5I_2+9.5I_3=0 \qquad \text{—(ii)}$

Also $\quad -4I_2-6(I_2+I_3)-2I_1+10=0$

or $\quad 2I_1+10I_2+6I_3=10 \qquad \text{—(iii)}$

Multiplying eqn.(i) by (2), we have

$$-2I_1+10I_2-4I_3=0$$

$$2I_1+10I_2+6I_3=10$$

Add, $\quad 20I_2+2I_3=10$

or $\quad 10I_2+I_3=5 \qquad \text{--(iv)}$

2 × eqn.(ii) gives $\quad -3I_1+15I_2+19I_3=0$

1.5 × eqn.(iii) gives $\quad 3I_1+15I_2+9I_3=15$

$$30I_2+28I_3=15$$

(3) × eqn.(iv) gives $\quad 30I_2+3I_3=15$

Subtracting, $\quad 25I_3=0 \quad$ or $\quad I_3=0$

Now, eqn.(iv) : $10I_2+I_3=5$

$\therefore \quad 10I_2+0=5$

or $\quad I_2=0.5\ A$

Eqn.(i) is $\quad -I_1+5I_2-2I_3=0$

or $\quad -I_1+5(0.5)-0=0$

or $\quad I_1=2.5\ A$

Thus

Current in arm CB, $I_1 = \mathbf{2.5\ A}$

Current in arm BO, $I_2 = \mathbf{0.5\ A}$

Current in arm BA, $I_1 - I_2 = \mathbf{2\ A}$

Current in arm AO, $I_3 = \mathbf{0}$

Current in arm AC, $I_1 - I_2 - I_3 = \mathbf{2A}$

Current in arm OC, $I_2 + I_3 = \mathbf{0.5A}$

Problem 1.22

Find the value of R and the current flowing through it in the circuit given below, when the current in the branch OA is Zero.

(Mar 94, B.U.)

Solution :

Let the current through the branches BA and BO be I_1 and I_2 respectively.

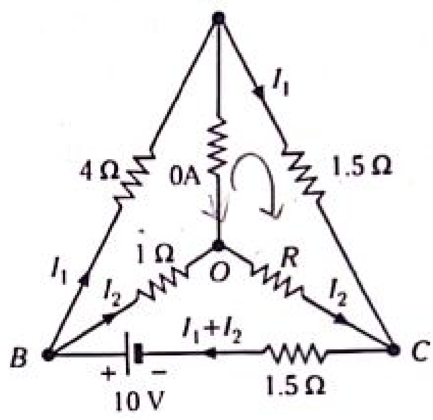

Fig. 1.34

Given that current in branch AO = 0.

Applying Kirchoff's First Law to junctions B, A and O, we obtain :

Current supplied by battery or current in branch CB is $= I_1 + I_2$

Current through branch AC = Current through branch BA = I_1, as branch AO carries zero current.

Next, we apply Kirchoff's Second Law to the loops BAO, ACO and OCB and obtain the following equations :

$$-4I_1 + 0 + 1.0\, I_2 = 0$$

or $\quad I_2 = 4I_1 \quad$ ---(i)

Also, $\quad -1.5I_1 + RI_2 = 0 \quad$ ---(ii)

Substituting the value of I_2 in eqn.(ii) above

$$-1.5I_1 + R(4I_1) = 0$$

or $\quad 4RI_1 = 1.5I_1$

or $\quad R = \mathbf{0.375\ \Omega}$

Also, $\quad -1.5\,(I_1 + I_2) + 10 - 1.0\, I_2 - RI_2 = 0$

Now $\quad R = 0.375\,\Omega \quad$ and $\quad I_2 = 4I_1$

$\therefore \quad -1.5\,(I_1 + 4I_1) + 10 - 1.0\,(4I_1) - 0.375\,(4I_1) = 0$

or $\quad -1.5(5I_1) + 10 - 4I_1 - 1.5I_1 = 0$

$$-7.5I_1 + 10 - 5.5I_1 = 0$$

or $\quad 13I_1 = 10$

$$I_1 = 0.77 \text{ A}$$

$\therefore \quad I_2 = 4I_1 = \mathbf{3.1A}$

1.9 Loop Current (Mesh Current) Method

When a continuous flow of current is assigned to each window or mesh in the network to complete its path in a closed loop, without the application of Kirchoff's Current Law at the principal nodes (since the assigned arbitrarily chosen window currents flow in a continuous path without splitting into the branch currents at the principal nodes, then it is called **mesh current method**).

The window currents can be arbitrarily assumed either in clockwise or in anticlockwise directions, but once chosen, it should remain the same. *Kirchoff's Voltage Law* is then applied to the two meshes, as is illustrated by the following problem.

Problem 1.23

In the network shown in Fig. 1.35 determine all branch currents and the voltage across the 5 Ω resistor, by mesh current method.

Solution :

Let the branch currents be I_a, I_b and I_c as in Fig. 1.35.

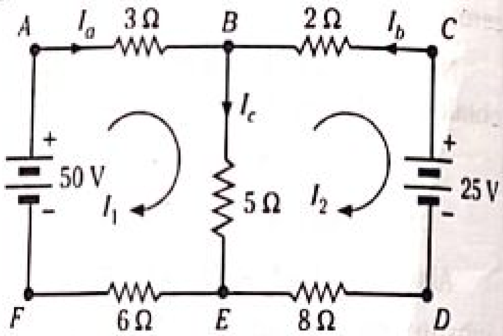

Fig. 1.35

Let the loop currents in the two meshes be I_1 and I_2.

Applying Kirchoff's Voltage Law to mesh *ABEF*,

$$50 - 3I_1 - 5(I_1 - I_2) - 6I_1 = 0$$

or $$14I_1 - 5I_2 = 50 \quad \text{---(i)}$$

Applying KVL to Mesh *BCDE*,

$$-2I_2 - 8I_2 - 5(I_2 - I_1) - 25 = 0$$

or $$-5I_1 + 15I_2 = -25 \quad \text{---(ii)}$$

Solving eqns.(i) and (ii) we get,

$$I_1 = 3.378\text{A} \text{ and } I_2 = -0.54 \text{ A}$$

The branch currents are

$$I_a = I_1 = \mathbf{3.378\ A}$$

$$I_b = -I_2 = \mathbf{0.54\ A}$$

$$I_c = I_1 - I_2 = 3.378 - (-0.54)$$

$$= \mathbf{3.918\ A}$$

As per KCL, at node *B*

$$I_c = I_a + I_b = 3.378 + 0.54 = 3.918\text{A}, \text{ as before.}$$

Voltage across the 5 Ω resistor = $5I_c$ = 5 × 3.918 = 19.59 Volts

1.10 Electrical Work

In an electrical circuit, there is movement of electrons which constitutes flow of current. This movement of electrons results in transfer of charge. *Electrical work is done when there is a transfer of charge*. The unit of such electrical work is *joule*.

One joule of electrical work done is that work done in moving a charge of 1 coulomb through a potential difference of 1 volt.

Thus, if V = Potential difference in volts

and Q = Charge in coulombs

then, electrical work, $W = V \times Q$ joules

Now, $I = \frac{Q}{t}$

Therefore $\boldsymbol{W = VIt}$ **joules**

where t = time in seconds

1.11 Electrical Power

The rate at which electrical work is done in an electric circuit is called ***electrical power***.

$$\therefore \text{ Electrical power } P = \frac{\text{electrical work done}}{\text{time}}$$

$$= \frac{w}{t} = \frac{VIt}{t}$$

$= VI$ joules/sec, *i.e.*, watts

The power consumed in an electrical circuit is one watt if potential difference applied across the circuit is 1 volt which causes 1 ampere of current to flow through the circuit.

$\boldsymbol{P = VI}$ **watts**

1 watt = 1 volt × 1 ampere

1 watt = 1 joule/sec

The unit of power i.e., *watt* is very small as compared with the power transfers in practical circuits. Hence power is generally expressed in bigger units like kilowatts and mega-watts.

1 kW = 1000 watts and 1 MW = 1×10^6 watts.

1.12 Electrical Energy (W)

Electrical energy is the total amount of electrical work done in an electrical circuit.

$\therefore$ Electrical energy = power × time

$$i.e., \quad W = VIt \text{ joules, } i.e., \text{ watt-secs}$$

The unit of electrical energy is joule or *watt-sec*.

Energy consumed by an electric circuit is 1 watt-sec or joule when it utilises power of 1 watt for 1 second.

As the watt-sec is a very small unit, electrical energy is measured in larger units, viz the watt-hour and kilowatt-hour (kWh).

1 watt-hour = 1 watt × 1 hour

or 1 Wh = 1 watt × 3600 sec = 3600 watt-sec *i.e.*, joules

and 1 kWh = 3600 × 1000 joules = 3.6×10^6 joules

When a power of 1 kW is utilised for 1 hour, the energy consumed is said to be **1 kWh**, which is the *commercial unit of energy*, based on which we pay our electricity bills. The commercial name of the unit kWh is **unit** itself.

1.13 Power and Energy using Ohm's Law

We have seen earlier that electrical power is given by

$$P = VI \text{ watts}$$

Now $V = IR$ .... Ohm's law

Substituting in the expression for power,

$$P = I^2 R = \frac{V^2}{R} \text{ watts}$$

Electrical energy is

$$W = VIt \text{ joules (or watt-secs)}$$

$$= I^2 Rt = \frac{V^2}{R} t \text{ joules (or watt-secs)}$$

The above expressions are used to calculate the power consumed in the various parts of an electric circuit.

Problem 1.24

A 250 V lamp is rated to pass a current of 0.30 A. Find its power output. Now, if a second similar lamp is connected in parallel with the first lamp, find the supply current required to give the same power output in each lamp.

$$P = VI = 250 \times 0.3 = \mathbf{75\ W}$$

When the second similar lamp is connected in parallel,

$$P = 75 + 75 = 150\ \text{W}$$

and $$I = \frac{P}{V} = \frac{150}{250} = \mathbf{0.60\ A}$$

Problem 1.25

In Problem 1.24, calculate the power if the lamps are connected in series.

In the case of one lamp,

$$R = \frac{V}{I} = \frac{250}{0.30} = 833.3\ \Omega$$

When both the lamps are connected in series, the total resistance is

$$R_t = 833.3 + 833.3 = 1666.60\ \Omega$$

$$I = \frac{V}{R_t} = \frac{250}{1666.60} = 0.15\ \text{A}$$

$$P = VI = 250 \times 0.15 = \mathbf{37.5\ W}$$

Problem 1.26

A current of 4 A passes through a 10 Ω resistor. Calculate :

(a) the power developed by the resistor

(b) the energy dissipated in 3 minutes.

(a) $P = I^2R = 4^2 \times 10 = \mathbf{160\ W}$

(b) $W = I^2Rt = 160 \times (3 \times 60) = \mathbf{28800\ J}$

i.e., 28800 watt-sec

Problem 1.27

A heater is given a current of 10 A from a 250 V source for 15 hours. etermine the energy consumed by the heater in kilowatt-hours.

$$P = VI = 250 \times 10 = 2500\ \text{W} = 2.5\ \text{kW}$$

$$W = Pt = 2.5\ \text{kW} \times 15\ \text{hours}$$

$$= \mathbf{37.50\ kWh}$$

Problem 1.28

For the circuit shown in Fig. 1.36 determine the power developed by each resistor.

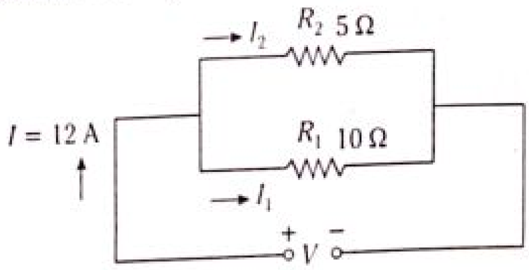

Fig. 1.36

$$I_1 = I.\frac{R_2}{R_1 + R_2} = 12 \times \frac{5}{5+10} = 4 \text{ A}$$

$$P_1 = I_1^2 R_1 = 4^2 \times 10 = \textbf{160 W}$$

$$I_2 = I - I_1 = 12 - 4 = 8 \text{ A}$$

$$P_2 = I_2^2 R_2 = 8^2 \times 5 = \textbf{320 W}$$

Problem 1.29

Two resistors connected in parallel across 100 V D.C. supply take 10 A from the line. The power dissipated in one resistor is 600 W. What is the current drawn when they are connected in series across the same supply?

(Mar 89, M.U.)

Solution :

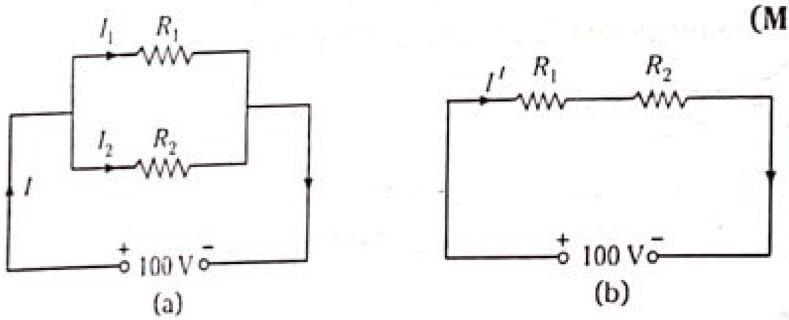

Fig. 1.37

Referring to Fig. 1.37(a),

$$I_1 = \frac{P_1}{V} = \frac{600}{100} = 6 \text{ Amps}$$

$$\therefore \quad I_2 = I - I_1 = 10 - 6 = 4 \text{ Amps}$$

$$R_1 = \frac{V}{I_1} = \frac{100}{6} = 16.66 \text{ ohms}$$

$$R_2 = \frac{V}{I_2} = \frac{100}{4} = 25 \text{ ohms}$$

Now, the two resistances are connected in series across the 100 V D.C. supply hown in Fig. 1.37(b).

The total resistance in this series circuit is = 16.66 + 25 = 41.66 ohms

$\therefore$ Current drawn, $I' = \frac{100}{41.66} = \mathbf{2.4\ Amps}$

4 Review Questions

Define the following terms and mention their units :

(i) Electric current (ii) Potential

(iii) Potential difference (iv) Resistance

(v) Electrical power (vi) Electrical energy

State and explain Ohm's Law. What are its limitations? (April 97, B.U)

State and explain Kirchoff's Laws as applied to D.C.circuits. (Aug/Sep 99, VTU)

Show that the equivalent resistance of two resistors connected in parallel is the ratio of the product of those two resistances divided by the sum of the two resistance values.

Write short notes on (a) Electrical Work
(b) Electrical Power
(c) Electrical Energy.

5 Exercises - Problems

Two coils are connected in parallel and a voltage of 200 V is applied to the terminals. The total current taken is 25 A and the power dissipated in one of the coils is 1500 W. What is the resistance of each coil ?

(KUD May, 1989; Bangalore Feb 1987).

Answer : a) 26.67 Ω ; (2) 11.43 Ω

Two resistors connected in parallel across 100 V d.c, supply take 10 A from the line. The power dissipated in one resistor is 600 W. What is the current drawn when they are connected in series across the same supply ?

[Mysore, March 89]

Answer : 2.4 A

3. Two batteries of 12 V and 12.5 V with internal resistances of 0.1 Ω and 0.2 Ω are connected in parallel. When a resistance of 1 Ω is connected across the above circuit, find the current through it.

(Bangalore Jan.'90)

Answer : 11.406 A

4. A resistor of 8 Ω is connected in series with a combination of 12 Ω and 24 Ω in parallel. The whole circuit is connected across 100 V supply. Find (i) current taken from the supply (ii) currents flowing in the 12 Ω and 24 Ω resistors.

[Gulbarga Nov. 87]

Answer : (i) 6.25A, (ii) 50 V (iii) 4.167 A and 2.083 A

5. Two batteries of 24 V and 20 V with internal resistances of 0.4 Ω and 0.25 Ω respectively are connected in parallel across a load of 4 Ohms. Calculate (i) the current supplied by each battery and (ii) Voltage across the load.

[KUD May 90; Mysore July 90]

Answer : 8.1482 A and –2.963 A, 20.741 V

6. A resistance R is connected in series with a parallel circuit comprising the resistances of 4 Ω and 6 Ω respectively. When the applied voltage is 15 V, the power dissipated in the 4 Ω resistor is 36 W. Calculate R.

[KUD, Nov 91]

Answer : 0.6 A

7. In the circuit shown in Fig. 1.38, find the current through R_L using the Branch Current Method.

Answer : 30.488 A

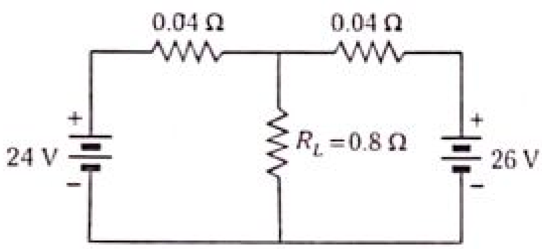

Fig. 1.38

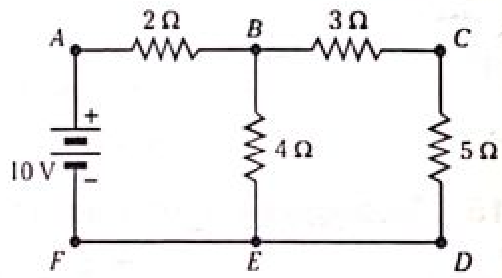

Fig. 1.39

8. Determine the current in the 3 Ω resistor of the network shown in Fig. 1.39, using branch current and mesh current analysis.

Answer : 0.667 A flowing from A to B

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$e = 1 \cdot \frac{13}{5 \times} \quad = \frac{10-(\cdot\cdot)}{0.01}$

MODULE

1

b) Electromagnetism & Electromagnetic Induction

1.16 Introduction

A piece of iron or steel etc., having the properties of attracting iron and pointing to the poles is called a *magnet*. A magnet which is freely suspended in the air, always points in the North - South direction. A magnet always attracts iron filings. In nature magnet is found in the form of iron ore, which is an oxide of iron (ferric oxide) and is known as magnetite. Such natural magnets are too weak to have most practical applications. Hence, artificial magnets, known as **electromagnets**, are built which have several practical applications. Materials such as iron, cobalt, nickel and their alloys are magnetic materials. When bars of such materials are wound with a coil and current is passed through them, they become electromagnets.

The strength of such an electromagnet depends on the number of turns in the coil and the magnitude of the current passing through it. Hence, by increasing the number of coils and the current passing through them, an electromagnet of any strength may be obtained.

1.17 Review of Field around a Conductor

When a wire carrying an electric current is placed above a magnetic needle (Fig. 1.40(a)), aligned with the normal direction of the latter, the needle is deflected clockwise or anticlockwise, depending on the direction of the current. We find that if we look along the conductor and if the current is flowing away from us, as shown by the cross inside the conductor (Fig. 1.40(b)); the magnetic field has a clockwise direction and the lines of magnetic flux may be shown by concentric circles around the wire.

The convention for direction of flow of current is shown in Fig. 1.40(c), where we have a conductor in which we have drawn an arrow indicating the direction of conventional current flow. But, when the conductor is viewed end on, the flow of current would be either towards us or away from us. If the current is flowing towards us, we indicate it by a dot, whereas if it is flowing away from us, it is represented by a cross.

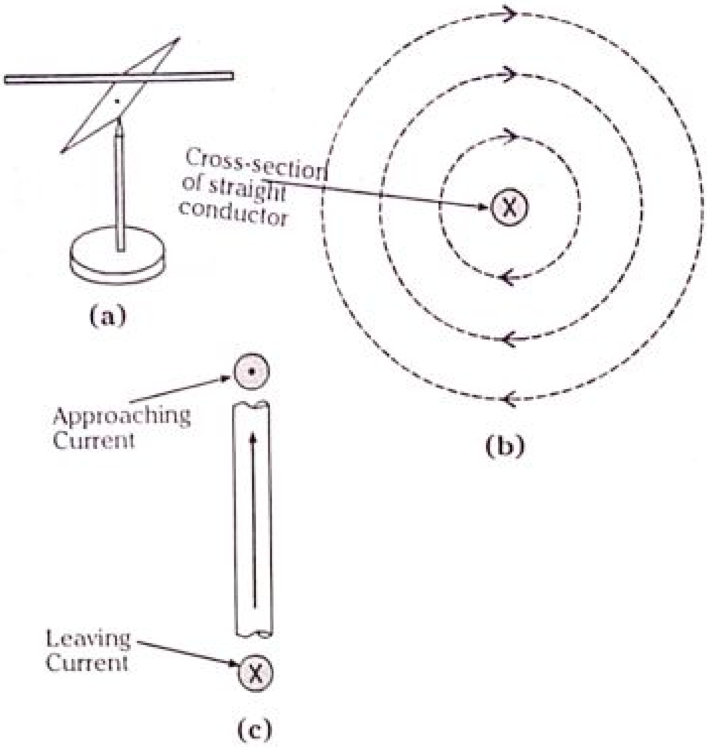


Fig. 1.40

1.18 Important Definitions

(i) Magnetic Force

A magnet has two opposite kinds of magnetism or polarities at its en
One polarity is called North Pole and the other is called South Pole.

Coulomb's First Law states that unlike poles attract each other and like pc
repel each other.

According to Coulomb's Second Law, the force between two magnetic pc
placed in a medium is

i) directly proportional to their pole strengths.
ii) inversely proportional to the square of the distance between them, a
iii) inversely proportional to the absolute permeability of the surround medium.

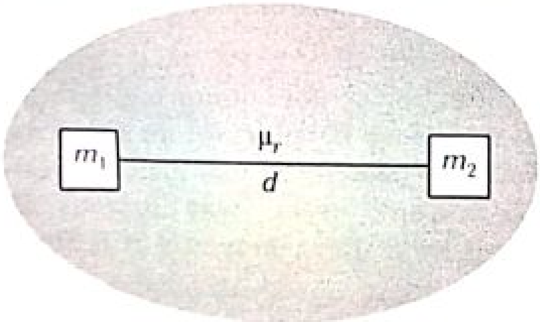


Fig. 1.41

If m_1 and m_2 are the magnetic pole strengths of the two poles (Fig. 1.41) and 'd' is the distance between them, and 'μ' the absolute permeability of the surrounding medium, then the force F is given by

$$F \propto \frac{m_1 m_2}{\mu d^2} \quad \text{or} \quad F = k\frac{m_1 m_2}{\mu d^2} \qquad \text{---(1)}$$

In the S.I. system of units, $k = \dfrac{1}{4\pi}$

$$\therefore \qquad \boldsymbol{F = \frac{m_1 m_2}{4\pi\mu d^2}} \textbf{ newtons}$$

$$\text{or} \qquad F = \frac{m_1 m_2}{4\pi\mu_o\mu_r d^2} \text{ newtons in a medium} \qquad \text{---(2)}$$

$$= \frac{m_1 m_2}{4\pi\mu_o d^2} \text{ newtons in air } (\because \mu_r = 1 \text{ for air}) \qquad \text{---(3)}$$

μ_o is the permeability of air (or evacuated space) and is equal to $4\pi \times 10^{-7}$ henry/metre and μ_r is the relative permeability of the medium with respect to air (or evacuated space).

In the SI System, m_1 and m_2 are taken in webers, 'd' in metres, and F given by expression (ii) will be in newtons.

Substituting $m_1 = m_2 = m$, $d = 1$, $\mu_r = 1$ and $F = 62{,}800$ newtons, in expression (ii), we get

$$62{,}800 = \frac{m \times m}{4\pi \times (4 \times 10^{-7}) \times 1 \times 1}$$

$$\text{or} \qquad m^2 = 1$$

$$\text{or} \qquad m = \pm 1 \text{ Wb}$$

Hence a unit magnetic pole is defined as that pole which, when placed in air or vacuum at a distance of one metre from a similar pole of the same strength, repels it with a force of 62,800 newtons.

(ii) Magnetic Field

The space around the poles of a magnet is called the magnetic field and is represented by magnetic lines of force. The magnetic field exists in the space around the magnet.

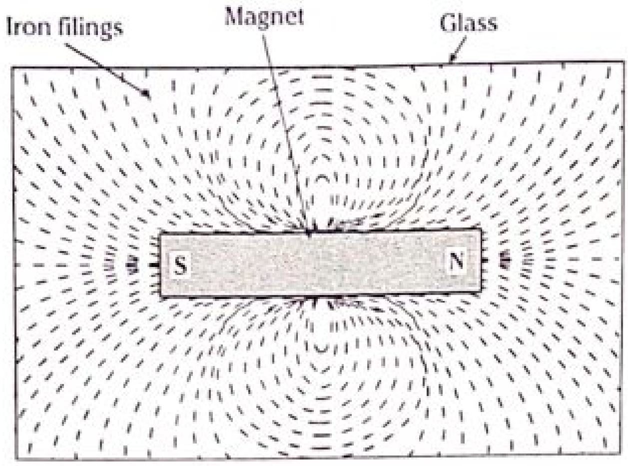


Fig. 1.42 Pattern of Iron Filings around a Magnet

The magnetic field is invisible but evidence of its force can be seen whe small Iron filings are sprinkled on a glass sheet over a bar magnet as shown i Fig. 1.42. It will be seen that many filings cling to the poles of the magnet showin that the field is strongest at the poles. Farther away from the poles, the filings ar less dense, indicating weaker field. Thus, it is evident that the magnetic field du to a magnet is strongest near the poles and goes on decreasing as we move awa from them.

Magnetic Lines of Force

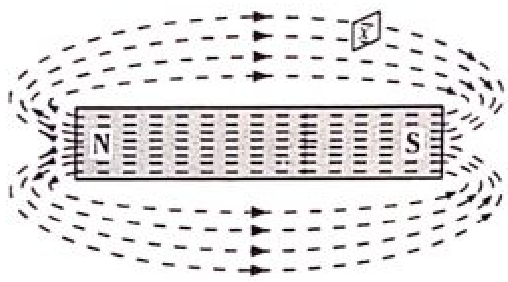


Fig. 1.43 Magnetic Lines of Force around a Bar Magnet

Fig. 1.43 shows the magnetic lines of force. The direction of each line of forc is from the N-pole to the S-pole outside the magnet, but from the S-pole to th N-pole inside the magnet. Close to the poles, where the field is strongest, the line are dense, but further away, the lines are less crowded, indicating lesser magneti field strength.

(iii) Magnetic Flux (ϕ)

The total number of magnetic lines of force in a magnetic field is called *magneti flux*. It is represented by the Greek Letter ϕ.

The unit of magnetic flux in the C.G.S. system is *maxwell*. One maxwell is equal to one magnetic line of force, *i.e.*,

1 maxwell = 1 line of force.

In the S.I. System, the unit of magnetic flux is *weber.*

1 weber = 10^8 magnetic lines of force or 10^8 maxwells.

(iv) Magnetic Flux Density (B)

The *magnetic flux density* at any point is given by the flux passing per unit area at that point, through a plane that is at right angles to the flux.

If ϕ Wb. is the total magnetic flux passing through an area of A m^2, then

$$\text{Flux density } B = \frac{\phi}{A} \text{ Wb/m}^2$$

We will consider the magnetic field due to a magnet, already shown in Fig. 2.3. It is required to find the flux density at a point X. Considering 1 cm^2 cross-sectional area at point X, we observe that two lines of force are passing through this area.

$$\therefore \quad \text{Flux Density at } X = \frac{\text{Flux passing}}{\text{Area}} = \frac{2 \text{ lines}}{1 \text{ cm}^2} = 2 \text{ lines/cm}^2$$

(v) Magnetic Circuit

The source of magnetic flux is either a permanent magnet or a current carrying coil. The lines of magnetic flux always follow a closed path. *A magnetic circuit is defined as the path which is followed by the magnetic flux*. Referring to an iron ring of Fig. 1.44, the magnetic flux is produced due to a current passing through a coil. The magnetic circuit of length 'l' metres is shown.

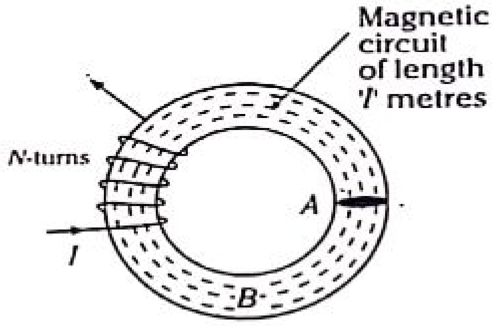
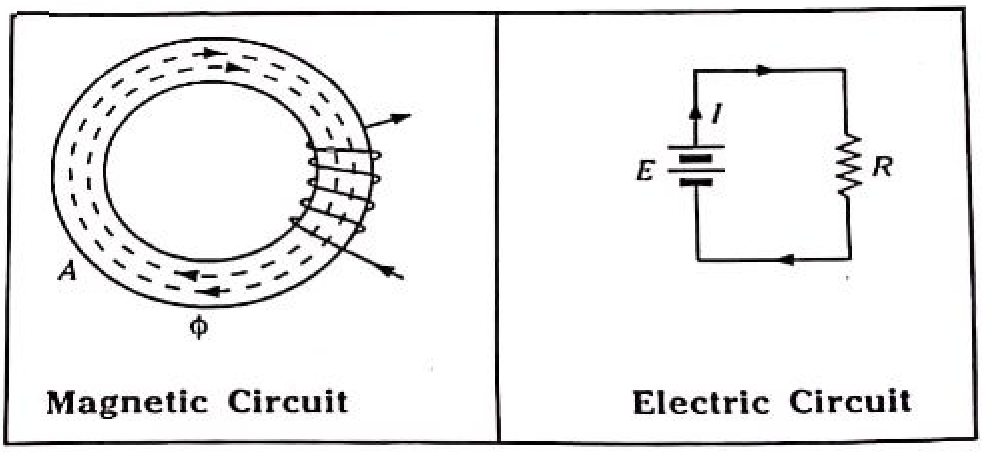


Fig. 1.44

(vi) Analogy between Magnetic & Electric Circuits



1. Flux = $\frac{\text{m.m.f}}{\text{reluctance}}$	1. Current = $\frac{\text{e.m.f}}{\text{resistance}}$
2. Flux ϕ (Webers)	2. Current I (amperes)
3. Flux Density B (Wb/m^2)	3. Current Density (A/m^2)
4. M.M.F (ampere turns)	4. E.M.F. (in volts)
5. Reluctance $S = \frac{l}{\mu_o \mu_r A}$	5. Resistance $R = \rho \frac{l}{A}$
6. Permeance $\left(= \frac{1}{\text{Reluctance}}\right)$	6. Conductance $\left(= \frac{1}{\text{Resistance}}\right)$
7. Reluctivity	7. Resistivity
8. Permeability $\left(= \frac{1}{\text{Reluctivity}}\right)$	8. Conductivity $\left(= \frac{1}{\text{Resistivity}}\right)$

(vii) Magnetomotive Force (M.M.F)

M.M.F is defined as the magnetic force, which creates magnetic flux . magnetic material.

Its unit is 'ampere turns' (AT)

$$\text{M.M.F} = N.I$$

where N = Number of turns in the coil
I = current through the coil

Another equation for M.M.F is

$$\text{M.M.F} = \text{Flux} \times \text{Reluctance}$$

or $NI = \phi \times S$

or $\phi = \frac{NI}{S}$ ---(4)

(viii) Reluctance (S)

Reluctance is the property of a magnetic material by virtue of which it opp the creation of magnetic flux in it. The unit is ampere-turn per weber (AT/ The reluctance of a magnetic material is directly proportional to the length o magnetic material and inversely proportional to the area of cross-section.

Fig. 1.44 shows a magnetic circuit (length of magnetic material) of length 'l' and area of cross-section 'A'.

$$S \propto \frac{l}{A} = \frac{1}{\mu} \cdot \frac{l}{A} = \frac{l}{\mu_o \mu_r A} \qquad \text{---(5)}$$

where μ = a constant known as the *absolute permeability* of the magnetic material.

$= \mu_o \mu_r$

where μ_o = permeability of free space or air and μ_r = relative permeability of the magnetic material.

From equations (4) and (5) we have,

$$\phi = \frac{NI}{\dfrac{l}{\mu_o \mu_r A}} \text{ Wb} \qquad \text{---(6)}$$

(ix) Permeability

The phenomenon of magnetism and electromagnetism depend upon a particular feature of the medium called its *permeability*, which can be defined as the *ability of a material to conduct magnetic flux through it*. Every medium possesses two permeabilities.

(a) absolute permeability (μ) and

(b) relative permeability (μ_r)

(a) Absolute permeability (μ): The absolute permeability of a magnetic material is defined as *the flux density induced in the magnetic material per unit magnetising force*. Hence,

$$\mu = \frac{B}{H} \text{ H/m} \qquad \text{---(7)}$$

where H = magnetising force.

(b) Relative permeability (μ_r): For defining the relative permeability of a magnetic material, the permeability of free space or air is taken as reference. Thus the relative permeability of free space or air is taken as unity.

Thus $\mu_r = 1$ for free space or air.

For any other magnetic material, the relative permeability is defined *as the ratio of the flux density induced in the magnetic material of a particular shape and size to the flux density induced in free space or air of the same shape and size, when the same magnetising force is applied*. Thus,

$$\mu_r = \frac{B}{B_o} \qquad \text{---(8)}$$

μ_r is dimensionless and hence it is a pure number. If the permeability of iron is 500, it means that iron is 500 times more magnetic than free space or air. Table 2.1 shows the relative permeability of the different magnetic materials.

Table 1.1 Relative permeabilities of some magnetic materials

Sl.No.	Magnetic Material	Relative permeability
1.	Mild Steel	2000
2.	Silicon Steel	7000
3.	Nickel	600
4.	Cobalt	250
5.	Permalloy (45 % Ni)	2700

(x) Magnetising Force or Magnetic Field Intensity

The force experienced on a unit (one weber) N-pole placed at a distance of 'd' metres from another pole of strength 'm' webers in a medium of relative permeability μ_r is given by

$$F = \frac{m \times 1}{4\pi\mu_0\mu_r d^2} \text{ newtons}$$

and this is called the magnetic field strength or field intensity (H) at that point. Its units are either newtons/weber or ampere-turns/metre.

Thus, the magnetic field strength or field intensity (also sometimes called magnetising force or intensity of magnetic field) at any point is defined as *the force experienced by a unit north pole, when placed at that point*.

$$\text{Thus} \qquad H = \frac{m}{4\pi\mu_0\mu_r d^2} \text{ newtons/weber}$$

The force experienced by a pole of 'm' webers placed in a uniform magnetic field of intensity H newtons/weber will be equal to mH newtons.

Magnetising force may also be defined as the number of ampere turns produced per unit length of the magnetic path. Thus,

$$H = \frac{NI}{l} \text{ AT/m} \qquad \text{---(9)}$$

(xi) Leakage Flux & Fringing Flux

Fig. 1.45 shows an iron ring with an air gap. When the magnetising winding is concentrated over a short length of the ring as shown, the entire flux set up due to the current passing through it does not reach the magnetic circuit. Some of the flux is present in and around the coil itself. This flux which is not doing any useful work is called *leakage flux* ϕ_l (Fig. 1.45).

If ϕ is the useful flux, *i.e.*, the flux carried by that part of the magnetic circuit in which the flux is really used, and ϕ_1 is the leakage flux, then the total flux is given by

$$\phi_t = \phi + \phi_1$$

Therefore, leakage factor is given by the ratio of

$$\frac{\text{Total flux}}{\text{Useful flux}} = \frac{\phi_t}{\phi} = K_l$$

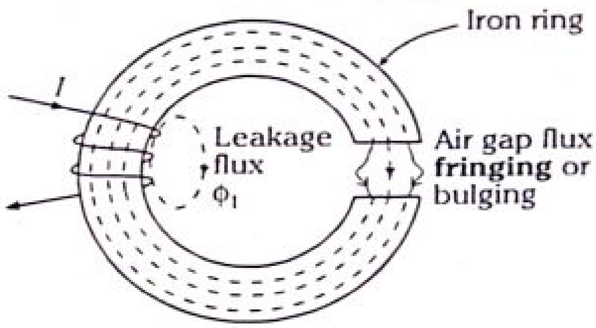


Fig. 1.45

The normal value of K_l is 1.1 to 1.2.

The useful flux crossing the air gap tends to bulge outwards at the ends of the airgap and the flux density in the airgap is reduced. This is called *fringing*. The longer the air gap the larger is the fringing.

1.19 Electromagnetic Induction

As an electric current passes through a conductor, magnetic field is set up which surrounds or links up with the conductor. Thus, magnetism can be created by means of an electric current. The converse of this is also true, *i.e.*, electricity can be created by magnetism by changing the magnetic flux linking with the conductor. This process is known as **electromagnetic induction.**

The e.m.f. produced is called *induced e.m.f.* and the resulting current is called *induced current*. Here, it is emphasised that the main requirement for electromagnetic induction is the *change in flux linking with the circuit*.

Demonstration

In order to show the phenomenon of electromagnetic induction, let an insulated coil wound with a large number of turns be connected to a galvanometer, as shown in Fig. 1.46.

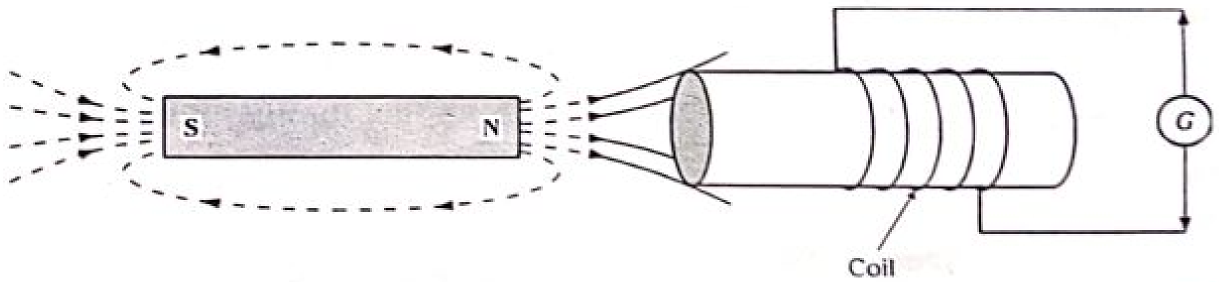


Fig. 1.46

a) If a permanent magnet NS is moved towards the coil, the pointer of the galvanometer will be deflected, indicating that there must be an e.m.f. induced in the coil. This e.m.f. is produced because the moving magnet is causing the change in flux linking the coil. *If the movement of the magnet is stopped, there is no change in flux even though the flux is linking with the coil* and hence, the deflection of the galvanometer is reduced to zero. An e.m.f. will also be induced if the magnet is kept stationary and the coil is moved.

Thus, we may conclude that, **whenever the flux linking a conductor changes, an e.m.f. is induced in it. This e.m.f. persists as long as the flux linking the conductor is changing.**

b) If the permanent magnet NS is now moved away from the coil, we will notice the deflection of the galvanometer pointer, though now in the reverse direction.

Thus, we may conclude that **the direction of the induced e.m.f. depends upon the direction of the magnetic flux and upon the direction in which the flux moves relative to the conductor.**

c) If the permanent magnet is moved towards or away from the coil with increased speed, the galvanometer pointer will be deflected to a greater extent, indicating greater induced e.m.f. in the coil. The greater the speed of the magnet, the more is the rate of change of flux linking the coil.

Hence, we see that the magnitude of the induced e.m.f. depends upon the rate of change of flux linking the conductor.

1.20 Faraday's Laws of Electromagnetic Induction

The phenomena mentioned above may be reflected in the form of two laws, called Faraday's Laws of Electromagnetic Induction.

First Law : *As per this Law, whenever the flux linking a coil or circuit changes, an e.m.f. is induced in it.*

Second Law : *The magnitude of the induced e.m.f. in a coil is directly proportional to the rate of change of flux linkages.*

The magnitude of flux linkages is determined by the number of times a circuit is linked by the flux. If there are N turns of a coil, then each flux line will link this circuit N times, *i.e.*,

Flux linkages = Flux × Number of turns.

Explanation : Suppose a coil has N turns and the flux through it changes from an initial value of ϕ_2 webers in t seconds.

Thus,

Initial flux linkages = $N\phi_1$

Final flux linkages = $N\phi_2$

$\therefore$ Rate of change of flux linkages $= \dfrac{N\phi_2 - N\phi_1}{t}$

As per Faraday's Second Law of Electromagnetic Induction, the e.m.f. 'e' is given by

$$e \propto \frac{N\phi_2 - N\phi_1}{t} \text{ volts}$$

$$= K\frac{N(\phi_2 - \phi_1)}{t} \text{ volts}$$

$$= \frac{N(\phi_2 - \phi_1)}{t} \text{ volts} \qquad \text{---(10)}$$

where the value of 'K' is unity.

Putting the above expression in differential form,

$$e = N\frac{d\phi}{dt} \text{ volts} \qquad \text{---(11)}$$

Normally, a minus sign is attached to the right-hand expression of eqn.(11) to indicate the fact that the induced e.m.f. sets up current in such a direction that the magnetic effect produced by it opposes the very cause producing it.

$$\therefore \qquad e = -N\frac{d\phi}{dt}$$

Problem 1.30

A coil of 1500 turns gives rise to a magnetic flux of 2.5 mWb, when carrying a certain current. If the current is reversed in 0.2 sec, what is the average value of the e.m.f induced in the coil ?

Solution :

With the reversal of current, there is a reversal of flux too.

$$d\phi = \text{change of flux} = \left(2.5\times 10^{-3}\right) - \left(-2.5\times 10^{-3}\right)$$

$$= 5\times 10^{-3} \text{ Wb}$$

$$e = -N\frac{d\phi}{dt} = -1500 \times \frac{5\times 10^{-3}}{0.2}$$

$$= \mathbf{-37.5 \text{ volts}} \quad \text{(Ans.)}$$

1.21 Direction of Induced E.M.F. and Current

There is a definite relationship between the direction of the induced current, the direction of the flux and the direction of motion of the conductor. The direction of the induced current may be found by the following two methods :

1. Lenz's Law
2. Fleming's Right Hand Rule.

1. Lenz's Law

This law gives the direction of induced e.m.f. and hence current and is stated as follows :

The direction of induced e.m.f. and hence current is such that it opposes the cause producing it.

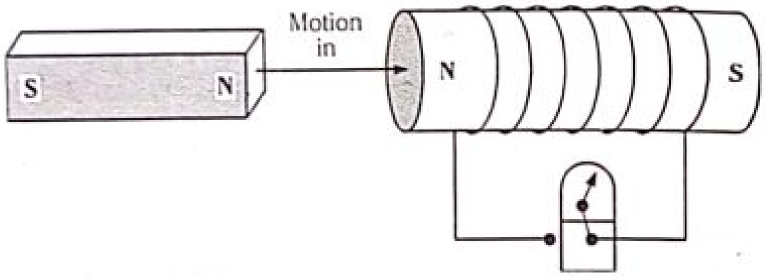


Fig. 1.47

Explanation : Let the *N*-pole of a magnet approach a coil as shown in Fig. 1.47. It is obvious that the flux linking the coil changes and consequently e.m.f. is induced in the coil. This induced e.m.f. sends current through the coil.

As per Lenz's Law, the direction of this induced current in this coil is such that it opposes the cause producing it. The cause which is producing the induced current is the motion of the magnet. Therefore, induced current should flow in such a direction in the coil so that it develops polarities which oppose the motion of the magnet. This is possible only if the left face of the coil becomes *N*-pole. Once we know the polarity of the coil face, the direction of the induced current is readily found by applying Fleming's Right Hand Rule.

It may be mentioned here that Lenz's Law is in accordance with the Law of Conservation of Energy. When the *N*-pole of the magnet is approaching the coil, the induced current flows in the coil in such a direction that its left face becomes the N-pole. The result is that the motion of the magnet is opposed. The mechanical energy spent in overcoming this repulsive force is converted into electrical energy which appears in the coil.

2. Fleming's Right-Hand Rule

This rule is used to find out the direction of induced e.m.f. and current in a coil. This is specially applicable when the conductor moves at right angles to the

magnetic field. This rule is stated as follows :

Hold the thumb and first finger of the right hand at right angles and bend the second finger so as to point at right angles to the plane of these two. *i.e.*, the three are at right angles to each other. *If the first finger points in the direction of the magnetic field, the thumb in the direction of motion of the conductor, then the middle finger points in the direction of the induced e.m.f.*

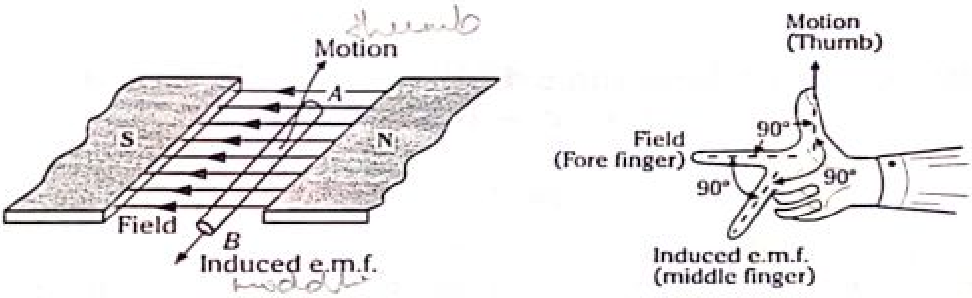


Fig. 1.48

Explanation : Consider a conductor *AB* moving upwards at right angles to a magnetic field, as shown in Fig. 1.48.

Applying Fleming's Right Hand Rule, it is obvious that the induced e.m.f. is as shown by the middle finger.

1.22 Fleming's Left-Hand Rule

When a current-carrying conductor is placed in a magnetic field, it experiences a force which acts in a direction perpendicular both to the direction of the current and the field. The magnitude of this force given by

$$F = BIl \text{ newtons}$$

where B = Flux density of the magnetic field (Wb/m^2)
I = current carried by the conductor (amperes)
l = Length of the conductor (metres)

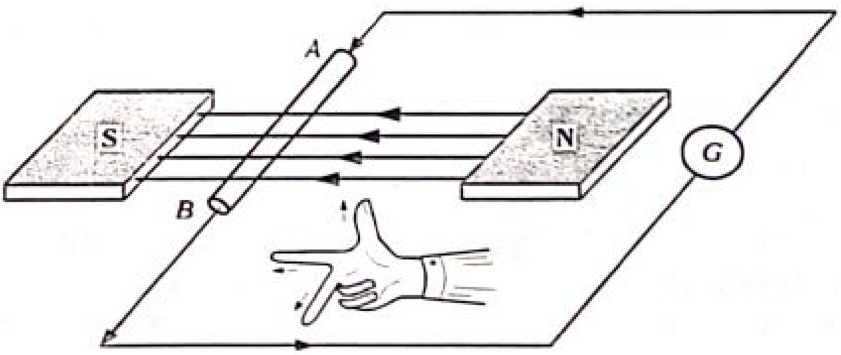


Fig. 1.49

The direction of the force is given by Fleming's Left-Hand Rule, which is as follows :

1. Hold the thumb, first finger and second finger of the left hand in such a way that they are at right angles to each other as shown in Fig 1.48.
2. If the forefinger represents the direction of the field and the second finger the direction of the current, then the direction of the force is indicated by the thumb.

1.22.1 Application of Fleming's Left Hand Rule to the Production of Torque in a D.C. Motor

Production of torque in relation to the operation of a d.c. motor is based on the Fleming's Left Hand Rule, as described below.

The operation of a d.c. motor is based on the principle that when a current carrying conductor is placed in a magnetic field, the conductor experiences a mechanical force. The direction of this force is given by *Fleming's Left Hand Rule*. We shall discuss this in detail.

Fig. 1.50 shows a 4-pole d.c. motor, where the field and armature circuits are connected across the d.c. mains supply. Suppose the armature conductors under the N-poles carry currents out of the plane of paper (shown by dots) and those under the S-poles carry currents into the plane of paper (as shown by crosses). By applying *Fleming's Left Hand Rule*, the force on each conductor tends to rotate the motor armature in the clockwise direction. All these forces add together to produce a driving **torque** which sets the armature rotating.

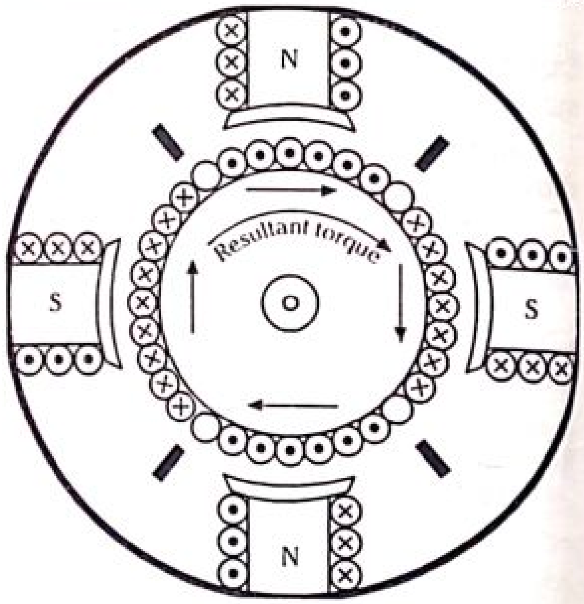


Fig. 1.50

1.23 Dynamically Induced E.M.F.

When the magnetic field is stationary and the conductor is in motion, the e.m.f. induced is called *dynamically induced e.m.f.*

Let us take a conductor of length l metres moving at right angles to a uniform magnetic field of $B\ Wb/m^2$ with a velocity of v metres/sec (Fig. 1.51(a)). Let the conductor move through a small distance dx in dt seconds. So, the area swept $= l \times dx$

$$\text{Flux cut} = \text{Flux density} \times \text{area swept}$$

$$= B \times l \times dx$$

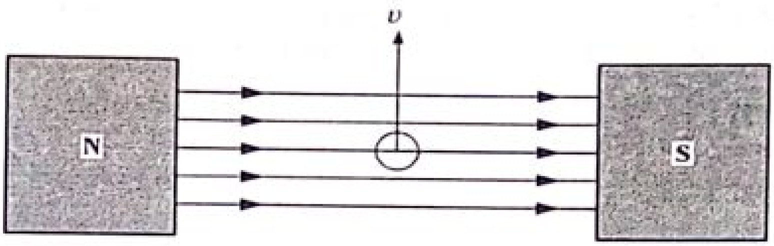


Fig. 1.51(a)

As per Faraday's Laws of Electromagnetic Induction, the e.m.f. 'e' induced in the conductor is given by

$$e = \frac{\text{Flux cut}}{\text{time}} = \frac{Bl\,dx}{dt}$$

$$= Bl\frac{dx}{dt}$$

or $\quad e = Bl\upsilon$ volts $\quad \left[\because \upsilon = \frac{dx}{dt}\right]$

2nd Case

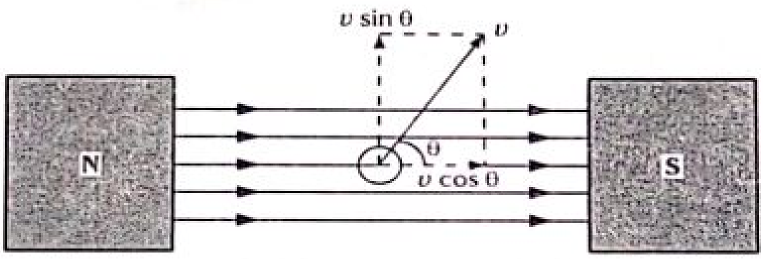


Fig. 1.51(b)

Suppose the conductor moves at an angle θ with the direction of the magnetic field as shown in Fig. 1.51(b), then velocity υ may be resolved into two components:

a) $\upsilon \cos\theta$, parallel to the field

b) $\upsilon \sin\theta$, perpendicular to the field.

The component $\upsilon \cos\theta$, being parallel to the field, does not induce any voltage. But, component $\upsilon \sin\theta$ produces e.m.f, and is given by :

$$e = Bl\upsilon \sin\theta \text{ volts}$$

The direction of the induced e.m.f. is found by following Fleming's Right Hand Rule.

Problem 1.31

A conductor of length 1 metre moves at right angles to a uniform magnetic field of flux density 1.5 Wb/m² with a velocity of 50 metres/second. Calculate

the e.m.f. induced in it. Find also the value of induced e.m.f. when the conductor moves at an angle of 30° to the direction of the field, and when it moves parallel to the field.

Solution :

(i) Here, $B = 1.5$ Wb/m^2, $l = 1$ m

So, $e = Blv$ volts

$= 1.5 \times 1 \times 50 = $ **75 volts**

(ii) In the second case,

$\theta = 30°$ $\therefore \sin 30° = 0.5$

$\therefore \quad e = Blv \sin 30°$

$= 75 \times 0.5 = 37.5$ V

(iii) When the conductor moves parallel to the field (lines of flux) the e.m.f. induced is zero.

Problem 1.32

A square coil of 10 cm side and with 100 turns, is rotated at a uniform speed of 1000 rev. per min, about an axis at right angles to a uniform magnetic field, having a flux density of 0.5 Wb/m^2. Calculate the instantaneous value of the induced e.m.f when the plane of the coil is (a) at right angles to the field (b) at 30° to the field and (c) in the plane of field.

Solution :

Speed = 1000 r.p.m.

$$\therefore \quad v = 1000 \times \frac{3.14 \times 0.1}{60} \text{ m/sec}$$

$$= 5.23 \text{ m/sec}$$

a) When the coil is rotated at right angles to the field, the coil sides do not cut any flux. So the e.m.f induced is zero.

b) When the coil is rotated at 30° to the field, the angle between the direction of motion of the conductor and the field is 60°. Two sides of the coil cut the flux.

$$\therefore \quad e = NBlv \sin 60° \times 2$$

$$= 100 \times 0.5 \times 0.1 \times 5.23 \times \sin 60° \times 2$$

$$= 45.29 \text{ volts (Ans)}$$

c) When the coil is rotated in the plane of the field, the conductors cut the flux at right angles, and hence maximum e.m.f is induced.

$$\therefore \quad e = NBlv \times 2 = 100 \times 0.5 \times 0.1 \times 5.23 \times 2$$

$$= 52.3 \text{ volts (Ans)}$$

Problem 1.33

A wire of length 50 cm moves at right angles to its length at 40 m/s in a uniform magnetic field of density 1 Wb/m^2. Calculate the e.m.f. induced in the conductor, when the direction of motion is (a) perpendicular to the field (b) inclined at 30° to the direction of the field.

Solution :

Given l = 50 cms = 0.5 m, v = 40 m/s;

Now, Induced e.m.f., $e = Blv \sin \theta$ volts

(a) When θ = 90°

$$e = 1 \times 0.5 \times 40 \times \sin 90^\circ$$
$$= 1 \times 0.5 \times 40 \times 1 = \mathbf{20\ Volts}$$

(b) When θ = 30°

$$e = 1 \times 0.5 \times 40 \times \sin 30^\circ$$
$$= 1 \times 0.5 \times 40 \times 0.5 = \mathbf{10\ Volts}$$

1.24 Statically Induced E.M.F.

When a conductor is stationary and the magnetic field is moving or changing, the e.m.f. induced is called *statically induced e.m.f.*

Statically induced e.m.f. may be :

a) Self-induced e.m.f, or

b) Mutually-induced e.m.f.

1.25 Self-induced E.M.F.

The e.m.f. induced in a coil due to the change of its own flux linked with it is called self-induced e.m.f.

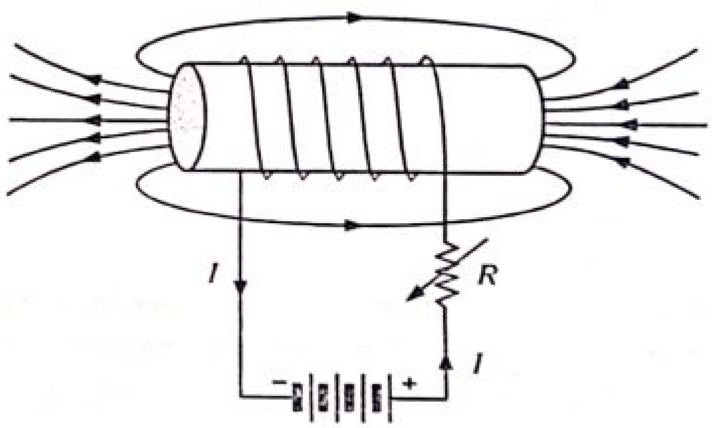


Fig. 1.52

Let us consider a coil of many turns as shown in Fig. 1.52. If the current through the coil is varied, then the flux linking the coil changes and hence e.m.f. is induced in the coil, which is called **self-induced e.m.f.**

Since, according to Lenz's Law, any induced e.m.f. acts to oppose the change that produces it, a self-induced e.m.f. is always in such a direction as to oppose the change of current in the coil or circuit in which it is induced. Further, the self-induced e.m.f. will persist so long as the current in the coil (and hence flux linking with it) is changing.

1.26 Self-Inductance or Coefficient of Self-Induction (*L*)

We can define this in any one of three following ways

i) First Method

The co-efficient of self-induction of a coil could be defined as the *weber turns per ampere in the coil.*

'Weber-turns' means the flux linkages of the coil and is given as

Weber-turns = flux in webers × number of turns with which the flux is linked.

Suppose a solenoid has N turns, and a current of I amperes is passing through it. If the flux produced is ϕ webers, then the weber-turns are $N\phi$.

Therefore, the weber turns per ampere are $\dfrac{N\phi}{I}$

By definition, $L = \dfrac{N\phi}{I}$

If, in the above expression, $N\phi$ = 1 Wb-turn, I = 1 ampere, then L = 1 henry (H).

So, a coil has a self-inductance of one henry if a current of 1 ampere flowing through the coil produces flux linkages of 1 Wb-turn in it.

So, $$L = \frac{N\phi}{I} \text{ henry} \qquad \text{---(12)}$$

ii) Second Method

Referring back to Eqn.(6) of Sec. 1.18(vii),

$$\phi = \frac{NI}{\dfrac{l}{\mu_o \mu_r A}} \text{ Wb} \qquad \text{---(13)}$$

where N = No. of turns of the coil, I = current carried by the coil

l = length (metres), A = Area of cross-section (m^2)

μ_o = absolute permeability of free space, μ_r = relative permeability

$$\therefore \quad \frac{\phi}{I} = \frac{N}{\dfrac{l}{\mu_o \mu_r A}}$$

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But $$L = N.\frac{\phi}{I} = N.\frac{N}{\frac{l}{\mu_o \mu_r A}} \text{ henry} \qquad \text{---(14)}$$

$$= \frac{N^2}{\frac{l}{\mu_o \mu_r A}} \text{henry} = \frac{\mu_o \mu_r A N^2}{l} \text{ henry} \qquad \text{---(15)}$$

We have seen in Sec 1.18(viii) that the term $\frac{l}{\mu_o \mu_r A}$ is known as **reluctance (S).** Hence, substituting in eqn.(14),

$$L = \frac{N^2}{S} \text{ Henry} \qquad \text{---(16)}$$

iii) Third Method

We have seen (while discussing the First Method) that

$$L = \frac{N\phi}{I} \quad \text{or} \quad N\phi = LI \quad \text{or} \quad -N\phi = -LI$$

Differentiating both sides, we have

$$-\frac{d}{dt}(N\phi) = -L\,.\,\frac{dI}{dt} \qquad \text{(we take } L \text{ to be a constant)}$$

$$-N\,.\,\frac{d\phi}{dt} = -L\,\frac{dI}{dt}$$

According to Faraday's Second Law of Electromagnetic Induction,

Self-induced e.m.f. $= -N\,\frac{d\phi}{dt}$

(The minus sign is attached to the R.H.S. to signify the fact that the induced e.m.f. sets up current in such a direction that the magnetic effect produced by it opposes the very cause producing it)

∴ Self induced e.m.f., $\boldsymbol{e_L = -L\,\frac{dI}{dt}}$

If $\frac{dI}{dt} = 1$ ampere/second and $e_L = 1$ volt, then $L = 1$ H.

Thus, a coil is said to have a self-inductance of one henry if one volt is induced in it when the current through it changes at the rate of one ampere/second.

Problem 1.34

A coil consists of 750 turns. A current of 10 A in the coil gives rise to a magnetic flux of 1200 μWb. Determine the inductance of the coil and the

average e.m.f. induced in the coil when this current is reversed in 0.01 sec.

Solution :

Given $N = 750$, $I = 10$ A

$\phi = 1200\ \mu$Wb $= 1200 \times 10^{-6}$ Wb

Also, $$\frac{dI}{dt} = \frac{[10-(-10)]}{0.01} = 2000 \text{ A/sec}$$

[Current of 10 A is reversed, *i.e.*, current changes from +10 A to −10 A]

(i) Self Inductance, $$L = \frac{N\phi}{I}$$

$$= \frac{750 \times (1200 \times 10^{-6})}{10} = \mathbf{0.09\ H}$$

(ii) Self induced e.m.f., $$e_L = L\frac{dI}{dt}$$

$$= 0.09 \times 2000$$

$$= \mathbf{180\ volts}$$

Problem 1.35

An iron-cored torroidal coil has 100 turns, a cross-sectional area of $10\ cm^2$ and a mean length of 314 cm. If the relative permeability of iron is 1000, calculate the inductance of the coil.

Solution :

Given $N = 100$; $A = 10\ cm^2 = 10 \times 10^{-4}\ m^2$

$l = 314$ cm $= 3.14$ m; $\mu_r = 1000$

Self-Inductance, $$L = \frac{\mu_o \mu_r A N^2}{l} \text{ Henry}$$

$$= \frac{(4\pi \times 10^{-7}) \times 1000 \times (10 \times 10^{-4}) \times 100^2}{3.14}$$

$$= 0.004 \text{ H or } \mathbf{4\ mH}$$

Problem 1.36

Calculate the approximate resistance and inductance of an air-cored solenoid, 100 cm long and 1 cm in diameter. The coil is made of copper wire having 2000 turns, a resistivity of $1.73 \times 10^{-2}\ \mu\Omega$–m and a diameter of 1 mm. Find the p.d. between the terminals of the solenoid, when a current of 2 A is changing at the rate of 12,000 A/sec.

Solution :

Resistance of the coil, $R = \rho \dfrac{l}{a}$ ---(i)

Length of the copper wire, $l = \pi \times (1 \times 10^{-2}) \times 2000$

$= 62.84$ m

Area of cross-section of the wire, $A = \dfrac{\pi \times (1 \times 10^{-3})^2}{4} = 0.785 \times 10^{-6}$ m

Given $\rho = 1.73 \times 10^{-2}\ \mu\Omega - m$

$= 1.73 \times 10^{-2} \times 10^{-6}\ \Omega - m$

Substituting in eqn.(i) above

$$R = (1.73 \times 10^{-8}) \times \frac{62.84}{0.785 \times 10^{-6}}$$

$= 1.384\ \Omega$

Inductance of the solenoid $L = \dfrac{N^2 A \mu_o \mu_r}{l}$

$$= \frac{2000^2 \times \frac{\pi \times 0.01^2}{4} \times (4\pi \times 10^{-7}) \times 1}{100 \times 10^{-2}}$$

$= 0.000395$ H $=$ 395 μH

The total p.d. across the coil is given by

$$V = V_R + V_L$$

$$= IR + L\frac{di}{dt}$$

$$= (2 \times 1.384) + (395 \times 10^{-6})\,12000$$

$= 2.768 + 4.740$

= 7.508 volts (Ans)

Problem 1.37

A circuit has 1000 turns, enclosing a magnetic circuit 20 cm² in section. With 4 A, the flux density is 1 Wb/m², and with 9 A, it is 1.4 Wb/m². Find the mean value of inductance between these current limits, and the induced e.m.f. if the current falls uniformly from 9 A to 4 A in 0.05 sec. (KUD. June, 1992)

Solution :

Given : No. of turns, $N = 1000$

X-sectional area, $A = 20\ cm^2 = 20 \times 10^{-4}\ m^2$

Case **1** : With $I_1 = 4\ A$, Flux Density $B_1 = 1\ Wb/m^2$, and

Case **2** : With $I_2 = 9\ A$, Flux density $B_2 = 1.4\ Wb/m^2$

$\therefore$ Flux $\phi_1 = B_1 A = 1 \times (20 \times 10^{-4}) = 2 \times 10^{-3}$ Wb

and Flux $\phi_2 = B_2 A = 1.4 \times (20 \times 10^{-4}) = 2.8 \times 10^{-3}$ Wb

Case 1 : Self Inductance $L_1 = N \dfrac{\phi_1}{I_1} = \dfrac{(1000 \times 2 \times 10^{-3})}{4} = 0.5\ H$

Case 2: Self Inductance $L_2 = N \dfrac{\phi_2}{I_2} = \dfrac{(1000 \times 2.8 \times 10^{-3})}{9} = 0.31\ H$

The mean value of self inductance, $L = \dfrac{L_1 + L_2}{2} = \dfrac{0.5 + 0.31}{2}$

$= \mathbf{0.405\ H}$

The current falls from 9A to 4A in 0.05 sec., then

$$\frac{dI}{dt} = \frac{9-4}{0.05} = 100\ A/sec$$

$\therefore$ Self-induced e.m.f $E = L \dfrac{dI}{dt}$

$= 0.405 \times 100 = \mathbf{40.5}$ **volts**

Problem 1.38

A coil of 150 turns is linked with a flux of 0.01 weber when carı current of 10 A. Calculate the inductance of the coil. If this current is unl reversed in 0.01 second, calculate the induced electromotive force.

Solution :

Given : $N = 150$ turns

$\phi = 0.01$ weber

$I = 10$ A

Now $L = \dfrac{N\phi}{I} = \dfrac{150 \times 0.01}{10}$

or $\quad L = \mathbf{0.15\ H}$

$$e_L = L\frac{dI}{dt}$$

$$= 0.15 \times \frac{[10-(-10)]}{0.01}$$

or $\quad e_L = \mathbf{300\ volts}$

Problem 1.39

A coil of 1000 turns is wound on a torroidal magnetic core having a reluctance of 10^6 AT/Wb, when the coil current is increasing at the rate of 200 A/sec. Determine the inductance of the coil and the e.m.f induced in it.

Solution :

Given : $N = 1000$

$S = 10^6$ AT/Wb

$$\frac{dI}{dt} = 200 \text{ A/sec}$$

Now, $L = \dfrac{N^2}{S} = \dfrac{1000^2}{10^6} = \mathbf{1\ Henry}$

$\therefore$ E.M.F induced in the coil, $e_L = L\dfrac{dI}{dt}$

$$= 1 \times 200 = \mathbf{200\ volts}$$

1.27 Mutually Induced E.M.F.

Let us consider two coils A and B placed close to each other so that the flux created by one coil completely links with the other coil. Let coil A have a battery and switch S and coil B be connected to a galvanometer G (Fig. 1.53).

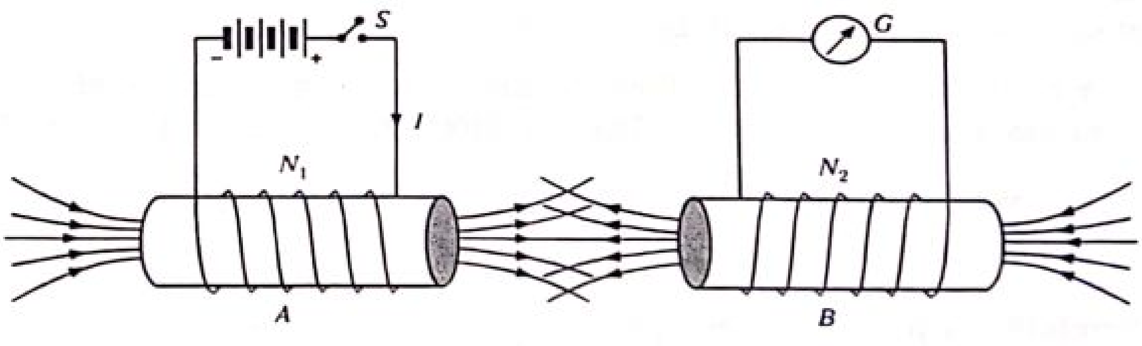

Fig. 1.53

When switch S is opened, no current flows through coil A, so no flux is created in coil A, *i.e.*, no flux links with coil B, and so the galvanometer indicates zero deflection. Next when the switch S is closed, current in coil A starts rising from zero value to a certain finite value, flux is produced during this period which increases with the increase in current of coil A; therefore flux linking with the coil B increases and an e.m.f. known as *mutually induced e.m.f.* is produced in coil B, which is indicated by the deflection of the galvanometer. As soon as the current in coil A reaches a certain finite value, the flux produced or flux linking with coil B becomes constant, so no e.m.f. is induced in coil B, and the galvanometer pointer gets back to zero. Now, if the switch S is opened, current will start decreasing, resulting in decrease in flux linking with coil B, an e.m.f. will be again induced but in a direction opposite to the previous one, this fact being shown by the galvanometer deflection in the opposite direction.

Hence, whenever the current in coil A changes, the flux linking with coil B changes and an e.m.f. known as mutually induced e.m.f. is induced in coil B.

The following points need to be noted :

i) The mutually induced e.m.f. in coil B persists so long as the current in coil A is changing. If current in A is constant, though the flux is linking the coils, there is no change in flux linkages and hence no e.m.f.

ii) The magnitude of mutually induced e.m.f. depends upon the number of turns of coil B and the amount of changing flux linked with it.

iii) The direction of mutually induced e.m.f. is such that it opposes the very cause producing it (Lenz's Law).

1.28 Mutual Inductance or Co-efficient of Mutual Induction (*M*)

We can define this in three ways as given :

i) First Method

Let us take two magnetically-coupled coils A and B, with N_1 and N_2 turns respectively, as shown in Fig. 1.53.

The coefficient of mutual inductance between two coils is defined as the weber turns in one coil due to one ampere current in the other.

Let a current of I_1 amperes flow in coil A and produce a flux of ϕ_1 webers in it. We will assume that this entire flux will link with the turns of coil B. Then, flux linkages *i.e.*, weber turns in coil B for unit current in coil A are $\frac{N_2 \phi_1}{I_1}$.

Therefore, as per definition, $$\boxed{M = \frac{N_2 \phi_1}{I_1}} \qquad \text{---(i)}$$

If weber-turns in coil B due to 1 ampere current in coil A, *i.e.*, $\frac{N_2 \phi_1}{I_1} = 1$, then we see that $M = 1$ H.

So, two coils have a mutual inductance of 1 henry if one ampere current, when flowing in one coil, produces flux linkages of one Wb-turn in the other.

ii) Second Method

Let us now obtain an expression for Coefficient of Mutual Inductance in terms of the dimensions of coils A and B.

$$\text{Flux in coil } A, \quad \phi_1 = \frac{N_1 I_1}{\frac{l}{\mu_o \mu_r A}} \text{ Wb}$$

$$\text{Flux per ampere,} \quad \frac{\phi_1}{I_1} = \frac{N_1}{\frac{l}{\mu_o \mu_r A}}$$

Suppose this entire flux links with the coil B, which has N_2 turns, the weber-turns in it due to the flux/ampere in coil A is

$$M = \frac{N_2 \phi_1}{I_1} = \frac{N_2 . N_1}{\frac{l}{\mu_o \mu_r A}}$$

$$\therefore \quad M = \frac{\mu_o \mu_r A N_1 N_2}{l} \text{ henry} \qquad \text{---(ii)}$$

$$\text{Also,} \quad M = \frac{N_1 . N_2}{\frac{l}{\mu_o \mu_r A}} = \frac{N_1 N_2}{\text{reluctance}} = \frac{N_1 N_2}{S} \text{ henry} \qquad \text{---(iii)}$$

iii) Third Method

We have seen, while discussing the First Method above, that

$$M = \frac{N_2 \phi_1}{I_1}$$

$$\therefore \quad N_2 \phi_1 = M I_1 \qquad \text{or} \qquad -N_2 \phi_1 = -M I_1$$

Differentiating both sides, we obtain

$$-\frac{d}{dt}(N_2 \phi_1) = -M . \frac{dI_1}{dt}$$

(here we take it for granted that M is constant).

But, $-\frac{d}{dt}(N_2 \phi_1) =$ mutually induced e.m.f. in the coil $B = e_M$

$$\therefore \qquad e_M = -M \frac{dI_1}{dt}$$

If $\frac{dI_1}{dt} = 1$ A/s; $e_M = 1$ volt, then $M = 1$ H

Thus, two coils have a mutual inductance of one henry if current changi the rate of 1 ampere per second in one coil induces an e.m.f. of one volt i other.

Problem 1.40

Two identical 1000 turn coils X and Y lie in parallel planes such that of the flux produced by one coil links the other. A current of 5 A in X proc in it a flux of 5×10^{-5} webers. If the current in X changes from +6 A to in 0.01 S, what will be the magnitude of the e.m.f. induced in Calculate the self-inductance of each coil and their mutual inductance.

(May/June 86,

Solution :

Flux produced per ampere of current in coil X :

$$\frac{\phi_1}{I_1} = \frac{5 \times 10^{-5}}{5} = 10^{-5} \text{ Wb/A}$$

Number of turns on coil X, N_1 = 1000

Number of turns on coil Y, $N_2 = N_1$ = 1000

Self-inductance of coil X (or Y),

$$L = N_1 \frac{\phi_1}{I_1} = 1000 \times 10^{-5} = \mathbf{0.01\ H}$$

Flux linking in coil Y per ampere of current in coil X,

$$\frac{\phi_2}{I_1} = \frac{0.6 \times 5 \times 10^{-5}}{5} \text{ Wb/A} = 0.6 \times 10^{-5} \text{ Wb/A}$$

Mutual Inductance between the coils,

$$M = N_2 \frac{\phi_2}{I_1} = 1000 \times 0.6 \times 10^{-5} = \mathbf{0.006\ H}$$

Rate of change of current in coil X,

$$\frac{dI_1}{dt} = \frac{6-(6)}{0.01} = 1200 \text{ A/S}$$

E.M.F. induced in coil Y,

$$e_2 = M \frac{dI_1}{dt} = 0.006 \times 1200 = \mathbf{7.2\ Volts}$$

Problem 1.41

Two identical coils of 1200 turns each are placed side by side such that 60 % of the flux produced by one coil links the other. A current of 10 A in the first coil sets up a flux of 0.12 mWb. If the current in the first coil changes from +10 A to −10 A in 20 m.sec., find (a) the Self-Inductances of the coils (b) the e.m.f's induced in both the coils.

Solution :

a) $$L_1 = L_2 = \frac{N_1 \phi_1}{I_1} = \frac{1200 \times (0.12 \times 10^{-3})}{10} = \mathbf{0.0144\ H} \quad \textbf{(Ans)}$$

b) $$e_1 = -L_1 \frac{dI_1}{dt} = -0.0144 \times \frac{[10-(-10)]}{20 \times 10^{-3}}$$

$$= \mathbf{-14.4\ V} \quad \textbf{(Ans)}$$

$$M = \frac{k_1 \phi_1 \times N_2}{I_1}$$

Here $k = 0.6$

$$\therefore \quad M = \frac{(0.6 \times 0.12 \times 10^{-3}) \times 1200}{10}$$

$$= 0.00864\ \text{H}$$

$$e_M = -M \frac{dI_1}{dt} = -0.00864 \times \frac{[10-(-10)]}{20 \times 10^{-3}}$$

$$= \mathbf{-8.64\ volts} \quad \textbf{(Ans)}$$

Problem 1.42

A cylinder, 50 mm in diameter and 1 m long is uniformly wound with 3000 turns in a single layer. A second layer of 100 turns of much finer wire is wound over the first one, near its centre. Calculate the mutual inductance between the coils.

Solution :

Given l = 1 m; N_1 = 3000 and N_2 = 100

Diameter = 50 mm = 50×10^{-3} m

$\therefore$ Cross-sectional area, $A = \frac{\pi}{4} d^2 = \frac{\pi}{4} (50 \times 10^{-3})^2\ \text{m}^2$

$$= \frac{25\pi}{4} \times 10^{-4}\ m^2$$

Let us take the cylinder material to be non-magnetic, so it is assumed $\mu_r = 1$.

$$\text{Now} \quad M = \frac{\mu_0 \mu_r A N_1 N_2}{l}$$

$$= \frac{(4\pi \times 10^{-7}) \times 1 \times 25\pi \times 10^{-4} \times 3000 \times 100}{(4 \times 1)}$$

$$= \mathbf{740.22\ \mu H}$$

Problem 1.43

If the mutual inductance between 2 coils is 0.2 H, calculate the e induced in one coil, when the current in the other coil is increased uniform rate from 0.5 to 3 A in 0.05 sec.

Solution :

Given $M = 0.2$ H

Rate of change of current, $\frac{dI}{dt}$, in the first coil

$$= \frac{(3-0.5)}{0.05} = 50 \text{ Amps}$$

∴ E.M.F induced in the second coil,

$$e_M = M\frac{dI}{dt} = 0.2 \times 50 = \mathbf{10\ Volts}$$

Problem 1.44

Two coils having 30 and 600 turns respectively are wound side by on an iron circuit of section 100 cm² and mean length 150 cm.

(a) **Estimate the mutual inductance between the coils, if the permeal of iron is 2000.**

(b) **A current in the first coil grows steadily from zero to 10 A in 0.01 Find the e.m.f induced in the other coil.**

Solution :

(a) Given $N_1 = 30$, $N_2 = 600$, $l = 150$ cm $= 1.5$ m

Cross-sectional Area, $A = 100\ cm^2 = (100 \times 10^{-4})\ m^2 = 10^{-2}\ m^2$

$\mu_r = 2.000$

We have $M = \frac{\mu_o \mu_r A N_1 N_2}{l}$

$$= \frac{(4\pi \times 10^{-7}) \times 2000 \times 10^{-2} \times 30 \times 600}{1.5}$$

or M = **0.302 H**

(b) Given, Rate of change of current in Coil-1,

$$\frac{dI_1}{dt} = \frac{(10-0)}{0.01} = 1000 \text{ A/sec}$$

$\therefore$ EMF induced in Coil-2, $E_M = M \frac{dI_1}{dt}$

$$= 0.302 \times 1000$$
$$= \mathbf{302 \text{ volts}}$$

Problem 1.45

If an e.m.f. of 5 V is induced in a coil when current in an adjacent coil varies at a rate of 80 A/s, what is the mutual inductance of the two coils ?

Solution : Given E_M = 5 V, $\frac{dI_1}{dt} = 80$ Amps/sec

Now, $E_M = M \frac{dI_1}{dt}$

or $M = \frac{E_M}{(dI_1/dt)} = \frac{5}{80}$ = 0.0625 H = **62.5 mH**

Problem 1.46

An air cored solenoid consists of 1500 turns of wire wound on a length of 60 cm. A search coil of 500 turns, enclosing a mean area of 20 cm², is placed centrally in the solenoid. Find (a) the Mutual Inductance of the arrangement and (b) the e.m.f. induced in the search coil, when the current in the solenoid is changing uniformly at the rate of 250 A/sec.

Solution :

$$M = \frac{\mu_o \mu_r A N_1 N_2}{l}$$

$$= \frac{(4\pi \times 10^{-7}) \times 1 \times (20 \times 10^{-4}) \times 1500 \times 500}{60 \times 10^{-2}}$$ [Note : μ_r = 1 for air]

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$$= 0.00314 \text{ H}$$

$$= \mathbf{3.14 \text{ mH}} \quad \text{(Ans)}$$

$$e_M = M\frac{dI_1}{dt} = 0.00314 \times 250$$

$$= \mathbf{0.785 \text{ V}} \quad \text{(Ans)}$$

Problem 1.47

Two identical coils *P* and *Q* having 1000 turns each lie in parallel planes such that 90 % of the flux produced by one coil links with the other. If a current of 4 A flowing in one coil produces a flux of 0.05 mWb in it, find the Mutual Inductance between the two coils. **[Mar 89, B.U.]**

Solution :

Given $N_1 = N_2 = 1000$ turns

$I_1 = 4$ A and $\phi_1 = 0.05$ mWb

$= 0.05 \times 10^{-3}$ Wb

90 % of the flux produced by one coil links the other coil,

$\therefore \quad k_1 = k_2 = 0.9 \qquad \left(\text{or } k = \sqrt{k_1 k_2} = 0.9\right)$

Mutual Inductance, $M = \dfrac{k_1 \phi_1 \times N_2}{I_1}$

$$\text{or} \quad M = \frac{\left(0.9 \times 0.05 \times 10^{-3}\right) \times 1000}{4}$$

$$= 0.01125 \text{ H}$$

$$= \mathbf{11.25 \text{ mH}}$$

Problem 1.48

'A' and 'B' are two coils, 'A' has 4000 turns and 'B' 3000 turns. When a current of 0.5 ampere flows in coil 'A', a flux of 100 μWb links coil 'A' and 60 % of this flux links coil 'B'. Find the Self-Inductance of coil 'A' and the Mutual Inductance.

Solution :

i) Self Inductance of coil 'A', $L_1 = \dfrac{N_1 \phi_1}{I_1}$

$$= \frac{4000 \times \left(100 \times 10^{-6}\right)}{0.5}$$

$$= \mathbf{0.8\ Henry\ \ (Ans)}$$

ii) Mutual Inductance, $M = k \ . \ N_2 \left(\frac{\phi_1}{I_1}\right)$

where $k = \frac{\phi_2}{\phi_1} = \frac{60}{100} = 0.6$

Thus, $M = \frac{0.6 \times 3000 \times \left(100 \times 10^{-6}\right)}{0.5}$

$$= \mathbf{0.36\ Henry\ \ (Ans)}$$

Problem 1.49

The self-inductance of a coil having 500 turns is 0.25 Henry. If 60 % of the flux is linked with a second coil of 10,000 turns, calculate i) Mutual Inductance of the two coils and ii) E.M.F. induced in the second coil when current in the first coil changes at the rate of 100 A/sec.

Solution :

Given $L_1 = 0.25$ H; $N_1 = 500$ turns

Now, $L_1 = \frac{N_1 \ \phi_1}{I_1}$ Henry

Flux/ampere in the first coil $\frac{\phi_1}{I_1} = \frac{L_1}{N_1} = \frac{0.25}{500} = 5 \times 10^{-4}$ Wb

Then the flux linking the second coil is 60 %

i.e., $k = 0.6$

i) Mutual Inductance $M = k . N_2 \frac{\phi_1}{I_1}$

or $M = 0.6 \times 10{,}000 \times 5 \times 10^{-4}$

$$= \mathbf{3\ Henry\ \ (Ans)}$$

ii) The e.m.f induced by Mutual Induction

$$e_M = M \frac{dI}{dt}$$

Given : $\frac{dI}{dt} = 100$ A/sec

$\therefore$ Substituting, $e_M = 3 \times 100$

$= \mathbf{300}$ **volts (Ans)**

Problem 1.50

Two coils having 30 and 600 turns respectively are wound side by side in a closed iron circuit of area of cross-section 100 sq.cm and mean length 200 cms. Estimate the mutual inductance between the coils if the relative permeability of the iron is 2000. If a current of zero ampere grows to 20 A in a time of 0.02 second in the first coil, find the e.m.f. induced in the second coil.

Solution :

Given : $N_1 = 30$

$N_2 = 600$

$A = 100 \text{ cm}^2$

$l = 200$ cm

$\mu_r = 2000$

$dI_1 = (20-0)\ A$

$dt = 0.02$ sec

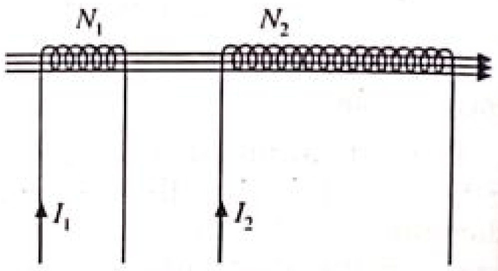


Fig. 1.54

Now, reluctance, $S = \frac{l}{\mu_o \mu_r A}$

$$= \frac{200 \times 10^{-2}}{\left(4\pi \times 10^{-7}\right) \times 2000 \times \left(100 \times 10^{-4}\right)}$$

$$= \frac{10^6}{4\pi} \text{ AT/Wb}$$

$$M = \frac{N_1 N_2}{S} = \frac{30 \times 600 \times 4\pi}{10^6} = \mathbf{0.226 \ H}$$

$$e_M = M \frac{dI_1}{dt} = 0.226 \times \frac{20}{0.02} = \mathbf{226 \ volts}$$

Problem 1.51

Two identical coils *A* and *B* lie in parallel planes. A current changing at the rate of 1500 Amperes/second in *A* induces an e.m.f. of 11.25 volts in *B*. Calculate the mutual inductance of the arrangement.

Solution :

Given : $\frac{dI_1}{dt} = 1500$ A/sec

$e_M = 11.25$ volts

Now, $e_M = M\frac{dI_1}{dt}$

or $11.25 = M \times 1500$

$\therefore$ $M = \frac{11.25}{1500} = \mathbf{7.5 \times 10^{-3}}$ **H**

1.29 Coefficient of Coupling

Let us take two magnetically coupled coils A and B, which have N_1 and N_2 turns respectively.

The Coefficient of Self-Induction of coil A, $L_1 = \frac{N_1^2}{\frac{l}{\mu_o \mu_r A}}$

The Coefficient of Self-Induction of coil B, $L_2 = \frac{N_2^2}{\frac{l}{\mu_o \mu_r A}}$

When current I_1 am peres flows in A, the flux produced,

$$\phi_1 = \frac{N_1 I_1}{\frac{l}{\mu_o \mu_r A}} \qquad \text{---(i)}$$

If a fraction k_1 of ϕ_1, *i.e.*, $k_1\, \phi_1$, is linked with coil B,

then $M = \frac{k_1 \phi_1 \times N_2}{I_1}$, where $k_1 \le 1$

Substituting the value of ϕ_1 as in expression (i),

$$M = k_1 \times \frac{N_1 N_2}{\frac{l}{\mu_o \mu_r A}} \qquad \text{---(ii)}$$

In a similar way, when a current of I_2 amperes flows in coil B, the flux produced in B is

$$\phi_2 = \frac{N_2\ I_2}{\frac{l}{\mu_o \mu_r A}}$$

Let a fraction k_2 of this flux, *i.e.*, $k_2\,\phi_2$ be linked with A.

$$\text{Then} \quad M = \frac{k_2\,\phi_2 \times N_1}{I_2} = k_2 \frac{N_1 N_2}{\frac{l}{\mu_o \mu_r A}} \qquad \text{---(iii)}$$

Multiplying Eqn.(ii) and (iii), we obtain

$$M^2 = k_1 k_2 \frac{N_1^2}{\frac{l}{\mu_o \mu_r A}} \times \frac{N_2^2}{\frac{l}{\mu_o \mu_r A}} = k_1\ k_2\ L_1\ L_2$$

If we substitute 'k' for $\sqrt{k_1 k_2}$, we have

$$M^2 = k^2\ L_1\ L_2$$

or

$$M = k\sqrt{L_1 L_2}$$

or

$$\boldsymbol{k} = \frac{M}{\sqrt{L_1 L_2}}$$

'k' is known as the Coefficient of Coupling, which shows the relation between Self and Mutual Inductance.

Problem 1.52

Two coils, A of 12,000 turns and B of 15,000 turns lie in parallel planes so that 45 % of the flux produced by Coil A links Coil B. A current of 5 A in A produces 0.05 mWb, while the same current in B produces 0.075 mWb. Calculate (1) Mutual Inductance, (2) The Co-efficient of Coupling.

Solution :

Given $N_1 = 12000,\ N_2 = 15{,}000$

k = Fraction of flux produced by coil A linking coil B = 45 %

(a) To find Mutual Inductance (M)

$$M = \frac{k\phi_1 \times N_2}{I_1} = \frac{\left(0.45 \times 0.05 \times 10^{-3}\right) \times 15000}{5}$$

$$= \mathbf{0.0675\ H\ or\ 67.5\ mH}$$

ϕ_1, ϕ_2

$\phi_{12} = 0.5$

(b) To find Coefficient of Coupling (k)

Self Inductance of Coil A, $L_1 = N_1 \dfrac{\phi_1}{I_1}$

$$\text{or} \quad L_1 = \frac{12{,}000 \times (0.05 \times 10^{-3})}{5} = 0.12 \text{ H}$$

Self Inductance of coil B, $\quad L_2 = N_2 \dfrac{\phi_2}{I_2}$

$$\text{or} \quad L_2 = \frac{15000 \times (0.075 \times 10^{-3})}{5}$$

$$= \mathbf{0.225 \ H}$$

$\therefore$ Coefficient of Coupling, $k = \dfrac{M}{\sqrt{L_1 L_2}}$

$$= \frac{0.0675}{\sqrt{(0.12 \times 0.225)}} = \mathbf{0.411}$$

Problem 1.53

Two coils *A* of 12000 turns and *B* of 18000 turns lie in parallel planes so that 60% of the flux produced in *A* links coil *B*. It is found that a current of 6 A in *A* produces a flux of 0.5 mWb while the same current in *B* produces 0.7 mWb. Determine

(i) Mutual Indutance and

(ii) Coefficient of coupling

Solution :

Given : $N_1 = 12000$

$N_2 = 18000$

$I_1 = 6\text{A}$

$I_2 = 6\text{A}$

$\phi_1 = 0.5 \text{ mWb}$

$\phi_2 = 0.7 \text{ mWb}$

$\phi_{12} = 0.6 \times \phi_1$

$= 0.6 \times 0.5 = 0.3 \text{ mWb}$

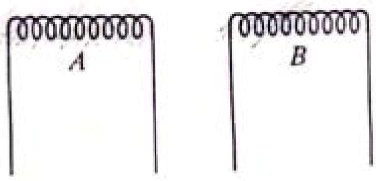


Fig. 1.55

$$M = \frac{N_1 \phi_{12}}{I_1} = \frac{12000 \times 0.3 \times 10^{-3}}{6}$$

$$= \mathbf{0.6 H}$$

$$L_1 = \frac{N_1 \phi_1}{I_1} = \frac{12000 \times 0.5 \times 10^{-3}}{6} = 1000 \times 10^{-3} \text{ H}$$

$$L_2 = \frac{N_2 \phi_2}{I_2} = \frac{18000 \times 0.7 \times 10^{-3}}{6} = 2100 \times 10^{-3} \text{ H}$$

Now $$K = \frac{M}{\sqrt{L_1 L_2}} = \frac{0.6}{\sqrt{(1000 \times 10^{-3})(2100 \times 10^{-3})}}$$ $k = \frac{0.6}{\sqrt{L_1 \cdot L_2}}$

or $K = \mathbf{0.41}$

Problem 1.54

Two 200-turn, air-cored solenoids, 25 cm long have a cross-sectional area of 3 cm² each. The Mutual Inductance between them is 0.5 μH. Find the Self-Inductance of the coils and the Coefficient of Coupling. (Jan 90, B.U.)

Solution :

Given $N_1 = N_2 = 200$ turns

$l = 25$ cm $= 0.25$ m

Cross -A = 3 cm² = 3×10^{-4} m² and

Mutual Inductance, $M = 0.5\ \mu\text{H} = 0.5 \times 10^{-6}$ H

Since the two coils are identical, having equal length and area, and the same number of turns, their self-inductances are also equal,

Self Inductance, $$L = \frac{\mu_o \mu_r A N^2}{l} \text{ Henry}$$

$$\therefore \quad L_1 = L_2 = \frac{(4 \times 10^{-7}) \times 1 \times (3 \times 10^{-4}) \times 200^2}{0.25}$$

$$= 6.0318 \times 10^{-5} \text{ Henry}$$

$$= \mathbf{60.0318\ \mu H}$$

Coefficient of Coupling, $$k = \frac{M}{\sqrt{L_1 L_2}} = \frac{M}{L_1} = \frac{M}{L_2} \qquad [\text{as } L_1 = L_2]$$

$$\therefore \quad k = \frac{0.5\ \mu H}{60.0318\ \mu H}$$

$$= 8.29 \times 10^{-3} \text{ or } \mathbf{0.00829}$$

Problem 1.55

Two coupled coils of Self-Inductances 0.8 H and 0.20 H, have a Co-efficient of Coupling 0.9. Find the Mutual Inductance and Turns Ratio.

Solution :

$$M = k\sqrt{L_1 L_2} = 0.9\sqrt{0.8 \times 0.2} = \mathbf{0.36\ H\ (Ans)}$$

$$M = N_2 \frac{k\phi_1}{I_1} \quad \text{---(i)}$$

Now, $$L_1 = \frac{N_1 \phi_1}{I_1}$$

or $$\frac{\phi_1}{I_1} = \frac{L_1}{N_1}$$

Substituting the above value in eqn.(i).

$$M = N_2 \frac{k\phi_1}{I_1} = N_2 k \cdot \frac{L_1}{N_1}$$

or $$0.36 = \frac{N_2 \times 0.9 \times L_1}{N_1}$$

$\therefore$ Turns ratio $$\frac{N_1}{N_2} = \frac{0.9 \times L_1}{0.36}$$

$$= \frac{0.9 \times 0.8}{0.36}$$

$$= \mathbf{2\ (Ans)}$$

Problem 1.56

The two windings of a transformer have inductances of 6 H and 0.06 H respectively with a Coefficient of Coupling k = 0.9. Find the e.m.f induced in both windings when the primary current increases at the rate of 100 amps/sec.

Solution :

i) **In the Primary,** the Self-Induced e.m.f $e_L = L\dfrac{dI_1}{dt}$ volts

Given L = 6 H and $\dfrac{dI_1}{dt} = 100$ A/sec

$\therefore \quad e_L = 6 \times 100 =$ **600 volts (Ans)**

ii) **The Coefficient of Mutual Inductance,** $M = k\sqrt{L_1 L_2}$

Given $k = 0.9$, $L_1 = 6$ H, $L_2 = 0.06$ H

$\therefore \quad M = 0.9\sqrt{6 \times 0.06} = \mathbf{0.54}$ **(Ans)**

iii) **The Secondary Mutually Induced e.m.f** $e_M = M\dfrac{dI_1}{dt}$ volts

Substituting, $e_M = 0.54 \times 100 =$ **54 volts (Ans)**

Problem 1.57

Two identical 750-turn coils A and B lie in parallel planes. A current changing at the rate of 1500 Amp/sec induces an e.m.f. of 11.25 volts in B. Calculate the mutual inductance of the arrangement. If the self-inductance of each coil is 15 mH, calculate the coefficient of coupling.

Solution :

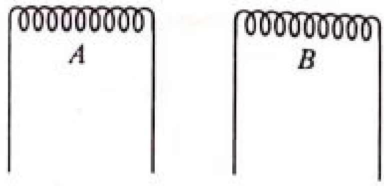


Fig. 1.56

Given : $N_A = N_B = 750$

$$\frac{dI}{dt} = 1500 \text{ A/sec}; \quad e_M = 11.25 \text{ volts}$$

$$L_A = L_B = 15 \text{ mH}$$

Now, $e_M = M\dfrac{dI}{dt}$

$$\therefore \quad M = e_M/(dI/dt) = \frac{11.25}{1500} = 7.5 \times 10^{-3} \text{ H}$$

Coefficient of Coupling,

$$K = \frac{M}{\sqrt{L_A L_B}} = \frac{7.5 \times 10^{-3}}{\sqrt{(15 \times 10^{-3}) \times (15 \times 10^{-3})}} = \mathbf{0.5}$$

1.30 Energy Stored in the Magnetic Field of an Inductor

In practice, an inductor has no resistance. Let us consider an electric circuit with an ideal inductor of inductance L. When the current increases from zero to a final steady value after a definite time lag, the self inductance opposes the growth of current. Some work has to be performed in increasing the current from zero to a maximum value against the induced e.m.f. This work done by the current *is stored up in the magnetic field as magnetic energy*.

When the current increases by dI amperes in dt seconds, the flux linkages of the coil increase, in which case the induced e.m.f is given by

$$L\frac{di}{dt} \text{ volts}$$

If the applied voltage is E, then it is equal to the induced e.m.f. Hence,

$$E = L\left(\frac{di}{dt}\right)$$

The work done in the small time interval dt seconds is given by

$$dw = E\,i\,dt \text{ Joules}$$

$$= \left[L\left(\frac{di}{dt}\right)\right] i.\,dt \qquad \text{---(i)}$$

or $$dw = L\,i\,di \text{ Joules}$$

From the instant the current is zero to its final steady state value of I amperes, the total work (or energy stored) is the summation of equation (i) over the interval O to I.

$$\therefore \quad W = \int_O^I dw = \int_O^I Li.di$$

$$W = \frac{1}{2} L I^2 \text{ Joules}$$

Energy stored in a magnetic field

This expression represents the energy stored in the magnetic field of the inductor.

We know that $L = \dfrac{\mu_o\, \mu_r\, AN^2}{l}$ henry (Refer Sec 1.26(ii) eqn (15))

Energy Density

Energy density is defiend as the energy stored per unit volume.

$$\text{i.e Energy Density} = \frac{1}{2} \times \left(\frac{\mu_o\, \mu_r\, AN^2}{l}\right) . I^2 \Big/ V$$

$$= \frac{1}{2} \times \left(\frac{\mu_o\, \mu_r\, AN^2 I^2}{l}\right) \Big/ Al \qquad \text{[Because Volume } V = A + l\text{]}$$

$$\text{or Energy Density} = \frac{1}{2} . \frac{\mu_o\, \mu_r\, N^2 I^2}{l^2} \qquad \text{---(ii)}$$

Now, Field Strength (or Field Intensity) H is given by $H = \dfrac{NI}{l}$.

Substituting in eqn (ii) above, we have

$$\text{Energy Density} = \frac{1}{2} \mu_o\, \mu_r\, H^2 \qquad \text{---(iii)}$$

Now $B = \mu H = \mu_o\, \mu_r\, H$

$$\therefore \quad \textbf{Energy Density} = \frac{1}{2} \text{BH} = \frac{1}{2} \frac{B^2}{\mu_o\, \mu_r}$$

Problem 1.58

Find the inductance of a coil of 200 turns wound on a paper core tube of 25 cm length and 5 cm radius. Also calculate energy stored in it if current rises from zero to 5 A (μ_r for paper = 1).

[B'lore Univ. Feb 1988]

Solution :

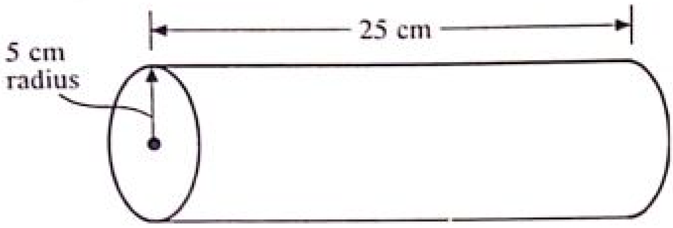


Fig. 1.57

$$\text{Area } A = \frac{\pi d^2}{4} = \frac{\pi}{4} \times (2 \times 0.05)^2 = 7.855 \times 10^{-3} \text{ m}^2$$

Now, reluctance, $S = \dfrac{l}{\mu_o \mu_r A}$

$$= \frac{0.25}{(4\pi \times 10^{-7}) \times 1 \times (7.855 \times 10^{-3})}$$

$$= 2.532 \times 10^7 \text{ AT/Wb}$$

Now, $L = \dfrac{N^2}{S}$ Henry [Refer Sec 1.26(ii) eqn (16)]

or $L = \dfrac{(200)^2}{2.532 \times 10^7}$

$$= 1.579 \times 10^{-3} \text{ Henry}$$

Energy stored in the coil,

$$W = \frac{1}{2} LI^2$$

$$= \frac{1}{2} \times (1.579 \times 10^{-3}) \times (5)^2$$

$$= \mathbf{0.01973 \text{ Joules}}$$

Problem 1.59

A coil consists of 750 turns and a current of 10 A in the coil gives rise to a magnetic flux of 1200 μWb. Calculate the e.m.f. induced and the energy stored when the current is reversed in 0.01 sec. [May 89, Gulbarga]

Solution :

Given $N = 750$, $I = 10$ A and $\phi = 1200\ \mu$Wb

$= 1200 \times 10^{-6}$ Wb

Also, rate of change of current,

$$\frac{dI}{dt} = \frac{10-(-10)}{0.01} = 2000 \text{ Amps/sec}$$

Now, Self Inductance, $L = \dfrac{N\phi}{I}$

$$= \frac{750\left(1200 \times 10^{-6}\right)}{10} = 0.09 \text{ H}$$

(a) Self-Induced e.m.f., $e_L = L\dfrac{dI}{dt}$

$$= 0.09 \times 2000 = \mathbf{180\ Volts}$$

(b) Energy stored, $W = \dfrac{1}{2}LI^2$

$$= \frac{1}{2} \times 0.09 \times 10^2 = \mathbf{4.5\ Joules}$$

Problem 1.60

An air-cored solenoid has a length of 50 cm and a diameter of 2 cm. Calculate its inductance if it has 1000 turns and also find the energy stored in it if the current rises from zero to 5 amps. (Feb 88, B.U.)

Solution :

No. of turns of coil, $N = 1000$

Length of solenoid: $l = 50$ cm $= 0.5$ m

Area of cross-section, $A = \dfrac{\pi}{4}\left(2 \times 10^{-2}\right)^2 = \dfrac{\pi}{10000}$ m^2

Relative permeability, $\mu_r = 1$

$\therefore$ Inductance of solenoid, $L = \dfrac{N^2 A \mu_o \mu_r}{l} = \dfrac{1000 \times 1000 \times \frac{\pi}{10000} \times \left(4\pi \times 10^{-7}\right) \times 1}{0.5}$

$$= \mathbf{0.0007\ H}$$

Energy stored in magnetic field $= \dfrac{1}{2}LI^2$

$$= \frac{1}{2} \times 0.0007 \times (5)^2 = \mathbf{0.00875\ J}$$

Problem 1.61

A solenoid 1 m in length and 10 cm in diameter has 5000 turns. Calcula (a) the Inductance (b) the energy stored in the magnetic field, when a curre of 2 A flows in the solenoid.

Solution :

(a) Given l = 1 m; N = 5000

$$A = \left(\frac{\pi}{4}\right) d^2 = \left(\frac{\pi}{4}\right) \times \left(10 \times 10^{-2}\right)^2 \text{ m}^2$$

$$= 0.007854 \text{ m}^2$$

Now, Self Inductance of the solenoid,

$$L = \frac{\mu_o \mu_r A N^2}{l} = \frac{\left(4\pi \times 10^{-7}\right) \times 1 \times 0.007854 \times (5000)^2}{1}$$

or L = **0.246 Henry** $[\mu_r = 1$ for air$]$

(b) Energy stored in the magnetic field, $W = \frac{1}{2} L I^2$ Joules

Given I = 2 A

$$\therefore \quad W = \frac{1}{2} \times 0.246 \times 2^2 = \mathbf{0.492 \text{ Joules}}$$

1.31 Review Questions

Q1. Derive an expression for the Energy Stored in a Magnetic Field. (Dec 81, May/June 86, June 89, B.U.)

Q2. Explain clearly the difference between Self and Mutual Inductance. (Aug 82, Sep/Oct 87, B.U.)

Q3. Explain the terms Magnetic Flux, Magnetomotive Force and Reluctance. Bring out the relation between them. (Dec 83, B.U.)

Q4. What is meant by (a) Self Inductance (b) Mutual Inductance. Define the unit in which each is measured. (Nov/Dec 84)

Q5. Explain the following terms :
(a) Absolute Permeability
(b) Relative Permeability (c) Reluctance (Oct 85, B.U.)

Q6. Define "Inductance". (June 86, B.U.)

Q7. Write a brief note on "Self and Mutual Inductances". (Jun/Jul 90, B.U.)

Q8. State and explain Fleming's Right Hand Rule.
(Mar/Apr 88, M.U.; Mar 89, M.U.)

Q9. Define the following terms :
i) M.M.F. (Mar/Apr 88, M.U.; Aug/Sep 89, M.U.)
ii) Magnetic Field Intensity. (Mar/Apr 88, M.U.)
iii) Flux Density (Aug/Sep 89, M.U.)
iv) Reluctance (Aug/Sep 89, M.U.)

Q10. Define 'Self-Inductance' of a coil and derive an expression for it.
(Mar/Apr 88, M.U.; Aug/Sep 90)

Q11. State and explain Lenz's Law.
(Apr/May 87, M.U.; Mar/Apr 88, M.U.; Aug/Sep 89 M.U.)

1.32 Exercises - Problems

1. Two coils having 100 and 200 turns respectively are wound side by side on a closed iron circuit of section 10 sq.cm and mean length 200 cm. Calculate the Coefficients of Self-Induction of the two coils and th e Mutual Induction between the two. Neglect leakage. Take $\mu_r = 2000$. If a current steadily grows from 0 to 1 A in 0.01 sec in the first coil, find the e.m.f. induced in the other coil. [KUD 1985]

Answer : 12.57 mH, 0.05H, 0.0251H, 2.5 V

2. At what rate is the current varying in a circuit having an inductance of 50 mH, if an e.m.f. of 16 V is induced ? [Gulbarga May 1989]

Answer : 320 A/sec

3. A field magnet coil wound with 2000 turns of wire produces a flux of 2.8 mWb, when carrying a current of 4 Amps. Estimate the inductance of the coil in Henrys.

[KUD 1988]

Answer : 1.4 Henry

4. An air-cored solenoid has a length of 50 cm and diameter of 2 cm. Calculate its inductance, if it has 1000 turns and also find the energy stored in it if the current rises from zero to 5 A.

[Bangalore Feb 88]

Answer : 0.79 mH, 9.875 mJoules

MODULE 2

(a) D.C. Machines

2.1 Introduction

DC machines are electrical machines which deal with conversion of one form of energy to another. A d.c. machine which converts mechanical energy into electrical energy is called a **d.c. generator**; while a d.c. machine which converts electrical energy into mechanical energy is known as a **d.c. motor**. Hence a d.c. generator can be used as a d.c. motor and vice-versa.

2.2 Working Principle of D.C. Machine as a Generator

Whenever a conductor is moved in a magnetic field such that it cuts across lines of flux, dynamically induced e.m.f. is produced in it according to *Faraday's Laws of Electromagnetic Induction*. The magnitude of this induced e.m.f. in the conductor is given by the equation,

$$e = B\,l\,v \sin\theta$$

where l = length of the portion of the conductor within the magnetic field, v = velocity of the conductor, B = magnetic flux density and θ = angle between direction of movement of the conductor and the direction of magnetic flux.

This e.m.f. causes a current to flow in the conductor if the circuit is closed. Thus, electrical power develops in the conductor. If the conductor does not move or if it is moved parallel to the lines of flux, no e.m.f. is induced in it, and hence no power is generated. Hence it is clear that, for the *generation of e.m.f. there should be relative motion between the conductor and the magnetic field*.

Hence, a generating action has the following requirements :

(i) The conductor (or coil), (ii) The flux, (iii) The relative motion between the conductor and the flux. In a practical generator, the conductors are rotated to cut the magnetic flux, keeping the flux stationary. In order to obtain a large voltage as the output, several conductors are joined together in a particular manner, to form a winding. Such a winding is known as the *armature winding of a d.c. machine*. The part on which this winding is placed is called the *armature* of a d.c. machine. The conductors situated on the armature are rotated by some external device called a *prime mover*. Some of prime movers used are steam engines, diesel engines, water turbines etc. The magnetic field is produced by a current-carrying winding known as *field winding*. The direction of the induced e.m.f. may be obtained by using *Fleming's Right Hand Rule*, as given below.

Fleming's Right Hand Rule (see Fig 1.47 Chap 1)

If three fingers of a right hand, namely the thumb, index finger and middle finger are outstretched so that every one of them is at right angles with the other two, and if in this position, the index finger points in the direction of lines of flux, the thumb in the direction of the relative motion of the conductor with respect to the flux, then *the outstretched middle finger gives the direction of the e.m.f induced in the conductor*. This Rule mainly gives the direction of current set up by the e.m.f. induced in the conductor when a closed path is provided to it.

2.3 Working Principle of D.C. Machine as a Motor

Its operation is based on the principle that when a current carrying conductor is placed in a magnetic field, the conductor experiences a mechanical force. The direction of this force is given by **Fleming's Left Hand Rule** which is as follows:

1. Hold the thumb, first finger and second finger of the left hand in such a way that they are at right angles at each other (see Fig. 1.48 Chap. 1).
2. If the forefinger represents the direction of the field and the second finger the direction of the current, then the direction of the force is indicated by the thumb.

The magnitude of this force is given by

$$F = B\,I\,l \text{ newtons}$$

where B = field strength in teslas (Wb/m^2)

I = current flowing through the conductor (Amps)

l = length of the conductor in metres

In a practical d.c. motor, the field winding produces the required magnetic field; the current carrying armature conductors are placed in this magnetic field and so experience a force. As conductors are placed in slots which are on the periphery, the individual force experienced by the conductors acts like a twisting or turning force on the armature which is called a **torque**. The torque is the product of the force and the radius at which this force acts. We shall now consider motoring action is some detail.

Consider a d.c. motor having North and South Poles, represented by N and S as shown in Fig. 2.1. Here, conductors are placed uniformly in the slots of the armature.

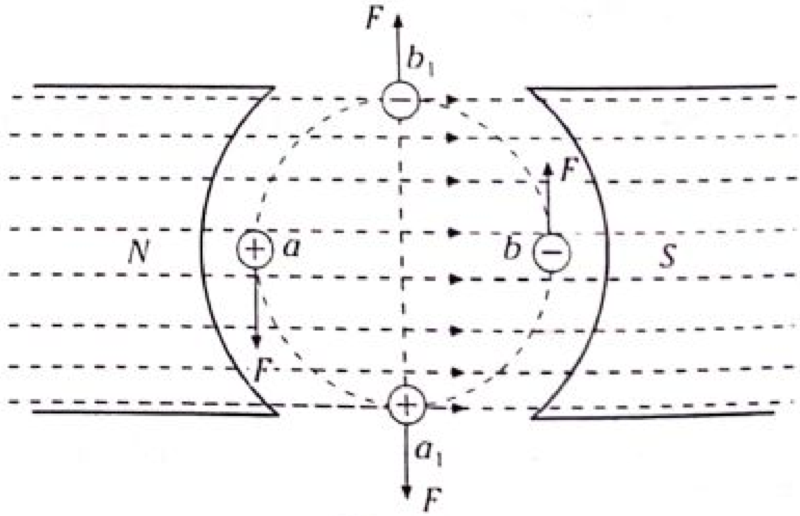


Fig. 2.1

For the purpose of explaining the principle of working of a d c motor, only two conductors a and b, which come under the influence of the North and South pole respectively, are considered. These two conductors are joined together by an end connection at the rear end of the armature, and to the commutator segments at the front end of the armature. When a d.c. supply is made available at the motor terminals, current passes through the conductors a and b via the commutator. The +ve sign marked on conductor a, shows that the current is flowing inwards and the –ve sign marked on conductor b shows that the current is flowing outwards. Horizontal dotted lines indicate the lines of magnetic force which originate from the North Pole N and terminate on the South Pole S as shown in Fig. 2.1.

As per Fleming's Left Hand Rule, the conductor a is subjected to a force F acting in the downward direction and the conductor b experiences an equal Force F acting in the upward direction. As the two conductors are connected together the two equal and opposite forces F acting on them constitute a couple, which rotates the armature in the anticlockwise direction through an angle of 90° and the conductor a and b occupy the positions a_1 and b_1 respectively.

In this new position, these conductors experience a force F in opposite directions along the *same line*, and hence the torque experienced by them is zero. Had the armature contained just these two conductors, the armature would have stopped in the position a_1, b_1. However, as the armature has several other conductors, which are uniformly distributed in the slots of the armature and which are interconnected, they experience a torque in the anticlockwise direction.

As it is necessary that the armature experiences a continuous anticlockwise torque, the direction of currents in the conductors a and b must be reversed as soon as they cross the positions a_1 and b_1 respectively. Otherwise the armature would experience a pulsating torque in the clockwise direction in the position a_1, b_1. This reversal of current in conductor a and b, after they cross the positions a_1 and b_1 respectively is brought about by the commutator, thus *making the armature experience a continuous anticlockwise torque, resulting in continuous rotation of the armature in the anticlockwise direction*.

2.4 Constructional Features of a D.C. Machine (together with functions of various parts)

Fig. 2.2 gives the cross-sectional view, showing the various parts of a 4-pole, practical d.c. generator. It consists of the following parts :

1. Field system : The object of the field system is to create a uniform magnetic field, within which the armature rotates. Electromagnets are generally preferred in comparison with permanent magnets because they are cheap, small in size, produce greater magnetic effect and the field strength can be easily varied by changing the magnetising current.

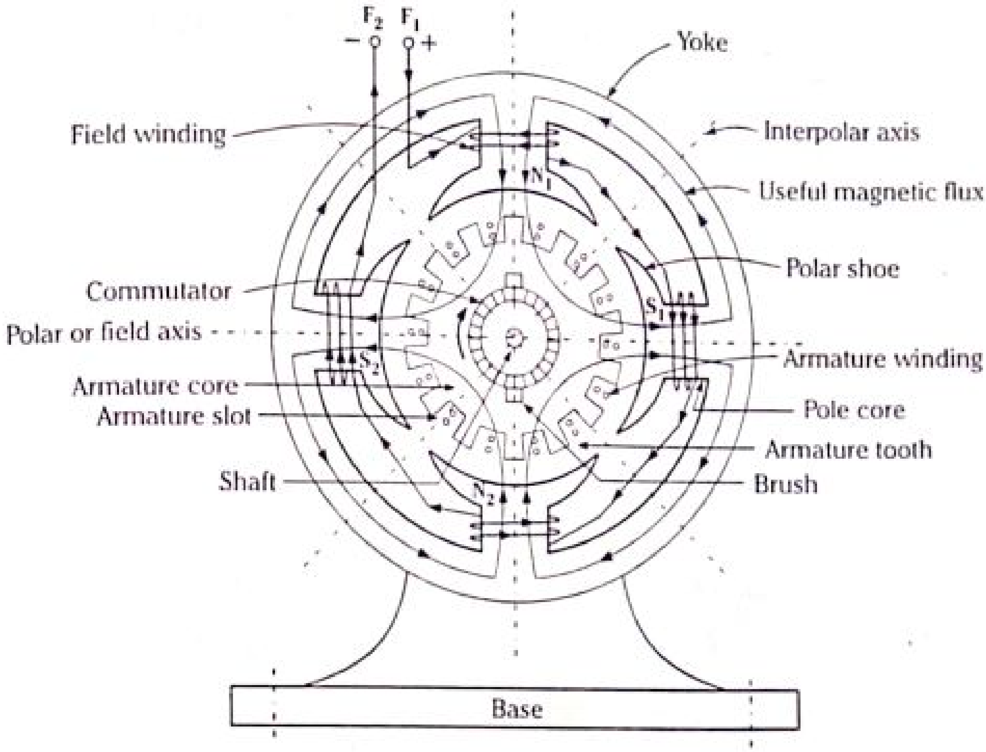


Fig. 2.2 Cross-section of a practical d.c. generator

The field system consists of the following parts :

(i) Yoke (ii) Pole cores (iii) Pole shoes (iv) Field coils

(i) Yoke : A cylindrical yoke is normally used which serves as a frame of the machine and has a twin function. Firstly, it provides mechanical support to the poles and protects the d.c. machine from harmful atmospheric elements like dust, moisture and gases like SO_2, acidic fumes, etc. Secondly, it offers a path of low reluctance to the magnetic flux produced by the poles. Such a low reluctance path is necessary so as to avoid wastage of power for providing the same flux. A large current and hence more power would have been necessary if the path had high reluctance, to produce the same flux.

For small machines, the yoke is made of cast iron. But for large machines, rolled steel, cast steel, or silicon steel is used as they have better magnetic properties (like higher permeability) than cast iron.

(ii) Pole Cores : Usually pole cores are of circular section. They are made of cast steel or wrought iron laminations which are rivetted together and bolted to the yoke. Each pole core carries coils of insulated copper wire through which the magnetising current flows.

(iii) Pole Shoes : The pole shoes are at the end of the poles and have the following functions :

(a) They support the field coils.

(b) They spread out the flux in the air gap.

(c) They are of large cross-section, due to which the reluctance of the path is reduced.

iv) **Field Coils :** These are coils of copper wire wound round the poles ; they are also called exciting coils. The object of these field coils or magnetising coils is to provide, under various conditions of operation, the number of ampere-turns of excitation required to give the proper flux through the armature, so as to induce the desired potential difference. All the field coils are connected in such a way that, when current flows through them, the adjacent poles have opposite polarities.

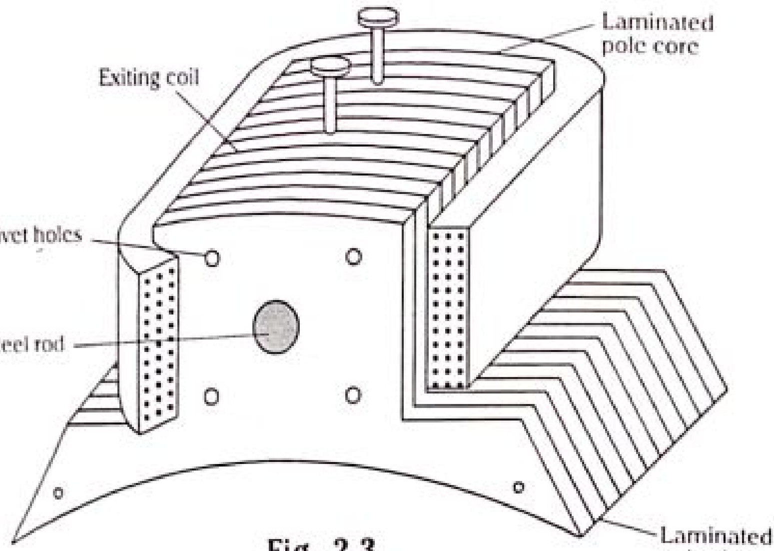


Fig. 2.3

There are different types of field construction, depending on the type of excitation. In a shunt field, many turns of fine wire are used, whereas in a series field a few turns of large cross-section are used; in a compound field, both shunt and series windings are used. Shunt coils are usually wound with double cotton-covered wires. The field coils after proper winding, are dipped in an insulating varnish and baked in an oven, which results in stiffness, rnechanical strength and good insulating properties to the windings.

While designing a generator, the number of poles required by the field structure depends on the speed of the armature, and the output for which the machine is designed. In a two-pole machine there are two voltage maximums per revolution of the armature, while in a four-pole machine there are four voltage maximums for one revolution. If the armature speed is kept constant, the number of poles determine the rate at which the individual coils cut the magnetic flux. Thus, the output voltage increases with the increase in the number of poles for a constant armature speed. A typical laminated pole core, pole shoe and exciting coil are shown in Fig. 2.3.

2. Armature Core and Armature Windings : The purpose of the armature core is to house the armature conductors and to provide a path of very low reluctance to the flux through the armature from a *N*-pole to a *S*-pole. The armature is mounted on a shaft so that when it is rotated, the conductors housed in it cut the magnetic flux and electric current is induced in it.

The armature core is made from high permeability silicon-steel stampings about 0.4 - 0.6 mm thick, each stamping being separated from its neighbouring one by thin paper or varnish as insulation.

High-grade steel is used for two reasons : firstly, to keep hysteresis loss low, which is due to cyclic change of magnetisation caused by the rotation of the core in the magnetic field; secondly, to reduce the eddy currents in the core which are induced by the rotation of the core in the magnetic field. By using stampings or laminations the path of the eddy currents is cut into several units. The laminations should be in such a direction that they are perpendicular to the paths of eddy currents and parallel to the flux.

Ventilating ducts are provided to dissipate the heat produced by hysteresis, eddy current losses *etc.* Air is drawn through these ducts by the fanning action of the armature.

A small air gap exists between the pole pieces and the armature, taking care to see that there is no rubbing in the generator. This gap should be kept very small so as to keep the field strength at a maximum.

In armatures of small diameters, holes are punched in the centre of the laminations for the shaft. However, in the large machines, a spider is provided, so that the angular laminations are rigidly fastened to the shaft. The latter construction permits air to flow freely between the radial arms to keep the armature ventilated and cooled.

Slots are stamped on the periphery of the laminations, in which conductors are placed. These armature slots, which are lined with tough insulating material, provide mechanical security to the armature winding and also give a shorter air-gap for the magnetic flux to cross between the pole face and the armature. Each slot has two circular conductors, insulated from each other. Conductors are the active part of the armature winding, which cut the flux and generate an e.m..f.

The dotted lines in Fig. 2.2 (only two per pole are shown for the sake of simplicity) represents the distribution of useful magnetic flux which leaves N_1 and divides, half going towards S_1 and half towards S_2. In a like manner, the flux leaving N_2 divides equally between S_1 and S_2.

Let us now see what happens when the armature revolves in a clockwise direction, as shown by the curved arrow in Fig. 2.2. As per Fleming's Right Hand rule, the e.m.f. generated in the conductors is directed towards the paper in those moving under the N poles and outwards from the paper in those moving under the S-poles. Taking the air-gap to be of uniform length, the e.m.f. generated in a conductor remains constant while it is moving under a pole face, and then quickly reduces to zero when the conductor is midway between the adjacent pole shoes.

Fig. 2.4 shows the variation of the e.m.f. generated in a conductor as it moves through two pole pitches (a pole pitch is the distance between the centres of two adjacent poles.) Thus, at instant A, the conductor is halfway between pole shoes

of say, N_1 and S_1 and BC depicts the e.m.f. generated during the movement of the conductor under the pole face of S_1, the e.m.f. being assumed positive when its direction is towards the paper in Fig. 2.2. At instant D, the conductor is halfway between the pole shoes of S_1 and N_2, and the next part $DEFG$ relates to the variation of e.m.f. as the conductor traverses the next pole pitch.

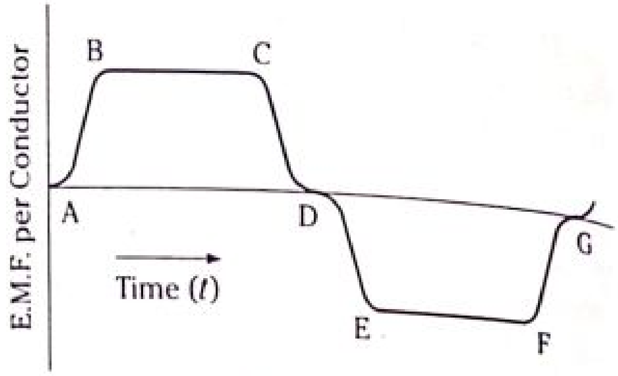


Fig. 2.4

Assuming constant speed, this variation during the interval AG in Fig. 2.4 is repeated indefinitely.

3. Commutator : A d.c. machine is required to produce a voltage that remains constant in direction and magnitude. A commutator converts the alternating e.m.f generated in the rotating armature conductors into a steady or direct voltage.

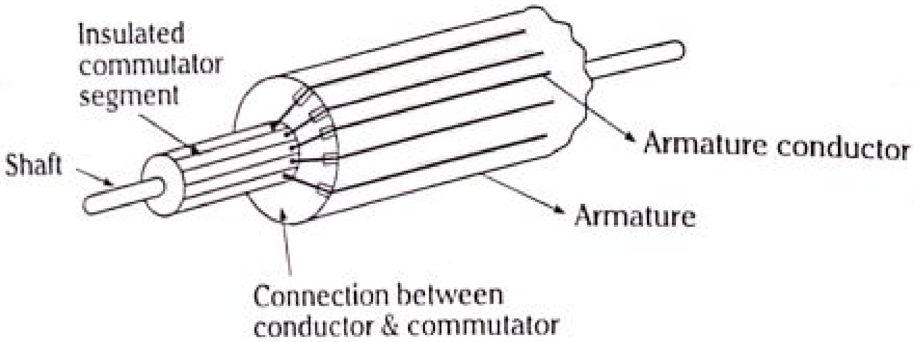


Fig. 2.5 Commutator

It is of cylindrical construction and is made up of wedge-shaped segments of high-conductivity, hard-drawn copper. These segments are insulated from each other by thin layers of mica (Fig. 2.5). The number of segments is equal to the number of armature coils. Each commutator segment is connected to the armature conductor by means of a copper lug or strip. These segments have V-grooves so as to prevent them from flying out due to centrifugal forces. These grooves are insulated by conical micanite rings.

4. Brushes and Brush-gear : The function of brushes is to collect current from the commutator. They are normally made of carbon and are in the shape of a rectangular block. These brushes are housed in brush-holders, which are usually of the box-type variety. The brush-holder is mounted on a spindle and the brushes can slide in the rectangular box open at both the ends. The brushes are made to press down on the commutator by means of a spring, whose tension can be adjusted by operating a lever. A flexible copper pigtail, mounted at the top of the brush, carries the current from the brushes to the holder. The number of brushes per spindle depends on the magnitude of the current to be collected from the commutator.

5. Bearings : Ball-bearings are usually used as they are more reliable. However, for heavy duty work, roller bearings are preferred. The ball and rollers are packed in hard oil for quieter operation and for minimizing wear of the bearings.

2.5 Armature Windings

We shall first discuss the meaning of the following terms used in respect of armature winding before we discuss the different types of windings.

2.5.1 Pole-pitch

This is the distance between two adjacent generator poles, which is also the total distance measured along the periphery of the armature divided by the number of poles.

It may also be defined as the number of armature conductors or armature slots per pole. If there are 64 conductors and 4 poles, the pole pitch is

$$= \frac{64}{4} = 16$$

2.5.2 Conductor

This is a length of wire, which when placed in a magnetic field, has an e.m.f induced in it. In Fig. 2.6 (a), *AB* or *CD* is a conductor.

2.5.3 Coil

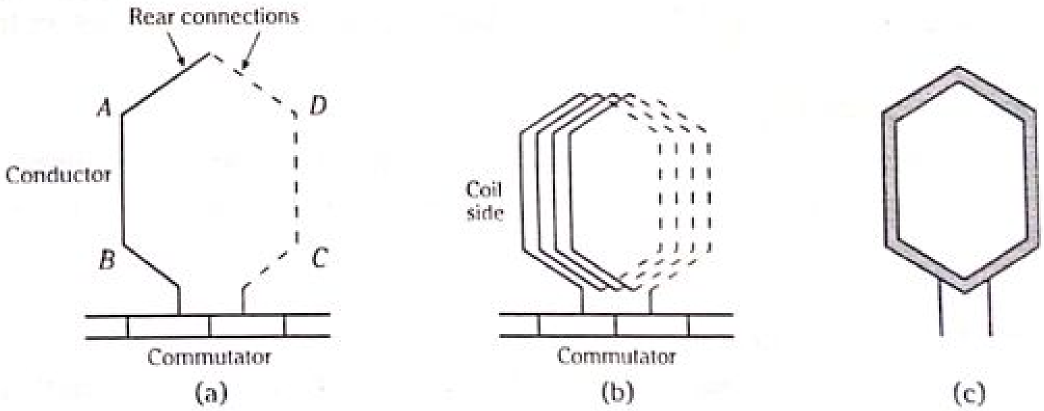


Fig. 2.6

Again referring to Fig. 2.6 (a), a coil (or winding element) is made up of the two conductors *AB* and *CD*, together with their end connections. A coil could be a single-turn coil, as in Fig. 2.6 (a), or a multiturn coil [Fig. 2.6 (b)]. The latter could have many conductors per coil side. In Fig. 2.6 (b) we depict 4 conductors per coil side. These conductors are wrapped in tape as in Fig. 2.6 (c) and placed in the armature slot. We observe that the beginning and end of the every coil is connected to a commutator bar.

2.5.4 Coil Pitch (Y_s)

The Coil Pitch or Coil Span is the distance, in terms of armature slots (or armature conductors) between two sides of a coil, measured along the periphery of the armature.

If the coil-pitch is equal to the pole-pitch, the winding is said to be *full-pitched*. This implies that the coil pitch is 180° electrical and the coil sides come under opposite poles, and so the e.m.f induced in the coil is the sum of the e.m.fs induced in the two coil sides. As an example, suppose there are 40 slots and 4 poles, then the coil pitch (or coil span) will be $\frac{40}{4} = 10$ slots.

If the coil span is less than the pole pitch, then we have a *fractional* pitched winding. Here, there is a phase difference between the e.m.f.s induced in the 2 coil sides; the total e.m.f will be vector sum of the e.m.f's in the two coil sides, and will be less than in the previous instance.

2.5.5 Pitch of a Winding (Y)

This is the distance around the armature between two successive conductors which are connected together. It is obvious that this is the distance between the starting points of two consecutive turns.

$$Y = Y_b - Y_f \qquad \text{for lap winding}$$
$$= Y_b + Y_f \qquad \text{for wave winding}$$

This is depicted in Figs. 2.7 and 2.8 of Lap and Wave Windings respectively.

2.5.6 Front Pitch (Y_f)

This is the length of the front (or commutator) end of the armature connection, measured in terms of the number of armature conductors. (Refer Fig. 2.7 and 2.8).

2.5.7 Back Pitch (Y_b)

This is the distance, measured in terms of armature conductors, that a coil advances at the back (opposite side of the commutator) of the armature (Fig. 2.7 and 2.8).

2.5.8 Resultant Pitch (Y_r)

This is the distance between the start of one coil and the start of the next one, to which it is connected (Fig. 2.7 and 2.8).

2.5.9 Commutator Pitch (Y_c)

This is the distance, measured in terms of commutator bars or segments between the segments to which the two ends of a coil are connected. Thus, again referring to Fig. 2.7 and 2.8, for a lap winding, Y_c is the difference between Y_b and Y_f, while in the case of wave winding it is the sum of Y_b and Y_f.

2.5.10 Lap and Wave Windings

The following two basic types of armature winding are most commonly used for drum type armature :

1. Lap Winding
2. Wave Winding

The difference between these two is in the arrangement of the end connections at the front or commutator end of the armature. However, the following rules are common to both types of windings :

a) The coils are wound into a closed circuit. While starting from a particular point, all the conductors should be traversed by the current before reaching the same point again without any discontinuity in between.

b) The windings are usually required to be full-pitched *i.e.*, the front pitch and the back pitch should be each almost equal to the pole-pitch. This will ensure enhanced e.m.f in the coils.

c) Both front pitch and back pitch should be odd, or else the coils cannot be readily placed on the armature. For example, if Y_b and Y_f are both even, then all the coil sides and conductors would lie either in the upper half of the slots or in the lower half. This will make it impossible for one side of the coil to lie in the upper half of one slot and the other side of the same coil to lie in the lower half of some other slot.

d) As the front ends of the conductors are joined to the segments in pairs, the number of commutator segments will be equal to the number of slots or coils (*i.e.*, half the number of conductors).

2.5.11 Lap Winding

We shall discuss the lap winding of the simplex type *i.e.*, using only single-turn coils, as shown in Fig. 2.7. In lap winding, the finishing end of one coil is connected to a commutator segment and to the starting end of the next coil coming under the same pole, and so on, till all the coils are connected. The salient features of lap-winding are as given below :

a) The front and back pitches are odd and have opposite signs. However, they cannot be equal. They differ by 2 or by a multiple of it.

b) The front and back pitch should be almost equal to the pole pitch.

c) The average pitch $Y_a = \frac{Y_b + Y_f}{2}$

d) Commutator Pitch $Y_c = \pm 1$

e) As the front-pitch (Y_f) and the back-pitch (Y_b) are odd, resultant pitch (Y_r) is even, as it is the arithmetical difference of two odd numbers.

f) In the case of a 2-layer winding, the number of slots is equal to the number of coils. The number of commutator segments is also the same.

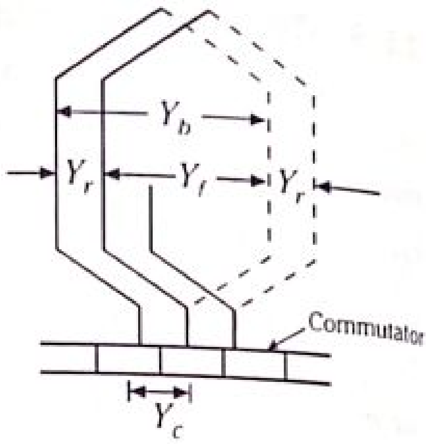


Fig. 2.7 Lap winding

g) The number of parallel paths in the armature = mP, where 'm' is the multiplicity of the winding and 'P' is the number of poles.

With reference to point (a) above, the difference between Y_b and Y_f should be 2 or a multiple of it; so $Y_b = Y_f \pm 2$. We have two cases :

Case 1 : $Y_b > Y_f$: This means that $Y_b = Y_f + 2$, and we have a right-handed or progressive winding *i.e.*, a winding which proceeds in a clockwise direction as seen from the commutator end. Thus $Y_c = +1$

$$\text{So,} \quad Y_f = \frac{Z}{P} - 1 \quad \text{and} \quad Y_b = \frac{Z}{P} + 1$$

where Z is the number of conductors

Case 2 : $Y_b < Y_f$: This means that $Y_b = Y_f - 2$, and we have a left handed or retrogressive winding *i.e.*, a winding which progresses in an anticlockwise direction as viewed form the commutator end. $Y_c = -1$.

$$\text{So,} \quad Y_f = \frac{Z}{P} + 1 \quad \text{and} \quad Y_b = \frac{Z}{P} - 1$$

It is apparent that, in either of the above cases, $\frac{Z}{P}$ must be even, to enable winding to be done.

2.5.12 Wave Winding

As shown in Fig. 2.8, conductor *AB*, which is under a *N*-pole, is connected to *CD* lying under *S*-pole, and then to *EF* lying under the next *N*-pole. Thus, the winding proceeds consecutively under every *N*-pole and *S*-pole (alternately) till it gets back to a conductor *YZ* lying under the original *N*-pole. As this winding progresses in one direction around the armature in a sequence of 'waves'. It is called 'wave winding'. We will deal with Simplex Wave Winding.

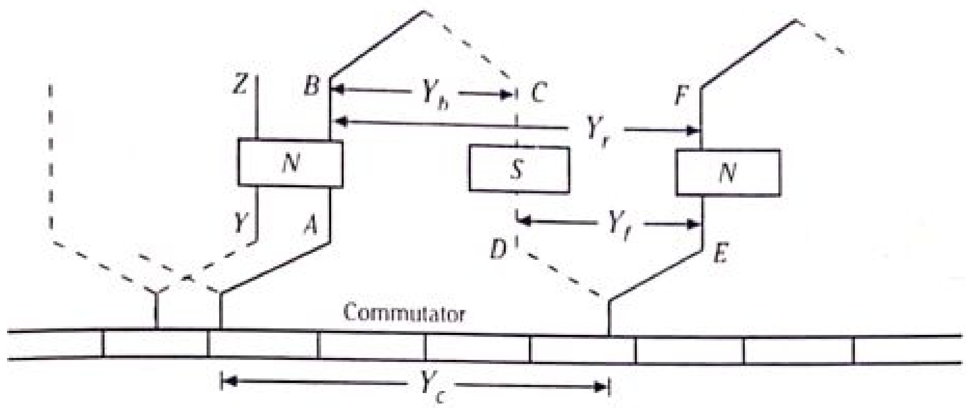


Fig. 2.8 Wave winding

If, after going around the armature once, the winding occupies a slot to the left of the starting point (*YZ* in Fig. 2.8), then the winding is known as *retrogressive*. If, on the other hand, it occupies one slot to the right, then it is *progressive*.

Suppose we have a 2-layer winding, and if conductor *AB* lies in the upper half of the slot, then passing once around the armature, the winding terminates at *YZ*, which must be at the upper half of the slot at the left or right. This implies that *AB* and *YZ* differ by two conductors (though they differ by one slot).

The salient features of a wave-winding are given below :

a) Both pitches Y_b and Y_f are odd and are of the same sign.

b) Y_b and Y_f are almost equal to the pole pitch and may be equal or differ by 2, under which condition they are respectively one more or one less than the average pitch.

c) The resultant pitch $Y_r = Y_f + Y_b$

d) Average pitch, $Y_a = \dfrac{Y_b + Y_f}{2}$

If Z = total no. of conductors or coil sides, then,

$$Y_a \times P = Z \pm 2 \quad \text{or} \quad Y_a = \frac{Z \pm 2}{P}$$

As P is always even and $Z = PY_a \pm 2$, 'Z' must always be even, which means that $\dfrac{Z \pm 2}{P}$ is an even integer.

The plus sign is used for a progressive winding and a minus sign for a retrogressive winding.

e) Commutator pitch, $Y_c = Y_a$

also, $Y_c = \dfrac{\text{No. of commutator bars} \pm 1}{\text{No. of pairs of poles}}$

f) We have seen in (d) above that

$$Y_a = \frac{Z \pm 2}{P} = \frac{\frac{Z}{2} \pm 1}{\frac{P}{2}} = \frac{\text{No. of commutator bars} \pm 1}{\text{No. of pairs of poles}}$$

g) Rearranging the above expression,

$Z = PY_a \pm 2$

$\therefore$ No. of coils, $N_c = \frac{Z}{2} = \frac{PY_a \pm 2}{2}$

h) The no. of armature parallel paths = $2\,m$.

2.6 D.C. Generator on Load

The symbolic representation of a D.C. generator with its armature and field windings is shown in Fig. 2.9. The field winding is connected to a D.C. Voltage source of voltage V_f, due to which a constant current of I_f amperes flows through the field winding. A magnetic flux ϕ is produced by the field winding. When the armature is rotated by means of a prime mover, the armature conductors cut the magnetic flux and so an e.m.f. E_g is generated. When a load resistance R_L is connected across the terminals of the generator, a load current I_L flows through it. V is the terminal voltage of the D.C. Generator.

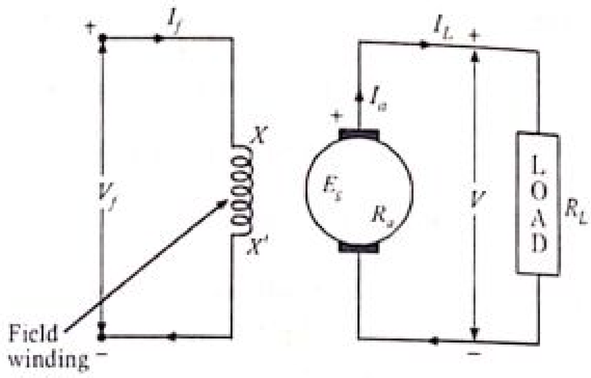


Fig. 2.9

The terminal voltage of the D.C. Generator will be slightly less than the generated voltage due to the following reasons :

(a) When a D.C. Generator is loaded, a load current I_L flows through the load, and an armature current I_a flows through the armature conductors. These conductors possess a small resistance R_a known as armature resistance, due to which a small voltage drop $I_a R_a$ occurs due to the current flowing through the armature conductors.

(b) When current flows through the armature conductors, the armature sets up its own flux, called armature flux. This armature flux has two effects on the main flux. It opposes the main flux and also distorts it. As a result, the main flux gets reduced and, in turn, the e.m.f. induced in the DC Generator also gets reduced. *This is known as armature reaction.* The reduction in the value of the generated voltage due to armature reaction is considered as a voltage drop due to armature reaction.

(c) The current flowing through the armature conductors reaches the load through the brushes which are placed over the rotating commutator. The contact between the commutator and the brushes has some resistance known as the brush contact resistance. Due to this resistance too, there is some voltage drop called the *voltage drop due to brush contact resistance*. This drop is expressed as volts per brush. The total voltage drop in a D.C. Generator due to brush contact resistance is twice the voltage drop per brush, as the generator has only two brushes.

∴ Generated e.m.f. = Terminal Voltage + Armature Resistance Drop + Armature Reaction Drop + Brush Contact Resistance Drop.

or $$E_g = V + I_a R_a + Ard + Bcd$$

or $$V = E_g - I_a R_a - Ard - Bcd$$

2.7 Types of D.C. Generators

D.C. Generators are generally classified according to the nature of excitation provided to the field windings. On this basis, D.C. generators can be broadly classified into the following two types :

(a) Separately excited D.C. generator

(b) Self excited D.C. generator

The self excited D.C. generator is further classified as

(i) D.C. shunt generator

(ii) D.C. series generator and

(iii) D.C. compound generator.

The D.C. compound generator is further classified as cumulatively compounded and differentially compounded generators. Depending on the way in which the field windings are connected to the armature, the *cumulatively compounded* and *differentially compounded generators* are further classified as long shunt and short shunt generators.

2.8 Separately Excited D.C. Generator

When the field coil is excited by an independent external source of D.C. voltage V_f, as shown in Fig. 2.10, the generator is called a *separately excited d.c. generator*. This voltage drives a current I_f through the field winding due to which a magnetic flux is produced. When the armature is rotated by a prime mover, the armature conductors cut the magnetic flux and hence an E.M.F. E_g is induced, which

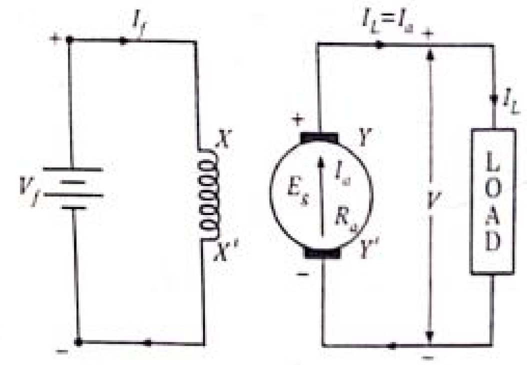


Fig. 2.10

is the generated voltage. When the load is connected across the armature terminals, current I_L flows through the load.

If V is the terminal voltage of the D.C. generator, then

Armature current $I_a = I_L$ Amps

and $V = E_g - I_a R_a - Ard - Bcd$ volts

X and X' represent the +ve and –ve terminals of the shunt field winding, Y and Y' represent the +ve and –ve terminals of the armature respectively.

Power developed, $P = E_g I_a$ Watts.

Power delivered to the load $= E_g I_a - I_a^2 R$

$$= I_a \left(E_g - I_a R_a\right) \text{ Watts.}$$

2.9 Self-excited Generator

When the field coils are excited by current supplied from the output of the generator itself, such a machine is known as a *self excited generator*. Due to residual magnetism, some flux is always present in the poles. With the rotation of the armature, some e.m.f., and consequently some induced current is produced, which is partly or fully passed through the field coils, thereby strengthening the magnetic field at the poles. The increased flux, in turn, generates a greater e.m.f., which again increases the current through the field coils, and so on, until the generator achieves its normal field strength. The different types of self-excited generators mentioned earlier are discussed below :

2.9.1 D.C. Shunt Generator

In this type of generator, the field windings are connected across or connected in parallel with the armature winding, so that the full voltage of the generator is applied across them.

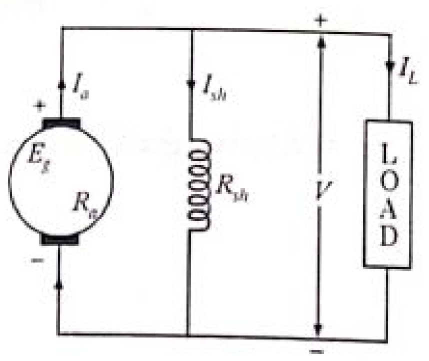


Fig. 2.11

The shunt field winding has several turns of fine wire having high resistance. So, only a part of the armature current flows through the field windings, the remaining current flowing through the load. Fig. 2.11 shows the connection diagram of this type of generator.

Armature current, $I_a = I_L + I_{sh}$

$$I_{sh} = \frac{V}{R_{sh}} \quad \text{where } R_{sh} = \text{Resistance of the field winding}$$

Terminal voltage $V = E_g - I_a R_a - Ard - Bcd$

Power develeloped, P = $E_g I_a$

Power given to the load = $V I_L$

2.9.2 D.C. Series Generator

Fig. 2.12 represents a D.C. Series Generator. This is called a series generator because the field winding is connected in series with the armature conductors. The voltage drop across this winding has to be very small, and hence its reistance has to be very less. Therefore, it is made of a few thick turns of copper. Whatever current flows through the load, the same current flows through the armature and the field windings.

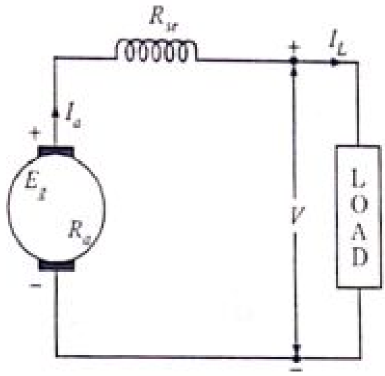


Fig. 2.12

Armature current, $I_a = I_{se} = I_L = I$ (say)

where I_{se} = current through the series field winding

Terminal voltage $V = E_g - I(R_a + R_{se}) - Ard - Bcd$

Power developed, $P = E_g I_a$

Power delivered to the load $= E_g I_a - I^2(R_a + R_{se})$

$$= I\left[E_g - I(R_a + R_{se})\right]$$

2.9.3 D.C. Compound Generator

A D.C. Compound Generator contains both shunt field winding and series field winding. If the two field windings are connected in such a way that the fluxes produced by them are in the same direction and are additive, then the generator is said to be *cumulatively compounded*. If the field windings are connected in such a way that the fluxes produced by them are in opposite direction, and the resultant flux is the difference between the two, then the generator is said to be *differentially compounded*.

Depending on how the series field winding is connected to the shunt field winding, we have *long shunt compound generator* and *short shunt compound generator*.

(a) Cumulatively Compounded D.C. Generator

(i) Long Shunt

Fig. 2.13 represents a long shunt cumulatively compounded D.C. generator.

Current I_a enters the +ve terminal of the series field winding and current I_{sh} enters the +ve terminal of the shunt field winding. Hence the fluxes produced by them will have the same direction and they are additive.

The total flux is given by,

$$\phi = \phi_{sh} + \phi_{se}$$

where ϕ = total flux

ϕ_{sh} = flux produced by the shunt field winding

ϕ_{se} = flux produced by the series field winding

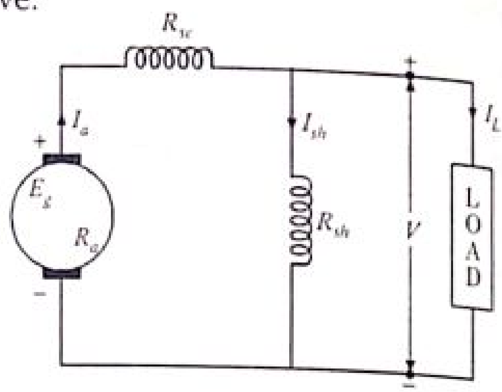


Fig. 2.13

Series field current, $I_{se} = I_a = I_{sh} + I_L$

Shunt field current, $I_{sh} = \dfrac{V}{R_{sh}}$

Terminal voltage $V = E_g - I_a(R_a + R_{se}) - Ard - Bcd$

Power developed, $P = E_g I_a$

Power delivered to load $= V I_L$

(ii) Short shunt

Fig. 2.14 shows a short shunt cumulatively compounded generator, where the shunt field winding is in parallel with the armature. For this generator,

Series field current, $I_{se} = I_L$

Shunt field current, $I_{sh} = \dfrac{V + I_L R_{se}}{R_{sh}}$

$$I_a = I_L + I_{sh}$$

Terminal voltage

$$V = E_g - I_a R_a - I_L R_{se} - Ard - Bcd$$

Power developed, $P = E_g I_a$

Power delivered to the load $= V I_L$

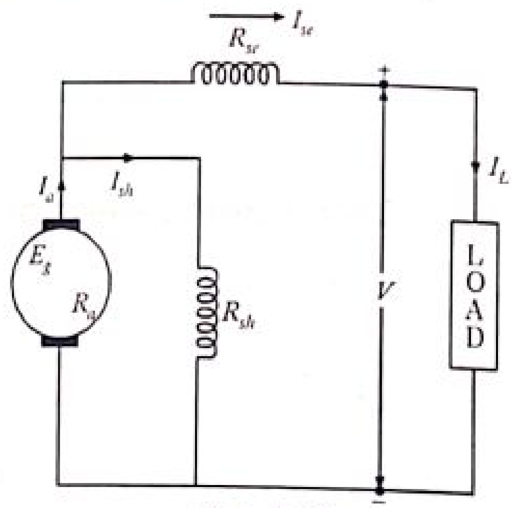


Fig. 2.14

(b) Differentially compounded D.C. Generator

(i) Long shunt

Fig. 2.15 shows a long shunt differentially compounded D.C. Generator. Current

I_a enters the negative terminal of the series winding and current I_{sh} enters the positive terminal of the shunt field winding. Hence the fluxes produced by them are in opposite directions. Hence, the resultant flux is given by

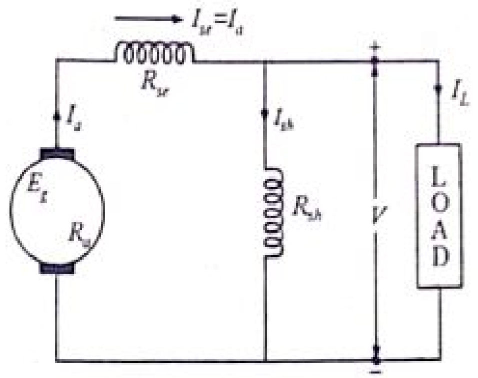


Fig. 2.15

$$\phi = \phi_{sh} - \phi_{se}$$

Shunt field current $\quad I_{sh} = \dfrac{V}{R_{sh}}$

Series field current, $\quad I_{se} = I_a = I_{sh} + I_L$

and terminal voltage $\quad V = E_g - I_a(R_a + R_{se}) - Ard - Bcd$

Power developed, $\quad P = E_g I_a$

Power delivered to load $= V I_L$

(ii) Short Shunt

Fig. 2.16 shows a short shunt differentially compounded D.C. compound generator. For this generator,

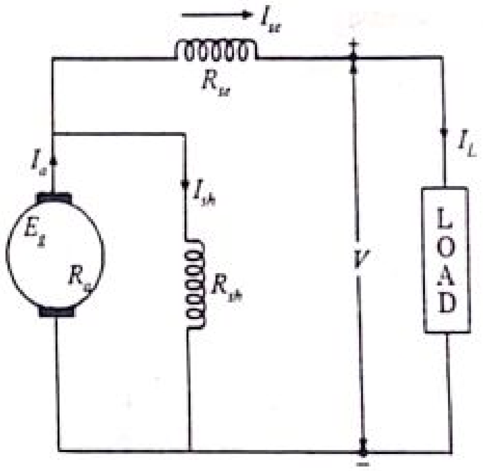


Fig. 2.16

$$I_{sh} = \frac{V + I_L R_{se}}{R_{sh}}$$

$$I_a = I_L + I_{sh}$$

and $\quad V = E_g - I_a R_a - I_L R_{se} - Ard - Bcd$

Power developed, $P = E_g I_a$

Power delivered to load $= V I_L$

2.10 E.M.F. Equation of D.C. Generator

Let ϕ = Flux/pole in webers

Z = Total number armature conductors or coil sides on armature

= No. of slots × No. of conductors/slot

P = No. of generator poles.

A = No. of parallel paths in the armature

N = Rotational speed of armature in revolutions per minute (r.p.m.)

E = e.m.f. induced in any parallel path in armature.

Generated e.m.f E_g = e.m.f. generated in any of the parallel paths. *i.e.*, E.

Average e.m.f. generated/conductor $= \dfrac{d\phi}{dt}$ volts (because $n = 1$)

During one revolution of armature in a P-pole generator, each armature conductor cuts the magnetic flux P times.

$\therefore$ Flux cut by one conductor in one revolution = ϕP webers.

Flux cut by each conductor per second = Flux cut by the conductor per revolution $\times$ no. of revolutions of armature per second.

$$= \phi P \times \frac{N}{60} \text{ webers}$$

As per Faraday's Laws of Electromagnetic Induction, E.M.F. generated per conductor $= \dfrac{d\phi}{dt} = \dfrac{\phi PN}{60}$ volts.

The number of conductors in series between a positive brush and a negative brush is equal to the total number of conductors divided by the number of parallel paths, or

No. of armature conductors per parallel path $= \dfrac{Z}{A}$

$\therefore$ The generated e.m.f, E_g (between the terminals)

= e.m.f generated per conductor $\times$ No. of conductors in each parallel path.

OR

$$E_g = \frac{\phi PN}{60} \times \frac{Z}{A} \text{ volts} \qquad \text{---(i)}$$

For a Simplex Wave-Wound Generator

No. of parallel paths, $A = 2$.

$\therefore$ Eqn (i) becomes $E_g = \dfrac{\phi PN}{60} \times \dfrac{Z}{2} = \dfrac{\phi ZPN}{120}$ volts

For Simplex Lap-Wound Generator

No. of parallel paths, $A = P$.

$\therefore$ Eqn (i) becomes $E_g = \dfrac{\phi PN}{60} \times \dfrac{Z}{P} = \dfrac{\phi ZN}{60}$ volts

So re-writing equation (i), the general equation for e.m.f generated is

$$E_g = \frac{\phi Z N}{60} \times \left(\frac{P}{A}\right) \text{ volts} \qquad \text{---(ii)}$$

$A = 2$, for simplex wave-winding
$= P$, for simplex lap-winding

Also putting eqn (ii) in another form,

$$E_g = \frac{1}{2\pi}\left(\frac{2\pi N}{60}\right)\phi Z\left(\frac{P}{A}\right) \text{ volts}$$

$$= \frac{\omega\,\phi\,Z}{2\pi}\left(\frac{P}{A}\right) \text{volts}$$

where ω is in rad/sec.

2.11 Simple Problems on D.C. Generators including those on E.M.F. Equation

Problem 2.1

In a given d.c. machine, if $P = 8$, $Z = 400$, $N = 300$ RPM and $\phi = 100$ milliweber, calculate E_g with winding (i) lap-connected (ii) wave-connected.
(Dec. 81, B.U.)

Solution :

$$E_g = \frac{\phi Z N}{60} \times \left(\frac{P}{A}\right) \text{volts}$$

For a simplex lap winding, the no. of parallel paths, $A = P$, and for a wave winding, $A = 2$.

$\therefore$ *For Lap-connected d.c. machine,*

$$E_g = \frac{\left(100 \times 10^{-3}\right) \times 400 \times 300}{60} \times \left(\frac{8}{8}\right) = \mathbf{200 \text{ volts}}$$

$\therefore$ *For Wave-connected machine*

$$E_g = \frac{\left(100 \times 10^{-3}\right) \times 400 \times 300}{60} \times \left(\frac{8}{2}\right) = \mathbf{800 \text{ volts}}$$

Problem 2.2

A 4-pole generator with wave wound armature has 51 slots, each having 24 conductors. The flux per pole is 0.01 Weber. At what speed must the armature rotate to give an induced e.m.f of 220 V ? What will be the voltage

developed if the winding is lap and the armature rotates at the same speed ? (Jan 93, B.U.)

Solution :

a) **Wave-Wound Armature**

$$\text{Speed } N = \frac{E_g \times 60 \times A}{\phi Z P}$$

Here No. of parallel paths $A = 2$

$$\therefore \quad N = \frac{220 \times 60 \times 2}{0.01 \times (51 \times 24) \times 4} = \mathbf{540 \ R.P.M}$$

b) **Lap-Wound Armature**

$$E_g = \frac{\phi Z N}{60} \times \left(\frac{P}{A}\right) \text{ volts}$$

Here, number of parallel paths, $A = P = 4$

Speed $N = 540$ R.P.M.

$$\therefore \quad \text{Voltage developed, } E_g = \frac{0.01 \times (51 \times 24) \times 540}{60} \times \frac{4}{4} = \mathbf{110 \ V}$$

Problem 2.3

An 8-pole lap-connected armature has 960 conductors, a flux of 40 m Wb per pole and a speed of 400 RPM. Calculate the e.m.f. generated. If the armature were wave-connected, at what speed must it be driven to generate 400 V ?

Solution :

a) **Lap-connected Armature**

$$E_g = \frac{\phi Z N}{60} \times \left(\frac{P}{A}\right) \text{volts} \qquad \text{---(i)}$$

Here, number of parallel paths, $A = P = 8$

$\phi = 40 \text{ m Wb} = 40 \times 10^{-3}$ Wb

$N = 400$; No. of conductors, $Z = 960$

$$\therefore \quad \text{E.M.F. generated, } E_g = \frac{(40 \times 10^{-3}) \times 960 \times 400}{60} \times \left(\frac{8}{8}\right) = \mathbf{250 \ V}$$

b) **Wave-connected Armature**

Eqn (i) above can be re-written as

$$N = \frac{E_g \times 60 \times A}{\phi Z P}$$

Here, No. of parallel paths = 2; E_g = 400 V

$$\therefore \quad N = \frac{400 \times 60 \times 2}{\left(40 \times 10^{-3}\right) \times 960 \times 8} = \mathbf{156\ R.P.M.}$$

Problem 2.4

An 8-pole lap-connected armature, driven at 400 RPM is required to generate 250 V. The useful flux per pole is 0.05 Wb. If the armature has 150 slots, calculate a suitable number of conductors per slot.

Solution :

As the winding is lap-wound, the number of parallel paths = No. of poles,

i.e., $A = P = 8$

Given : E_g = 250 and N = 400 RPM

$$E_g = \frac{\phi Z N}{60} \times \left(\frac{P}{A}\right)$$

Thus $$250 = \frac{0.05 \times Z \times 400}{60} \times \left(\frac{8}{8}\right)$$

or $Z = 750$

$$\therefore \quad \text{No. of conductors per slot} = \frac{750}{150} = 5$$

Problem 2.5

A 110-V d.c. shunt generator delivers a load current of 50 amps. The armature resistance is 0.2 ohm and the field circuit resistance is 55 ohms. The generator, rotating at a speed of 1800 r.p.m. has 6 poles lap wound and has a total of 360 conductors. Calculate

i) the no-load voltage in the armature

ii) the flux per pole. **(Feb/Mar 83, B.U.)**

Solution :

Load current, I = 50 A

$$I_{sh} = \frac{110}{55} = 2\text{ A}$$

$$I_a = I + I_{sh} = 50 + 2 = 52\text{ A}$$

$$I_a R_a = 52 \times 0.2 = 10.4\text{ V}$$

$$\therefore \quad E_g = V + I_a R_a$$

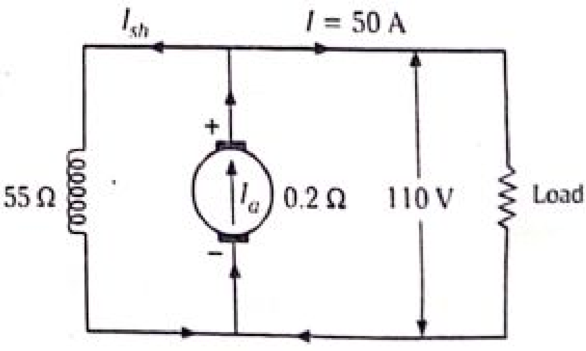


Fig. 2.17

$$= 110 + 10.4$$

$$= \mathbf{120.4\ V}$$

$$E_g = \frac{\phi Z N}{60} \times \left(\frac{P}{A}\right) \text{volts}$$

Given $P = 6$; $N = 1800$; $Z = 360$

For lap winding, $A = P = 6$

$$\therefore \quad 120.4 = \frac{\phi \times 360 \times 1800}{60} \times \left(\frac{6}{6}\right)$$

$$\therefore \quad \phi = \frac{120.4 \times 60}{360 \times 1800} = \mathbf{0.011\ Wb}$$

Problem 2.6

A 4-pole lap-wound 750 r.p.m. shunt generator has an armature resistance of 0.4 ohm and field resistance of 200 ohms respectively. The armature has 720 conductors and the flux per pole is 3×10^{-2} weber. If the load resistance is 10 ohms, determine the terminal voltage. (Apr. 85, B.U.)

Solution :

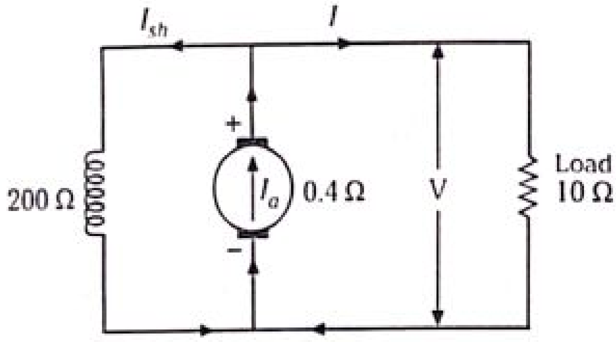


Fig. 2.18

$$E_g = \frac{\phi Z N}{60} \times \left(\frac{P}{A}\right) \text{volts}$$

$$= \frac{\left(3 \times 10^{-2}\right) \times 720 \times 750}{60} \times \left(\frac{4}{4}\right) \text{volts} = 270 \text{ V}$$

As seen from Fig. 2.18, the shunt and load resistances are in parallel with each other.

$\therefore$ Their combined resistance $= \dfrac{10 \times 200}{210} = 9.52\ \Omega$

Total circuit resistance $= R_a + 9.52$

$$= 0.4 + 9.52$$
$$= 9.92\ \Omega$$

$\therefore$ Armature current, $I_a = \dfrac{270}{9.92} = \mathbf{27.2\ A}$

Armature drop $= 27.2 \times 0.4$
$= 10.88$ volts

$\therefore$ Terminal voltage $= 270 - 10.88$
$= \mathbf{259.12\ V}$

Problem 2.7

A shunt generator supplies a load of 10 kW at 200 V through a pair of feeders of total resistance 0.05 Ω. Armature resistance is 0.1 ohm. Shunt field resistance is 100 ohms. Find the terminal voltage and generated e.m.f. of the generator. (Aug/Sep 99, VTU)

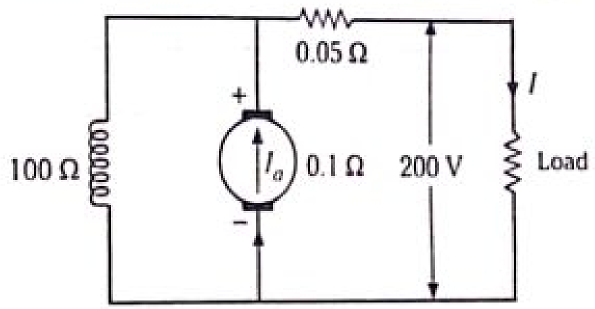


Fig. 2.19

Solution :

Load Current, $I = \dfrac{10,000}{200} = 50\ A$

Drop in the feeders $= 0.05 \times 50 = 2.5\ V$

$\therefore$ Terminal Voltage, $V = 200 + 2.5 = \mathbf{202.5\ V}$ (Ans)

$$I_{sh} = \frac{202.5}{R_{sh}} = \frac{202.5}{100} = 2.025\ A$$

$$I_a = I + I_{sh} = 50 + 2.025 = 52.025\ A$$

Generated e.m.f

$$E_g = V + I_a R_a$$
$$= 202.5 + (52.025 \times 0.1)$$
$$= \mathbf{207.7\ V}\ \text{(Ans)}$$

Problem 2.8

A four-pole lap wound shunt generator delivers 200 amperes at terminal voltage of 250 volts. It has a field and armature resistance of 50 ohms and 0.05 ohm respectively. Neglecting brush drop determine

i) armature current (ii) the current per armature parallel path
iii) e.m.f. generated (iv) power developed. (Mar/Apr 88, M.U.)

Solution :

The generator circuit is given in Fig. 2.20.

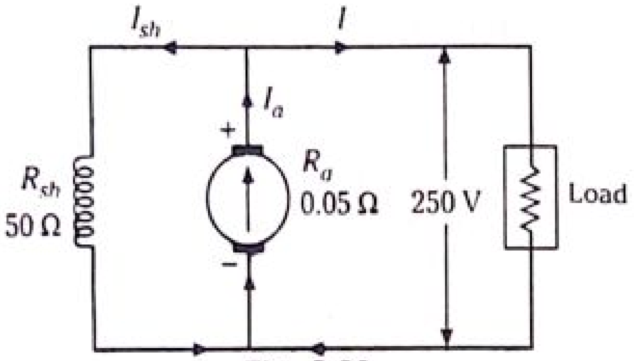


Fig. 2.20

Current through the shunt field winding is :

$$I_{sh} = \frac{250}{50} = 5 \text{ A}$$

Load current, I = 200 A

i) Armature current, $I_a = I + I_{sh}$

$$= 200 + 5 = \mathbf{205\ A}$$

In a lap-wound generator, the number of parallel paths = the number of poles (*i.e.*, A = No. of poles = 4).

ii) Current per armature parallel path is

$$\frac{I_a}{A} = \frac{205}{4} = \mathbf{51.25\ A}$$

Armature voltage drop

$$I_a R_a = 205 \times 0.05 = \mathbf{10.25\ A}$$

iii) E.M.F. generated, E_g = terminal voltage + armature voltage drop.

$$= 250 + 10.25$$
$$= \mathbf{260.25\ V}$$

iv) Power developed, $P = E_g I_a$

$$= 260.25 \times 205$$
$$= 53351 \text{ W} = \mathbf{53.35\ kW}$$

Problem 2.9

A 4-pole, 100 V shunt generator with lap-connected armature, having field and armature resistance of 50 Ω and 0.1 Ω respectively, supplies sixty, 100 V, 40 W lamps. Calculate the total armature current, the current per path and the generated e.m.f. Allow a contact drop of 1 volt per brush.

(Aug/Sep 89, M.U.,)

Solution :

Total load = 60 × 40 = 2400 W

V = voltage across load = 100 V

∴ Current through load, $I = \frac{2400}{100} = 24$ A

$$I_{sh} = \frac{V}{R_{sh}} = \frac{100}{50} = 2 \text{ A}$$

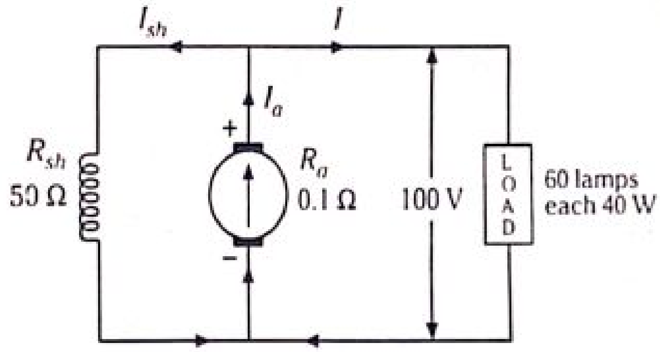


Fig. 2.21

i) Armature current, $I_a = I + I_{sh}$

$= 24 + 2 = \mathbf{26\ A}$

No. of paths in a lap winding = No. of poles

i.e., $A = P = 4$

ii) Current per path $= \frac{I_a}{A} = \frac{26}{4} = \mathbf{6.5\ A}$

Armature drop, $I_a R_a = 26 \times 0.1 = 2.6$ V

Brush drop = 2 × 1 = 2V

iii) Generated e.m.f., $E_g = V + I_a R_a$ + brush drop

$= 100 + 2.6 + 2$

$= \mathbf{104.6\ V}$

D.C. Motor

2.12 Back E.M.F. in a D.C. Motor

As soon as the armature of a D.C. Shunt Motor (described in Sec. 2.13) starts rotating, dynamically induced e.m.f. is produced in the armature conductors. The direction of this induced e.m.f as found by Fleming's Right-Hand Rule, is such that it opposes the applied voltage (Fig. 2.22). This induced e.m.f is known as ***Back e.m.f.*** E_b. It has the same value as that of the motionally induced e.m.f. in the generator.

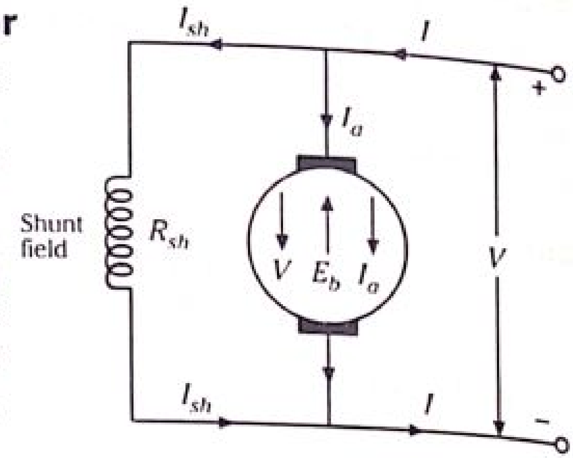


Fig. 2.22

So, $E_b = \frac{\phi Z N}{60} \times \frac{P}{A}$ volts, where N is in RPM

The applied voltage V has to force current through the armature against this back e.m.f. The electrical work performed in over-coming this opposition is converted into mechanical energy developed in the armature.

2.12.1 Value of Back e.m.f. ; Voltage & Current Relations

The back e.m.f. E_b is always less than the applied voltage V, although the difference is small when the motor is running under normal conditions.

The net voltage across the armature circuit = $V - E_b$

If armature resistance is R_a,

$$\text{Armature current, } I_a = \frac{\text{Net voltage in armature circuit}}{\text{Armature Resistance}}$$

$$\text{or} \quad I_a = \frac{V - E_b}{R_a}$$

Since V and R_a are usually fixed, the value of E_b will determine the armature current drawn by the motor. If the speed of the motor is high, $E_b = \left(\frac{\phi Z N}{60} \times \frac{P}{A}\right)$ is large and hence the motor will draw less armature current and vice-versa.

2.12.2 Significance of Back-e.m.f.

Due to the presence of back e.m.f., the d.c. motor becomes a self-regulating machine *i.e.*, the motor is made to draw as much armature current as is just sufficient to develop the torque required by the load.

When the motor runs on no load, a small amount of torque is required to

overcome the friction and windage losses. So, a small amount of armature current flows and the back e.m.f is almost equal to the applied voltage.

If load is suddenly brought on to the motor, the first effect is to slow down the armature. Therefore, the speed at which the armature conductors move through the field is reduced and so there is a fall in the back e.m.f. E_b. The decreased e.m.f. allows a larger current to flow through the armature, and a larger current means an increased driving torque. Thus, the driving torque increases as the speed of the motor reduces, and the motor will stop slowing down when the armature current is just sufficient to produce the reduced torque required by the load.

Taking another case, when the load on the motors is decreased, the driving torque is momentarily in excess of the requirement, so that the armature is accelerated. As the armature speed increases, the back e.m.f. E_b also increases and causes the armature current I_a to decrease. The motor will stop accelerating when the armature current is just sufficient to produce the reduced torque required by the load.

So, we conclude that back *e.m.f. in a d.c. motor regulates the flow of armature current i.e., it automatically alters the armature current to meet the load requirement.*

2.13 Types of D.C. Motors and their Representation

Depending upon how the field winding is placed in relation to the armature, d.c. motors are of three types; **Shunt Motors**, **Series Motors** and **Compound Motors**.

(i) D.C. Shunt Motors

Fig. 2.23 symbolically represents a D.C. Shunt Motor, where

V = applied voltage

E_b = back e.m.f.

I_a = armature current

R_a = armature circuit resistance

Voltage V has to

a) overcome the back e.m.f E_b and

b) supply the armature ohmic drop $I_a R_a$.

Thus, *voltage equation* of a motor is given by

$$V = E_b + I_a R_a \qquad \text{---(i)}$$

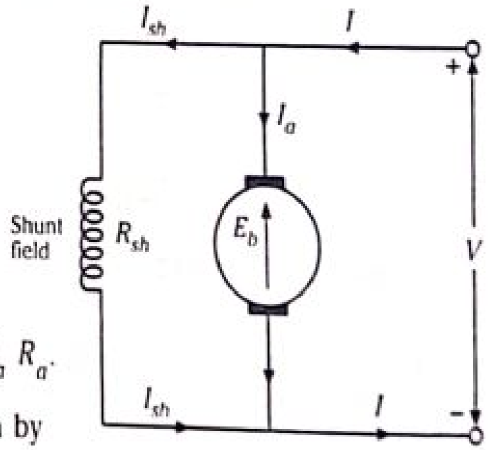


Fig. 2.23

Since the emf E_b generated in the armature of the motor is in opposition to the applied voltage V, it is called back e.m.f., which is explained in detail in the earlier Sec. 2.12.

Multiplying both L.H.S. and R.H.S. of eqn(i) by I_a, we obtain

$$V I_a = E_b I_a + I_a^2 R_a$$

$V I_a$ = electrical input to the armature (*armature input*)

$E_b I_a$ = electrical equivalent of mechanical power developed by the armature (*total armature output*)

$I_a^2 R_a$ = Electrical power lost in the armature (*armature copper loss*).

Thus, out of the armature input, a small portion (about 5 %) is wasted as $I_a^2 R_a$ loss and the remaining portion $E_b I_a$ is converted into mechanical energy within the armature.

The motor develops mechanical power given by

$$P_m = V I_a - I_a^2 R_a$$

If we differentiate both L.H.S. and R.H.S. with respect to I_a, and equate to zero, we have

$$\frac{dP_m}{dI_a} = V - 2 I_a R_a = 0$$

or $$I_a R_a = \frac{V}{2} \qquad \text{---(ii)}$$

But, we have seen that $V = E_b + I_a R_a$ ---(iii)

∴ Substituting the value of $I_a R_a$ given in eqn (ii) in eqn (iii), we obtain

$$V = E_b + \frac{V}{2} \quad \text{or} \quad \boldsymbol{E_b = \frac{V}{2}}$$

Thus, mechanical power developed by a motor is maximum when the back e.m.f. is equal to half the applied voltage.

This is a purely theoretical condition. In practice, the current will be far greater than the normal current of the motor. Besides, half the input is wasted in the form of heat and other losses, bringing down the motor efficiency to less than 50 %.

(ii) D.C. Series Motors

In a series wound motor, the field winding is connected in series with the armature as shown in Fig. 2.24. The series field winding consists of a few turns of thick wire having low resistance. It is apparent that the same current flows through both the field winding and the armature. If the mechanical load on the motor increases, the armature current also increases. Therefore, the flux in the series motor increases with the increase in armature current and vice-versa.

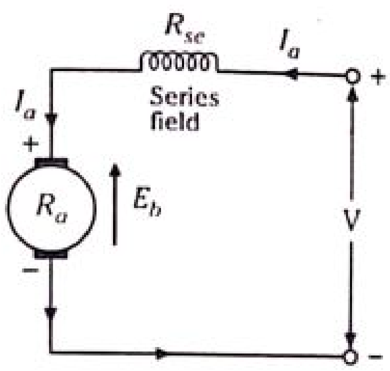


Fig. 2.24

From the circuit diagram of Fig. 2.21, we have

(i) $V = E_b + I_a(R_a + R_{se})$

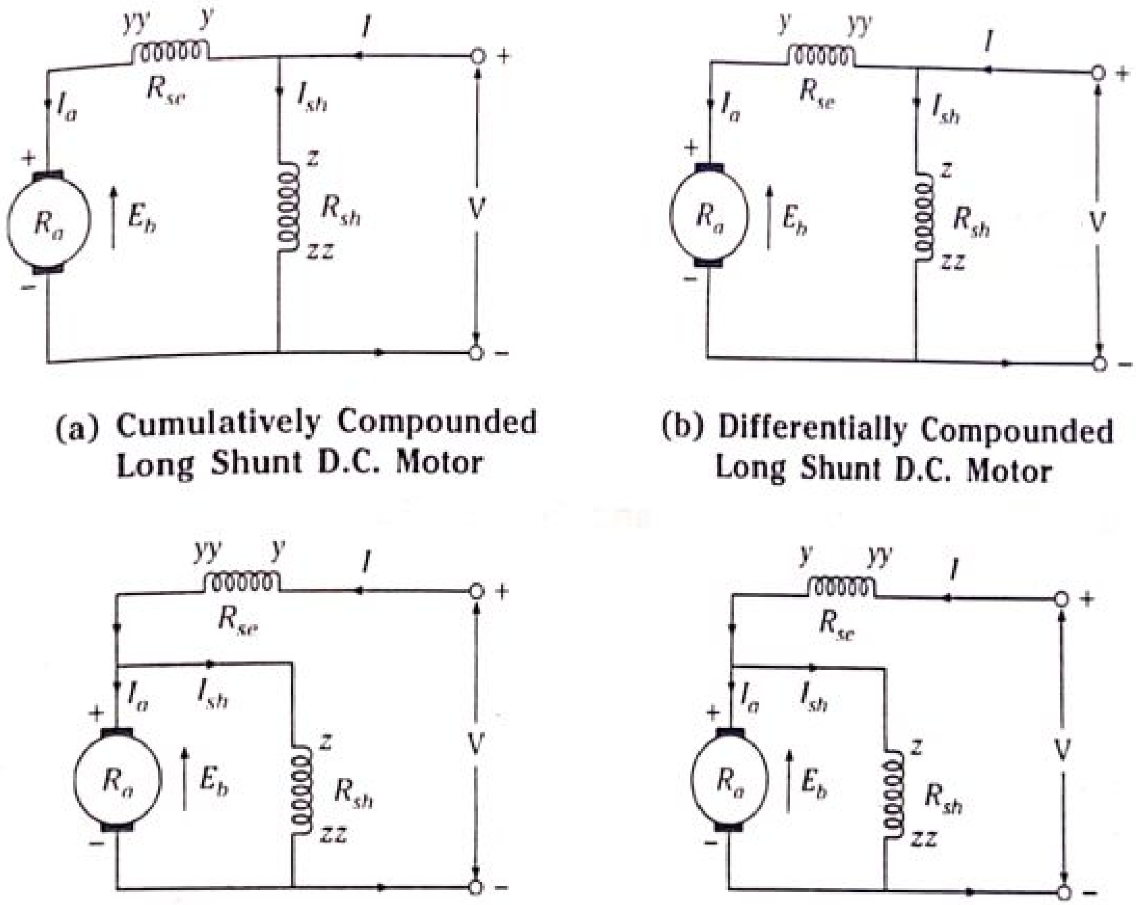
where V = Applied voltage

E_b = Back e.m.f

R_a = armature resistance

R_{se} = series field resistance

(iii) D.C. Compound Motor



(a) Cumulatively Compounded Long Shunt D.C. Motor

(b) Differentially Compounded Long Shunt D.C. Motor

(c) Cumulatively Compounded Short Shunt D.C. motor

(d) Differentially Compounded Short Shunt D.C. motor

Fig. 2.25

A D.C. Compound Motor has a shunt field winding and a series field winding. If the fluxes ϕ_{sh} produced by the shunt field winding and ϕ_{se} produced by the series field winding are in the same direction and are additive, then the motor is said to be *cumulatively compounded*. If the two fluxes oppose each other, then the motor is said to be *differentially compounded*. Depending on the way in which the two field windings are connected, the compound motors can be either *long shunt* or *short shunt*. The four types of D.C. compound motors are shown in Figs. 2.25(a), (b), (c) and (d).

In the case of cumulatively compounded motors, it is seen that the currents enter the positive terminals of the two field windings and hence the fluxes produced by them are in the same direction and they are additive. For the differentially compounded motors, the current through the series winding enters the negative terminal and the current through the shunt field winding enters the positive terminal. Hence the two fluxes produced are in opposite directions and hence they oppose each other.

The following equations are applicable to cumulatively or differentially compounded long shunt D.C. Motor.

$$I_{sh} = \frac{V}{R_{sh}}, \quad \text{and} \quad I_a = I - I_{sh}$$

Also, $E_b = V - I_a(R_a + R_{se})$.

For a cumulatively or differentially compounded short shunt D.C. Motor, we have the following equations :

$$I_{sh} = \frac{V - I R_{se}}{R_{sh}} \quad \text{and} \quad I_a = I - I_{sh}$$

Also, $E_b = V - I R_{se} - I_a R_a$.

2.14 Production of Torque and Torque Equations

2.14.1 What is Torque?

The measure of causing the rotation of a wheel or the turning or twisting moment of a force about an axis is called the torque. It is measured by the product of force and the radius at which this force acts.

Let us take a wheel of radius R metres acted upon by a circumferential force F newtons, making it rotate at N r.p.s. (Fig. 2.26).

So, torque $T = F \times R$ newton-metres

Work done by this force in one revolution

$= \text{Force} \times \text{distance}$

$= F \times 2\pi R$ joules

Work done per second

$W = F \times 2\pi R \times N$ joules/second

$= (F \times R) \times 2\pi N$ joules/second

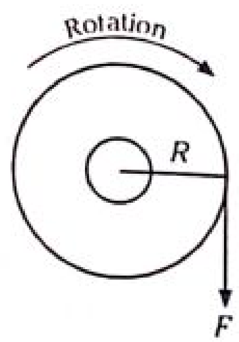


Fig. 2.26

But $2\pi N$ = angular velocity

$= \omega$ (radians/second)

Also, torque $T = F \times R$

$\therefore$ Work done per second, $W = T \times \omega$ joules/second

Also, power developed, $P = T \times \omega$ watts

where T = torque in newton-metres

ω = angular velocity in radians/second

2.14.2 Armature Torque

Let T_a = torque developed by a motor armature (N-m)

N = speed of rotation (r.p.s.)

Angular velocity, $\omega = 2\pi N$ rad/sec

$\therefore$ Power developed, $P = T_a \times \omega$

$$= T_a \times 2\pi N \text{ watts} \quad \text{---(i)}$$

The electrical power converted into mechanical power in the armature

$$= E_b I_a \text{ watts} \quad \text{---(ii)}$$

Equating eqns (i) and (ii),

$$T_a \times 2\pi N = E_b I_a \quad \text{---(iii)}$$

But we know that $E_b = \phi Z N \times \frac{P}{A}$ volts

So, $T_a \times 2\pi N = \phi Z N \times \frac{P}{A} \times I_a$

or $T_a = \frac{\phi Z I_a}{2\pi} \times \frac{P}{A}$ N-m

or $$\boldsymbol{T_a = 0.159 \phi Z I_a \times \frac{P}{A}} \text{ N-m} \quad \text{---(iv)}$$

$$= \left(\frac{0.159}{9.81}\right) \phi Z I_a \times \frac{P}{A} \text{ kg-m}$$

$$= 0.0162 \phi Z I_a \times \frac{P}{A} \text{ kg-m}$$

Since Z, P and A are constant for a particular machine, $\boldsymbol{T_a \propto \phi I_a}$. ---(v)

Hence, torque in a d.c. motor is directly proportional to flux per pole and armature current.

a) **For a series motor,** flux ϕ is directly proportional to armature current I_a, prior to saturation, as full armature current flows through the field windings.

$\therefore \quad T_a \propto I_a^2$

b) **In the case of a shunt motor,** ϕ is almost constant, so

$T_a \propto I_a$

From eqn (iii) we have

$$T_a = \frac{1}{2\pi} \cdot \frac{E_b I_a}{N} \text{ N-m}$$

$$= 0.159 \frac{E_b I_a}{N} \text{ N-m}$$

$$= \left(\frac{0.159}{9.81}\right) \frac{E_b I_a}{N} \text{ kg-m}$$

$$= 0.0162 \frac{E_b I_a}{N} \text{ kg-m}$$

If N is in r.p.m., then armature torque (refer eqn (iii))

$$T_a = \frac{E_b I_a}{\frac{2\pi N}{60}}$$

$$= \frac{60 E_b I_a}{2\pi N}$$

$$= 9.55 \frac{E_b I_a}{N} \text{ N-m}$$

2.14.3 Shaft Torque

The torque which is available at the motor shaft for doing useful work is called shaft torque (T_{sh}).

The total torque T_a developed in the armature is not available at the shaft, as part of it is lost in overcoming the iron and frictional losses. Therefore, shaft torque T_{sh} is somewhat less than the total armature torque T_a.

$$\text{Output} = T_{sh} \times 2\pi N \text{ watts}$$

where T_{sh} is in N-m, and N is in r.p.s.

$$\therefore \quad T_{sh} = \frac{\text{output in watts}}{2\pi N} \text{ N-m}$$

If N is in r.p.m., then

$$T_{sh} = \frac{\text{output in watts}}{\frac{2\pi N}{60}} \text{ N-m}$$

$$= \frac{60}{2\pi} \cdot \frac{\text{output in watts}}{N}$$

$$= 9.55 \frac{\text{output in watts}}{N} \text{ N-m}$$

2.15 Speed of a D.C. Motor

From the voltage equation (i) of Sec. 2.12(i), we get

$$E_b = V - I_a R_a \quad \text{or} \quad \frac{\phi Z N}{60}\left(\frac{P}{A}\right) = V - I_a R_a$$

$$\therefore \quad N = \frac{V - I_a R_a}{\phi} \times \left(\frac{60\,A}{Z\,P}\right) \text{ r.p.m}$$

Now $V - I_a R_a = E_b$

$$\therefore \quad N = \frac{E_b}{\phi} \times \left(\frac{60\,A}{Z\,P}\right)$$

As, Z, A and P are constant for a particular machine, the quantities within the brackets above can be considered as constant K

$$\therefore \quad N = K\,\frac{E_b}{\phi} \quad \text{or} \quad N \propto \frac{E_b}{\phi}$$

Thus, speed is directly proportional to the back e.m.f. E_b and inversely proportional to the flux ϕ.

2.15.1 For Series Motor

Let N_1, I_{a1} and ϕ_1 be the speed, armature current and flux per pole in the first case.

Let N_2, I_{a2} and ϕ_2 be the speed, armature current and flux per pole in second case

Then, using the above relation, we get

$$N_1 \propto \frac{E_{b1}}{\phi_1}, \quad \text{where} \quad E_{b1} = V - I_{a1}\,R_a$$

and $$N_2 \propto \frac{E_{b2}}{\phi_2}, \quad \text{where} \quad E_{b2} = V - I_{a2}\,R_a$$

$$\therefore \quad \frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{\phi_1}{\phi_2}$$

Before saturation of the magnetic poles occurs, $\phi \propto I_a$

$$\therefore \quad \frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{I_{a1}}{I_{a2}}$$

2.15.2 For Shunt Motor

In this case too, the same equation is used

i.e., $$\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{\phi_1}{\phi_2}$$

If $\phi_1 = \phi_2$, then $$\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}}$$

2.16 D.C. Motor Characteristics

D.C. motor characteristics depict the relationships between the following quantities :

i) ***Torque and armature current or*** T_a/I_a **Characteristic** (also called electrical characteristic).

ii) ***Speed and armature current*** *i.e.*, N/I_a **Characteristic**.

iii) ***Speed and torque or*** N/T_a **Characteristic** (also called mechanical characteristic). This can also be ascertained from (i) and (ii) above.

2.16.1 Characteristics of Series Motors

In a series wound motor, the field winding is connected in series with the armature as shown in Fig. 2.27. The series field winding consists of a few turns of thick wire having low resistance. It is apparent that the same current flows through both the field winding and the armature. If the mechanical load on the motor increases, the armature current also increases. Therefore, the flux in the series motor increases with the increase in armature current and vice-versa.

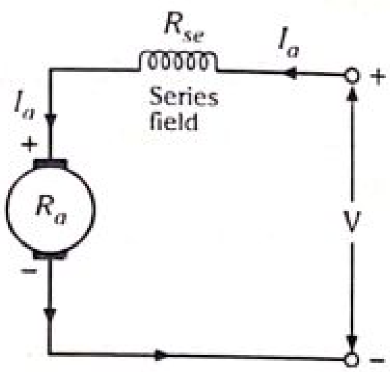


Fig. 2.27

1. T_a/I_a **Characteristic** : We know that $T_a \propto \phi I_a$ (Eqn(v) of Sec 2.14.2). Also, in a series motor, as the field windings also carry armature current, $\phi \propto I_a$ till the point of magnetic saturation is reached. Thus $T_a \propto I_a^2$.

At light loads, I_a and hence ϕ, is small. But, as I_a increases, T_a increases as the square of the current in a parabolic manner, till the point of saturation A is reached (see Fig. 2.28).

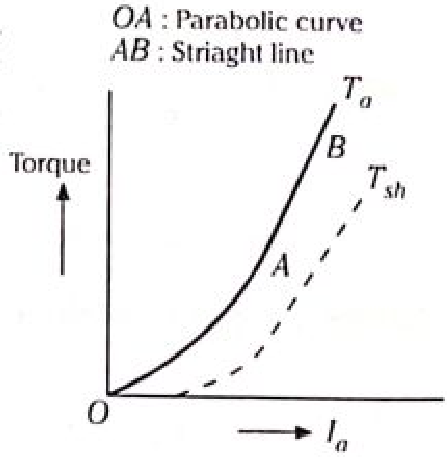


Fig. 2.28

After saturation, ϕ is practically independent of I_a, hence $T_a \propto I_a$, and so the characteristic becomes a straight line. (portion AB of the characteristic).

The shaft torque T_{sh} is less than the armature torque because of stray losses, a dotted curve depicting it in Fig. 2.28.

So, we reach the conclusion that, on heavy loads, before the onset of magnetic saturation, the armature torque is proportional to the square of armature current. Therefore, if a large starting torque is required for accelerating heavy masses quickly; (*e.g.* in electric locomotives, hoists *etc.*) series motors are ideal.

2. **N/I_a Characteristic :** Looking back at Sec 2.15, we know that changes in speed can be determined from the formula :

$$N \propto \frac{E_b}{\phi}$$

Variation of E_b for different load currents is so negligible that E_b may be treated as a constant. If I_a is increased, flux ϕ too increases. So, *speed is inversely proportional to the armature current*, as shown in Fig. 2.29.

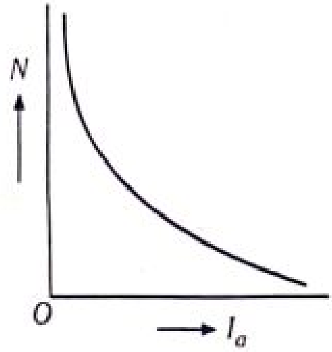


Fig. 2.29

When there is a heavy load, I_a is large. But when the load, and consequently I_a, slumps to a low value, the speed becomes dangerously high. Hence, a series motor should invariably be started with some mechanical load on it, to prevent excessive speed and damage due to the heavy centrifugal forces produced.

3. **N/T_a Characteristic :** The N/T_a characteristic of a series motor is shown in Fig. 2.30. From the curve, it is apparent that the series motor develops a high torque at low speed and vice-versa. This is because an increase in torque requires an increase in armature current, which is also the field current. The result is that the flux is strengthened and hence speed drops $\left(\text{as } N \propto \frac{1}{\phi}\right)$. Similarly, at low torque, the motor speed is high.

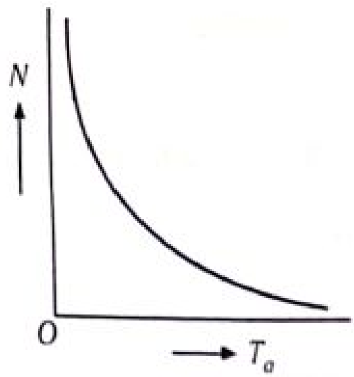


Fig. 2.30

2.16.2 Characteristics of Shunt Motors

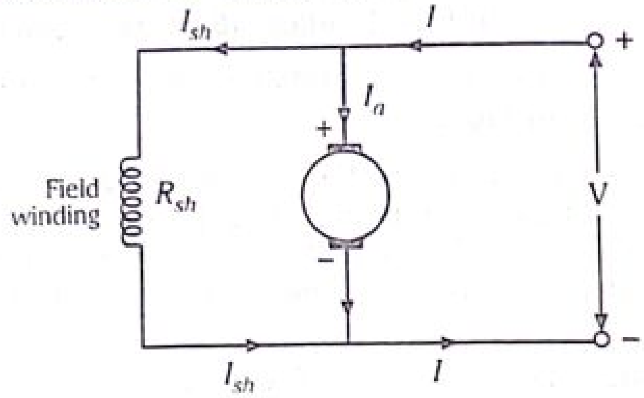


Fig. 2.31

In the shunt wound motor, the field winding is connected in parallel with the armature, as shown in Fig. 2.31. The line current I divides into two parallel paths : I_{sh} flows in the shunt field circuit and I_a in the armature circuit. It is to be kept in mind that the field current is constant, since the field winding is directly connected to the supply voltage V, which is assumed to be constant. Hence, the flux in a shunt motor is approximately constant,

1. T_a/I_a Characteristic : As we have assumed flux ϕ to be practically constant (neglecting armature reaction), we see that $T_a \propto I_a$. This implies that this characteristic is practically a straight line through the origin. (Fig. 2.32).

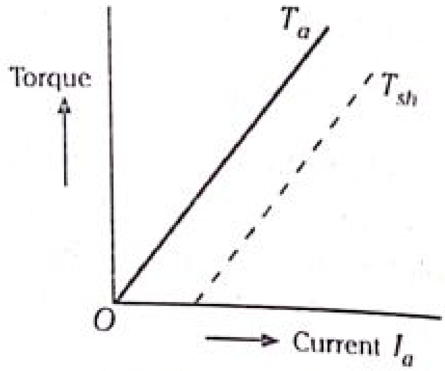


Fig. 2.32

The shaft torque *vs* armature current is also shown (dotted). It is clear from the curve that a larger armature current is required to start a heavy load. Therefore, a shunt motor should not be started on heavy load.

2. N/I_a Characteristic : We have seen that $N \propto \frac{E_b}{\phi}$. As ϕ is assumed to be constant, $N \propto E_b$. As E_b is also practically constant, the speed too is practically constant (Fig. 2.33), as indicated by dotted line AB.

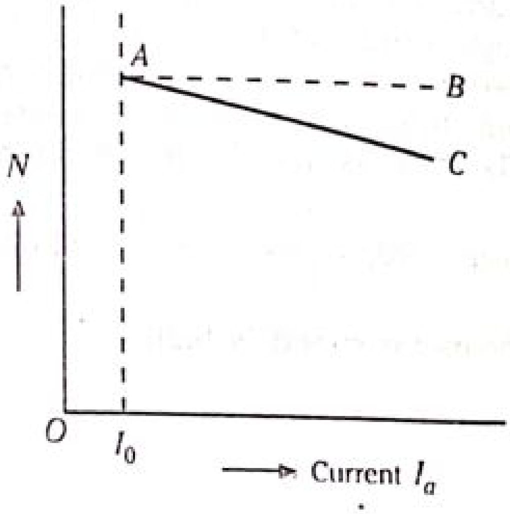


Fig. 2.33

However, to be accurate, both E_b and ϕ decrease with increasing load. But E_b decreases somewhat more than ϕ so that, all considered, there is some decrease in speed, the drop ranging from 5 to 15 % of full load, depending

on certain other conditions. Thus, the actual speed curve will be somewhat drooping, as shown by line AC.

It may be noted that the characteristic does not have a point of zero armature current, because a small current (no-load current I_o) is necessary to maintain the rotation of the motor at no-load.

As there is no marked change in the speed of a shunt motor, during the transition from no-load to full load, it may be connected to loads which can be suddenly disconnected without fear of excessive speeding. Because of this virtue of constant speed, shunt motors can be usefully employed for driving shafts, lathes, machine tools and other applications where an approximately constant speed is desired.

3. N/T_a Characteristic :

This curve is obtained by plotting the values of N and T_a for various armature currents I_a. It may be seen that speed falls somewhat as the load torque increases (Fig. 2.34).

This characteristic can be also be deduced from the other two characteristics just described.

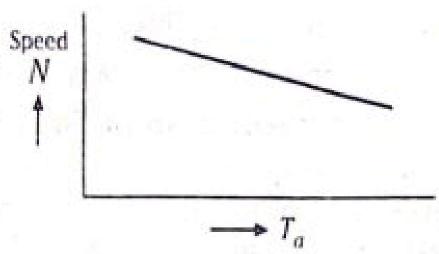


Fig. 2.34

The N/T_a characteristic is of great importance in determining which type of motor is best suited to drive a given load.

2.16.3 Characteristics of Compound Motors

(a) Characteristic of Cumulative Compound Motors

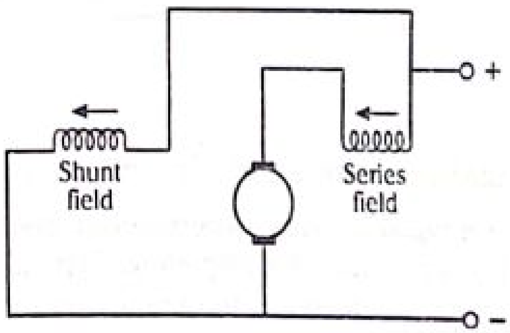


Fig. 2.35

Fig. 2.35 shows the connections for this type of motor. As the armature current is increased, the series flux increases, thus increasing the total flux of the motor. As a result of this, the torque is increased. The increase of torque T_a with armature current I_a is shown by the T_a/I_a characteristic curve OA of Fig. 2.36. This increase of T_a with I_a is greater than what it is in the case of a shunt motor (dotted curve OB) and less than what it is in the case of a series motor (dotted curve OC). Obviously, this type of motor develops a high torque with sudden increase in load.

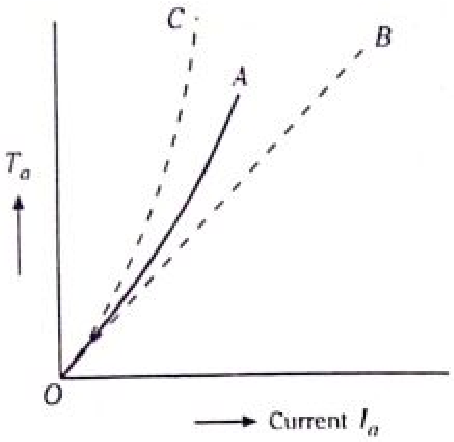


Fig. 2.36

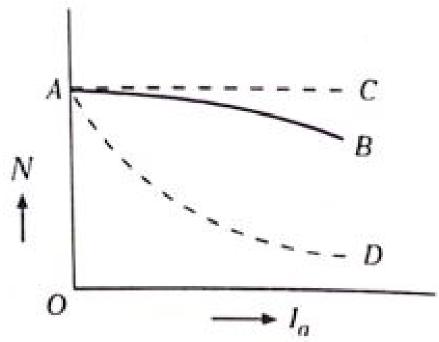


Fig. 2.37

We have just discussed that, with the increase of I_a, the series flux, and hence total flux, increases. This leads to decrease in motor speed, starting from a particular value given by the point A at no-load. The variation of N with I_a is given by the N/I_a *characteristic* AB of Fig. 2.37. Again, it must be borne in mind that this decrease in speed is greater than what it would be in the case of a shunt motor (given by the dotted curve AC), but less than what it would be in the case of a series motor. (shown by dotted line AD).

As series excitation assists shunt excitation, the N/T_a *characteristic* curve AB will lie between that of a shunt motor (dotted line AC) and of a series motor (dotted line AD) as shown in Fig. 2.38.

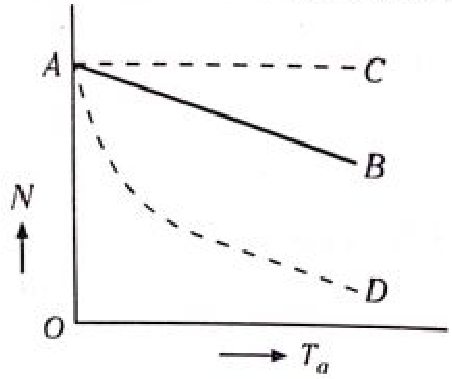


Fig. 2.38

Applications of Cumulative Compound Motors :

These motors are employed in cases where series characteristics are desired and where the load may be removed completely. Typical examples are certain types of coal-cutting machines, which are frequently required to make deep cuts. The shunt windings will ensure that the speed will not become excessive, and the series windings will cater to heavy loads. In many instances, flywheels are used in conjunction with such motors, where temporary heavy loads, occurs, as in the case of rolling mills. The flywheel supplies its stored energy when the motor tends to slow down due to sudden heavy load. When the load is removed, the motor speed increases and the flywheel again stores up kinetic energy, which is required to be released again when the situation demands.

(b) Characteristics of Differential Compound Motors

The connection diagram is shown in Fig. 2.39. Since the series field opposes the shunt-field, the total motor flux is decreased as load is applied to the motor. This results in the motor speed remaining almost constant or even increasing with increase in load $\left(\text{as } N \propto \dfrac{E_b}{\phi}\right)$. This is shown by the N/I_a *characteristics* of Fig. 2.40 (curve *AB*). Point *A* indicates no-load speed. For purposes of comparison curve *AC* is for shunt motor, where the speed remains practically constant with increase in load. Curve *AD* is for series motor, where speed reduces with load.

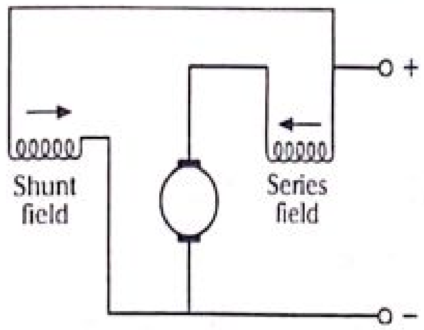


Fig. 2.39

Again, the total field is the difference between the shunt and series fields. So T_a does increase with I_a (curve *OA*), but not as rapidly as would be the case for a series motor (curve *OB*). Curve *OC* gives this relationship for a shunt motor. These T_a/I_a **characteristics** for the differential compound motor are given in Fig. 2.41.

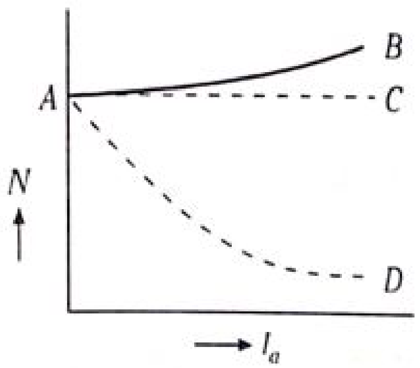


Fig. 2.40

Now, coming to the N/T_a **characteristics** for the differential compound motor (Fig. 2.42), there is a slight increase in motor torque with increase in speed, as indicated by curve *OA*. For comparison purposes, *OB* and *OC* are the curves for shunt and series motors respectively.

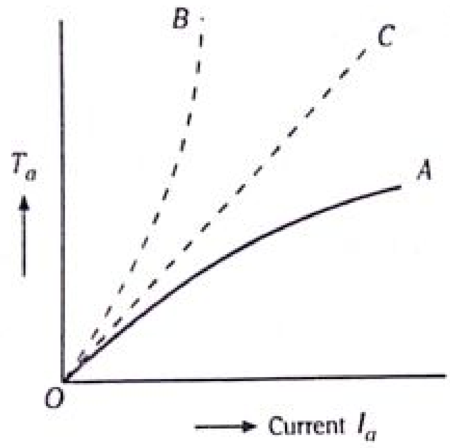


Fig. 2.41

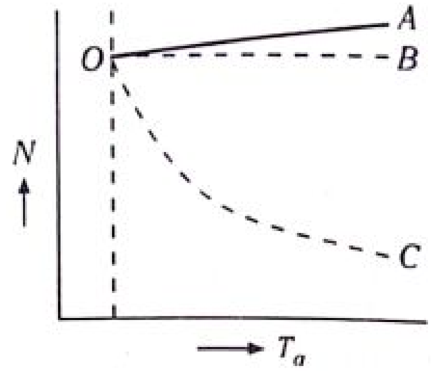


Fig. 2.42

2.17 Applications of D.C. Motors

Some applications of the 3 types of D.C. motors, viz, shunt, series and compound motors have already been mentioned under their respective headings. However, we may summarize the applications as follows :

Shunt Motors

1. Blowers and fans.
2. Centrifugal and Reciprocating pumps.
3. Lathes.
4. Machine tools.
5. For driving constant speed line shafting.

Series Motors

1. Traction purposes : Electric Locomotives, Trolley cars, *etc.*
2. Hoists and Cranes
3. Conveyors.

Cumulative Compound Motors (Also see last paragraph of Sec. 2.12.3)

1. Elevators
2. Conveyors
3. High-torque loads of intermittent nature
4. Punches
5. Shears
5. Heavy machine tools
7. Heavy planers.

2.18 Need of a Starter for D.C. Motors

We have already seen that the armature current drawn by a d.c. motor depends upon the applied voltage and the back e.m.f $\left(\text{as } I_a = \dfrac{V - E_b}{R_a}\right)$. This back e.m.f. prevents the armature current from being excessive, although the armature resistance is very small. The value of the back e.m.f. depends upon motor speed because it is caused by the armature conductors cutting the magnetic lines of force.

When the motor is at rest, there is no back e.m.f. ; therefore, if the motor is connected directly across the supply mains, a heavy current $\left(I_a = \dfrac{V}{R_a}\right)$ will flow through the armature conductors and damage them as resistance of motor armature is very low. As an example, a 5 H.P., 220 V shunt motor has a normal full load current of 30 A and an armature resistance of 0.4 ohm. If this motor is directly connected to the supply, it will take an armature current of $\dfrac{220}{0.4} = 550$ A, which is 18.33 times the full load current. This excessive starting current will not only damage the armature winding due to excessive heating effects, but will also blow out fuses and damage brushes *etc*.

In order to avoid excessive current at *starting*, a variable resistance called a *starting resistance* is inserted in series with the armature and is slowly cut out as the motor gains speed and sets up the back e.m.f., which then regulates the speed.

2.19 Connections of Starting Resistance to D.C. Motors

The starting resistance is connected to the three main types of D.C. motors as follows :

a) D.C. Series Motor :

In this case, the armature, the field and the starting resistance are all connected in series, as shown in Fig. 2.43. At start, the moveable arm is at position *A*, so that the entire resistance is in the circuit. When this arm is gradually moved towards position *B*, the resistance is gradually cut out of the circuit and the motor accelerates. At position *B* the entire resistance is cut out of the circuit and the motor attains full speed.

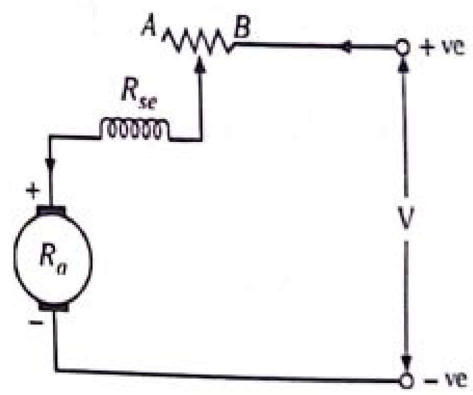


Fig. 2.43

b) D.C. shunt Motor :

Here, the starting resistance is connected in series with the armature only, as shown in Fig. 2.44. Full shunt field is established right from the start and maintained during the starting period (as the shunt field winding is always connected across the supply voltage), while the starter resistance is gradually decreased and finally cut out of the circuit.

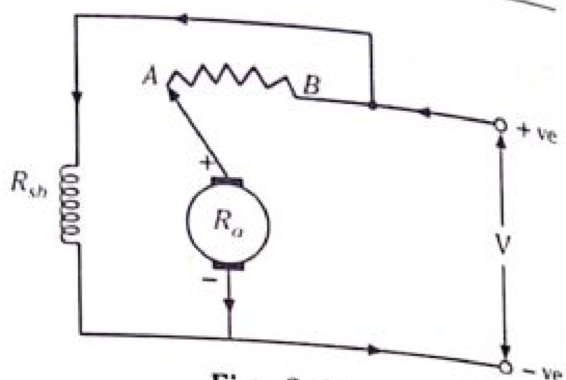


Fig. 2.44

c) D.C. Compound Motor :

As in the case of shunt motor, full shunt field is established right from the beginning and maintained throughout the starting period while the starter resistance which is in series with the armature and the series field, is progressively decreased (Fig. 2.45).

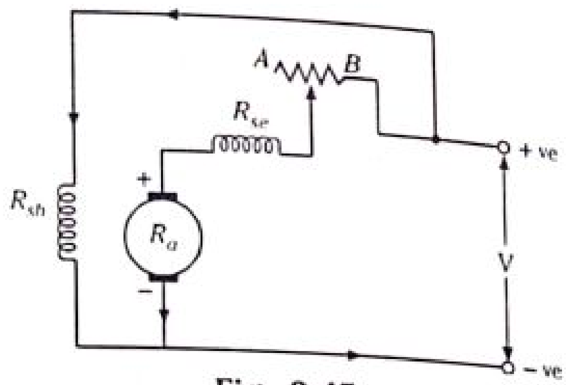


Fig. 2.45

2.20 Illustrative Examples/Problems on D.C. Motors

Problem 2.10

A 4-pole, 250 volt, series motor has a wave-connected armature with 1254 conductors. The flux per pole is 22 mWb when the motor is taking 50 A. Armature resistance is 0.2 Ω and series field resistance is 0.2 Ω. Calculate the speed. (May/June 86, B.U., MQP-2, B.U., MQP-4, B.U.)

Solution :

$$E_b = \left(\frac{\phi Z N}{60}\right) \times \frac{P}{A} \text{...........} N \text{ is in r.p.m.} \quad \text{.... Sec. 2.12}$$

Now, $E_b = V - I_a(R_a + R_{se}) = 250 - 50\,(0.2 + 0.2)$

$$= 250 - 20 = 230 \text{ V}$$

$$\therefore \quad 230 = \left(22 \times 10^{-3}\right) \times 1254 \times \frac{N}{60} \times \frac{4}{2} \qquad (A = 2 \text{ for wave-winding})$$

$$\therefore \quad N = \frac{230 \times 2 \times 60}{22 \times 10^{-3} \times 1254 \times 4}$$

$$= \mathbf{250.1 \text{ r.p.m.}}$$

Problem 2.11

A 250-V shunt motor takes a total current of 20 A. Resistance of the shunt field is 200 Ω and of the armature 0.3 Ω. Find the current in the armature and the back e.m.f. **(Dec 86, B.U.)**

Solution :

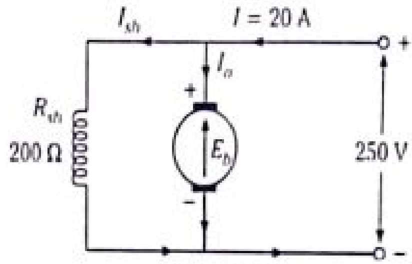


Fig. 2.46

Line current, $I = 20$ A ; $V = 250$ volts

Shunt Field Resistance, $R_{sh} = 200\ \Omega$

$\therefore$ Shunt field current, $I_{sh} = \dfrac{V}{R_{sh}} = \dfrac{250}{200} = 1.25$ A

Armature current, $I_a = I - I_{sh} = 20 - 1.25 = \mathbf{18.75\ A}$

Armature resistance, $R_a = 0.3\ \Omega$

Back e.m.f., $E_b = V - I_a R_a$ .... (Sec. 2.12)

$$= 250 - (18.75 \times 0.3) = 250 - 5.625$$

$$= \mathbf{244.375\ Volts}$$

Problem 2.12

A shunt generator delivers 100 kW at 250 V, when running at 400 r.p.m. The armature resistance is 0.01 ohm and field resistance is 100 ohms. If the same machine is run as a shunt motor with an input of 100 kW at 250 V, calculate the speed of the machine as a motor. Contact drop per brush is 1 V. **(Sep/Oct 83, B.U.; MQP-5, B.U.)**

Solution :

As Generator [Fig. 2.47 (a)]

Load Current, $I = \dfrac{100{,}000}{250} = 400$ A

Shunt current, $I_{sh} = \dfrac{250}{100} = 2.5$ A

$I_a = I + I_{sh} = 400 + 2.5 = 402.5$ A

$I_a R_a = 402.5 \times 0.01 \approx 4$ V

Brush drop $= 2 \times 1 = 2$ V

Induced e.m.f. in armature

$$E_g = V + I_a R_a + \text{drop in brushes} = 250 + 4 + 2 = 256 \text{ V}$$

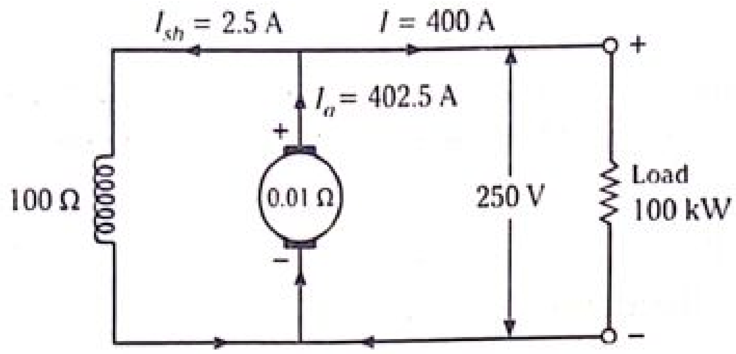


Fig. 2.47 (a) Generator

It is apparent that, if this machine is to run as a motor at 400 r.p.m., it will have a back e.m.f. of 256 V induced in its armature. E_{b1} = 256 V ; N_1 = 400 r.p.m.

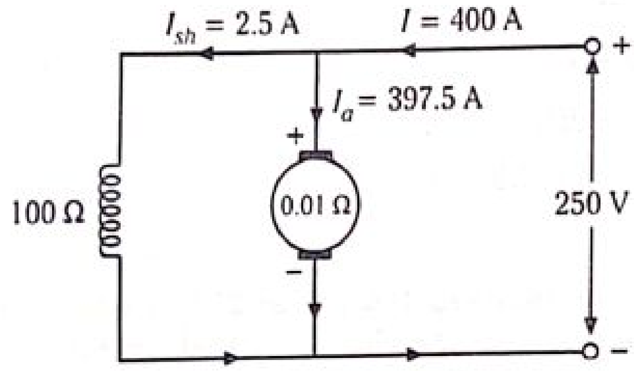


Fig. 2.47 (b) Motor

As Motor [Fig. 2.47 (b)]

Input line current, $I = \dfrac{100,000}{250} = 400$ A

Shunt current, $I_{sh} = \dfrac{250}{100} = 2.5$ A

$I_a = I - I_{sh} = 400 - 2.5 = 397.5$ A

$I_a R_a = 397.5 \times 0.01 = 3.97$ V

Brush drop = 2 × 1 = 2 V (as before)

$\therefore \quad E_{b2} = 250 - (3.97 + 2) \approx 244$ V

N_2 = ?

Now, $\dfrac{N_2}{N_1} = \dfrac{E_{b2}}{E_{b1}} \times \dfrac{\phi_1}{\phi_2}$ .... Sec. 2.15.2

However, as I_{sh} is constant in both cases, $\phi_1 = \phi_2$

$$\therefore \quad \frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \quad \text{or} \quad \frac{N_2}{400} = \frac{244}{256}$$

$$\therefore \quad \mathbf{N_2 = 381.25 \ RPM}$$

Problem 2.13

A 120 V, DC shunt motor has an armature resistance of 0.2 Ω and a field resistance of 60 Ω. It runs at 1800 RPM, when it takes a full load current of 40 A. Find the speed of the motor when it is operating with half full-load.

(July 89, July 93, B.U.)

Solution :

$V = 120$ volts; $R_a = 0.2\ \Omega$; $R_{sh} = 60\ \Omega$

Shunt field current, $I_{sh} = \frac{V}{R_{sh}} = \frac{120}{60} = 2\text{A}$

Full Load

Full-load current, $I = 40$ A

Armature current, $I_a = I - I_{sh} = 40 - 2 = 38$ A

Back e.m.f. $E_b = V - I_a R_a = 120 - (38 \times 0.2) = 112.4$ V

Half load

Current on half load, $I_h = \frac{1}{2} \times 40 = 20$ A

Armature current, $I_{ah} = I_h - I_{sh} = 20 - 2 = 18$ A

Back e.m.f., $E_{bh} = V - I_{ah} R_a = 120 - (18 \times 0.2) = 116.4$ V

Now, $\frac{N}{N_h} = \frac{E_b}{E_{bh}}$ (flux remains constant and hence not considered)

$$\frac{1800}{N_h} = \frac{112.4}{116.4}$$

$$\therefore \quad \text{Speed on half-load, } N_h = \frac{1800 \times 116.4}{112.4} = \mathbf{1864 \ RPM}$$

Problem 2.14

A 4-pole, 500 V, shunt motor has 720 wave connected conductors on its armature. The full load armature current is 60 A, and the flux per pole 0.03 Wb. The armature resistance is 0.2 Ω and the contact drop is 1 volt per brush. Calculate the full-load speed of the motor. (Dec. 83, K.U.)

Solution :

EMF generated, $E_b = \dfrac{\phi Z N}{60} \times \dfrac{P}{A}$ ....Sec. 2.12

Given,

$P = 4$

$Z = 720$

$\phi = 0.03$ Wb/pole

For wave-connected conductors,

No. of parallel paths, $A = 2$

$$\therefore \quad E_b = \frac{0.03 \times 720 \times N}{60} \times \frac{4}{2} = 0.72 \text{ N} \qquad \text{---(i)}$$

Now,

$$E_b = V - [I_a R_a + \text{Brush drop}]$$

Now,

brush drop = 1 volt per brush (given)

$\therefore$ Total brush drop = $1 \times 2 = 2$ volts

Given

$V = 500$ volts and Armature resistance $R_a = 0.2$ ohms

$$\therefore \quad E_b = 500 - [(60 \times 0.2) + 2] = 486 \text{ V}$$

Substituting in Eqn (i) above,

$486 = 0.72$ N

$\therefore$ N = **675 RPM**

Problem 2.15

Find the useful flux per pole of a 250 V, 6 pole shunt motor having a two circuit connected armature winding with 220 conductors. At normal working temperature, the overall armature resistance including brushes is 0.2 Ω. The armature current is 13.3 A at the no-load speed of 908 rpm.

(Mar 99, V.T.U.)

Solution :

Given $V = 250$ V

No. of poles, $P = 6$

Armature Resistance, $R_a = 0.2\ \Omega$

No. of conductors, $Z = 220$

$N = 908$ rpm

This is a wave winding ; hence the number of parallel paths, $A = 2$

Now, $E_b = V - I_a R_a$

$= 250 - (13.3 \times 0.2)$

$= 247.34$ Volts

Also $$E_b = \frac{\phi Z N}{60}\left(\frac{P}{A}\right)$$

or $$247.34 = \frac{\phi \times 220 \times 908}{60} \times \left(\frac{6}{2}\right)$$

$\therefore$ Useful Flux per Pole, $$\phi = \frac{2 \times 247.34 \times 60}{220 \times 908 \times 6}$$

$$= \mathbf{0.0247\ Wb}$$

Problem 2.16

A d.c. shunt motor runs at 750 r.p.m. from 250 V supply and takes a full load line current of 60 A. Its armature resistance is 0.4 ohm and the field resistance is 125 ohms. Assuming 2 V brush drop and negligible armature reaction effect, find the no-load speed for a no-load current of 6 amperes.

(Sep/Oct 87, B.U.)

Solution :

We use the formula :

$$\frac{N}{N_o} = \frac{E_b}{E_{bo}} \times \frac{\phi_o}{\phi}$$

N = Speed on load = 750 r.p.m.

E_b = Back e.m.f. under FL conditions

E_{bo} = Back e.m.f. under no-load conditions

ϕ = flux under full load conditions

ϕ_o = flux under no-load conditions

R_a = 0.4 Ω

R_{sh} = 125 Ω

No-load current, I_o = 6 amps

Full-load current, I = 60 A

No-load speed, N_o = ?

The effect of armature reaction is negligible, so $\phi = \phi_o$

$$I_{sh} = \frac{250}{125} = 2\ A$$

Full load current, I = 60 A

$\therefore$ Armature current under full load conditions

$$I_a = I - I_{sh} = 60 - 2 = 58 \text{ A}$$

$$I_a R_a = 58 \times 0.4 = 23.2 \text{ V}$$

$$\therefore \quad E_b = V - [I_a R_a + \text{brush drop}] = 250 - [23.2 + 2] = 224.8 \text{ V}$$

Armature current under no-load conditions

$$I_{ao} = I_o - I_{sh} = 6 - 2 = 4 \text{ A}$$

$$I_{ao} R_a = 4 \times 0.4 = 1.6 \text{ V}$$

$$\therefore \quad E_{bo} = V - [I_{ao} R_a + \text{brush drop}]$$

$$= 250 - [1.6 + 2] = 246.4 \text{ V}$$

$$\frac{N}{N_o} = \frac{E_b}{E_{bo}} = \frac{224.8}{246.4}$$

$$\therefore \quad N_o = \frac{N \times 246.4}{224.8} = \frac{750 \times 246.4}{224.8} = \mathbf{822 \text{ r.p.m.}}$$

Problem 2.17

A 230 V d.c. shunt motor takes a no-load current of 2 A, and runs at 1100 r.p.m. If the full-load current is 40 A, find the speed at full load. Assume the flux remains constant and armature resistance is 0.25 ohms.

(July 88, B.U.)

Solution :

$$\frac{N}{N_o} = \frac{E_b}{E_{bo}} \times \frac{\phi_0}{\phi_1}$$

But $\phi_0 = \phi_1$ (given that flux remains constant)

$N_o = 1100$ r.p.m.

Armature current on no-load, $I_{ao} = 2\text{A}$

Armature current on full load, $I_a = 40$ A

$$E_{bo} = V - I_{ao} R_a = 230 - (2 \times 0.25) = 229.5 \text{ V}$$

$$E_b = V - I_a R_a = 230 - (40 \times 0.25) = 220 \text{ V}$$

$$\therefore \quad \frac{N}{1100} = \frac{220}{229.5}$$

$$\therefore \quad \text{Speed at full load, } N = \frac{220 \times 1100}{229.5} = \mathbf{1054.5 \text{ r.p.m.}}$$

Problem 2.18

Determine the total torque developed in a 250 V, 4-pole d.c. shunt motor with lap winding, accommodated in 60 slots, each containing 20 conductors. The armature current is 50 A and the flux per pole is 23 mWb.

(Mar 89 M.U.)

Solution : (Ref Fig. 2.48)

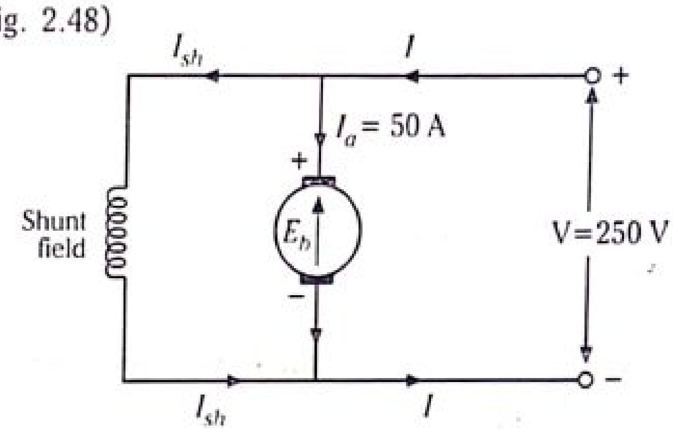


Fig. 2.48

Using the Eqn(iv) of Sec. 2.14.2

$$\text{Armature Torque, } T_a = 0.159\ \phi Z I_a \times \left(\frac{P}{A}\right) \text{ N-m} \qquad \text{---(i)}$$

Flux per pole, ϕ = 23 mWb = 23×10^{-3} Wb.

Total number of conductors,

Z = No. of conductors in one slot × No. of slots

i.e., $Z = 20 \times 60 = 1200$

No. of poles, $P = 4$

For lap winding, $A = P = 4$

Armature Current $I_a = 50$ A

Substituting the above values in eqn (i),

$$T_a = 0.159 \times \left(23 \times 10^{-3}\right) \times 1200 \times 50 \times \left(\frac{4}{4}\right) \text{ N-m}$$

$$= \mathbf{219.5\ N\text{-}m}$$

Problem 2.19

A 250-V shunt motor has an armature resistance of 0.5 ohm and a field resistance of 250 ohms. When driving at 600 r.p.m. on load, the torque of which is constant, the armature takes 20 A. If it be desired to raise the speed from 600 to 800 r.p.m., what resistance must be inserted in the shunt field circuit, assuming the magnetisation curve to be a straight line ?

(Aug 94, B.U.)

Solution :

Case 1 :

Field current $I_{sh1} = \frac{V}{R_{sh1}} = \frac{250}{250} = 1\text{ A}$

Given armature current $I_{a1} = 20$ A

Back e.m.f. $E_{b1} = V - I_{a1} R_a$

where $V = 250$ Volts and Armature Resistance $R_a = 0.5\ \Omega$

$\therefore \quad E_{b1} = 250 - (20 \times 0.5) = 240$ V

Case 2 :

Let now the armature current $= I_{a2}$ (after speed has been increased)

$\therefore \quad E_{b2} = V - I_{a2} R_a$

$= 250 - 0.5\, I_{a2}$

Now, $\quad I_{sh1}\, I_{a1} = I_{sh2}\, I_{a2}$

or $\quad 1 \times 20 = I_{sh2} \times I_{a2}$

$\therefore \quad I_{sh2} = \frac{20}{I_{a2}}$

Also, $\quad \frac{N_1}{N_2} = \frac{E_{b1}\, I_{sh2}}{E_{b2}\, I_{sh1}}$

where N_1 = the speed in case 1 = 600

and where N_2 = the speed in case 2 = 800

or $\quad \frac{600}{800} = \frac{\frac{240 \times 20}{I_{a2}}}{(250 - 0.5\, I_{a2}) \times 1}$

On simplification, we get

$$-0.5\, I_{a2}^2 + 250\, I_{a2} - 6400 = 0$$

If $f(x) = ax^2 + bx + c$, the roots are given by

$$\left\{ \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \right\}$$

$$\text{so,} \quad I_{a2} = \frac{-250 \pm \sqrt{250^2 - (4 \times 0.5 \times 6400)}}{2 \times (-0.5)}$$

$$= \frac{-250 \pm \sqrt{62500 - (4 \times 0.5 \times 6400)}}{1}$$

$$= \frac{-250 \pm \sqrt{49700}}{-1}$$

$$= \frac{-250 \pm 223}{-1} = \frac{-250 + 223}{-1} = 27 \text{ A}$$

$$\therefore \quad I_{sh2} = \frac{20}{I_{a2}} = \frac{20}{27} = 0.74 \text{ A}$$

$$\therefore \quad R_{sh2} = \frac{V}{I_{sh2}} = \frac{250}{0.74} = 338 \text{ ohms}$$

The field resistance R_{sh1} = 250 ohms.

∴ The resistance to be inserted in the shunt field circuit

= − 250 = **88 ohms.**

Problem 2.20

A 250-volt d.c. shunt motor takes 6 amperes line current on no-load and runs at 1000 r.p.m. The field resistance is 250 ohms and armature resistance 0.2 ohm. If the full-load line current in 26 amperes, calculate the full load speed, assuming constant air gap flux. (Apr/May 87, M.U.)

Solution :

The formula used :

$$\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{\phi_1}{\phi_2} \qquad \text{Sec. 2.15.2}$$

where

N_1, E_{b1} and ϕ_1 refer to no-load conditions

N_2, E_{b2} and ϕ_2 refer to full-load conditions

Now, as air gap flux is assumed to be constant, $\phi_1 = \phi_2$

$$\text{Thus,} \quad \frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \qquad \text{---(i)}$$

$$I_{sh} = \frac{V}{R_{sh}} = \frac{250}{250} = 1 \text{ A}$$

The armature current at no-load,

$$I_{a1} = 6 - 1 = 5 \text{ A}$$

The armature current at full-load,

$$I_{a2} = 26 - 1 = 25 \text{ A}$$

It may be noted here that the field current, I_{sh}, is the same under both no-load and full-load conditions.

$$E_{b1} = V - I_{a1} R_a$$
$$= 250 - (5 \times 0.2) = 249 \text{ V}$$

$$E_{b2} = V - I_{a2} R_a$$
$$= 250 - (25 \times 0.2) = 245 \text{ V}$$

Substituting in eqn (i)

$$\frac{N_2}{1000} = \frac{245}{249}$$

$$\therefore \quad N_2 = 984 \text{ R.P.M.}$$

Problem 2.21

A Shunt motor running light at 440 volts takes a current of 3.0 Amperes. The field resistance is 600 ohms, and armature resistance 0.4 ohm, and speed 1000 r.p.m. Calculate its speed when taking a current of 30 Amperes from the line, neglecting armature reaction.

Solution :

Shunt field current, $I_{sh} = \dfrac{V}{R_{sh}} = \dfrac{440}{600} = 0.73 \text{ A}$

$\therefore$ No load armature current, $I_{ao} = 3 - 0.73 = 2.27 \text{ A}$

No load back e.m.f, $E_{bo} = V - I_{ao} R_a$

$$= 440 - (2.27 \times 0.4) = 439.1 \text{ V}$$

Loaded condition

Armature current on load, $I_{a1} = I_L - I_{sh}$

$$= 30 - 0.73 = 29.27 \text{ A}$$

Back e.m.f on load, $E_{b1} = V - I_{a1} R_a$

$$= 440 - (29.27 \times 0.4)$$
$$= 428.3 \text{ V}$$

No load speed, $N_o = 1000$ r.p.m.

$$\frac{N}{N_o} = \frac{E_{b1}}{E_{bo}} \quad \text{or} \quad N = \frac{N_o \times E_{b1}}{E_{bo}}$$

$$= \frac{1000 \times 428.3}{439.1} = \mathbf{975 \ R.P.M}$$

Problem 2.22

A four-pole d.c. shunt motor takes 22.5 amperes from a 250 V supply. R_a = 0.5 ohm and R_f = 125 ohms. The armature is wave-wound with 300 conductors. If the flux per pole is 0.02 Wb, calculate (i) the speed (ii) torque developed (iii) power developed.

(Aug/Sep 89 M.U.)

Solution :

The shunt motor circuit is given in Fig. 2.49.

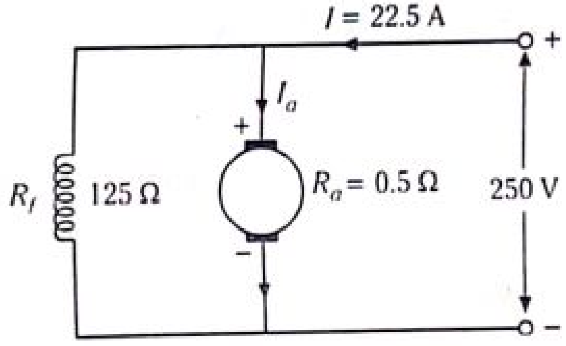


Fig. 2.49

Given voltage V = 250 V and I = 22.5 A

$$I_{sh} = \frac{V}{R_f} = \frac{250}{125} = 2 \text{ Amps}$$

$$I_a = I - I_{sh} = 22.5 - 2 = 20.5 \text{ Amps}$$

$$Z = 300$$

For wave-wound armature,

No. of parallel paths, A = 2

$$E_b = V - I_a R_a$$

$$= 250 - (20.5 \times 0.5)$$

$$= 239.75 \text{ V}$$

i) Speed $N = \frac{E_b}{\phi} \times \left(\frac{60\,A}{Z\,P}\right)$ (Refer Sec. 2.15)

$$= \frac{239.75 \times 60 \times 2}{0.02 \times 300 \times 4}$$

$$= \mathbf{1199\ R.P.M}$$

ii) Torque $T_a = 0.159\ \phi\ Z I_a \times \frac{P}{A}$ N-m

$$= 0.159 \times 0.02 \times 300 \times 20.5 \times \left(\frac{4}{2}\right)$$

$$= \mathbf{39.1\ N\text{-}m}$$

iii) Power developed $= E_b\, I_a = 239.75 \times 20.5$

$$= 4915\ \text{W} = \mathbf{4.915\ kW}$$

Problem 2.23

A 4-pole D.C. Shunt motor takes 22 A from 220 V supply. The armature and field resistances are respectively 0.5 Ω and 100 Ω respectively. The armature is lap connected with 300 conductors. If the flux per pole is 20 mWb, calculate the speed and gross torque. (Mar 95, B.U.)

Solution :

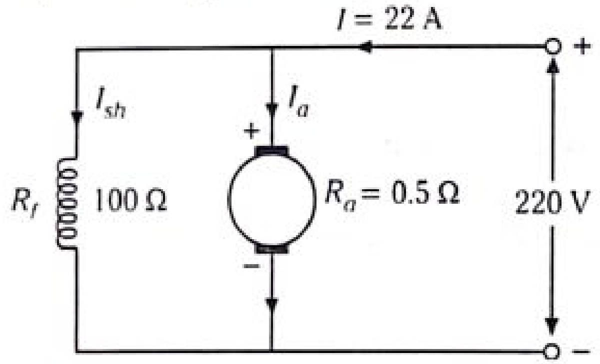


Fig. 2.50

The Shunt motor circuit is given in the above figure.

Given voltage $V = 220$ V and $I = 22$ A

$$I_{sh} = \frac{V}{R_f} = \frac{220}{100} = 2.2 \text{ Amps}$$

$\therefore$ $I_a = I - I_{sh} = 22 - 2.2 = 19.8$ Amps

$Z = 300$ and $\phi = 20$ mWb $= 0.02$ weber

For lap-wound armature, $A = P = 4$

$$E_b = V - I_a R_a$$
$$= 220 - (19.8 \times 0.5)$$
$$= 220 - 9.9 = 210.10 \text{ Volts}$$

i) Speed $N = \dfrac{E_b}{\phi} \times \left(\dfrac{60\ A}{Z P}\right)$ (Refer Sec. 2.15)

$$= \frac{210.10}{0.02} \times \frac{60 \times 4}{300 \times 4}$$
$$= \mathbf{2100\ RPM}$$

ii) Torque $T_a = 0.159\ \phi\ Z\ I_a \times \dfrac{P}{A}$ Newton-metres

$$= 0.159 \times 0.02 \times 300 \times 19.8 \times \left(\frac{4}{4}\right)$$
$$= \mathbf{18.9\ Newton\text{-}metres}$$

Problem 2.24

A 230 volts shunt motor has an armature resistance of 0.6 ohm. If the full-load armature current is 30 Amps and no load armature current is 4 Amps, find the change in back e.m.f. from no load to full load.

(Aug/Sep 89, M.U.)

Solution :

$V = 230$ V

$R_a = 0.6$ ohm

Full-load armature current, $I_{a1} = 30$ A

No-load armature current, $I_{a0} = 4$ A

i) *Back e.m.f. on no-load*

$$E_{bo} = V - (I_{ao} \times R_a)$$
$$= 230 - (4 \times 0.6) = 227.6 \text{ V}$$

ii) *Back e.m.f. on full load*

$$E_{b1} = V - (I_{a1} \times R_a)$$
$$= 230 - (30 \times 0.6) = 212 \text{ V}$$

$\therefore$ Change in back e.m.f from NL to FL

$$= 227.6 - 212 = \mathbf{15.6\ V}.$$

2.21 Review Questions

1. Explain the principle of operation of a DC Machine as
   (a) a generator
   (b) a motor

2. Explain with a neat sketch the construction features of a D.C. Machine and mention the functions of each part.

3. With usual notations derive an expression for the induced E.M.F of a D.C. Machine. **(Sept/Oct 87, B.U. Mar 95, B.U.)**

4. What is 'back e.m.f? Explain its significance.

5. What are the various types of D.C. Motors? Give their circuit representation.

6. Derive an expression for the torque developed by a D.C. Motor.

7. Draw & explain the following in respect of D.C. shunt and series motors :
   (i) Torque vs. speed characteristics
   (ii) Torque vs. armature current characteristics. **(Mar 94, B.U.)**

8. Sketch and explain the speed-load and torque load characteristics of d.c. cumulatively compound and differentially compounded motors and comment on the shape of the characteristics. Indicate where such motors we ideally used.
   (May/June 86, B.U.; Aug 94, B.U.)

9. Explain the principle of production of torque in d.c. motors.

10. Why is a starter needed for D.C. Motors? Explain in brief.

2.22 Exercises - Problems

1. A 4-pole 1500 RPM d.c. generator has a lap-wound armature having 200 conductors. What should be the flux per pole to develop 250 V ?
   [Oct 85, K.U.]
   Answer : 0.05 Wb.

2. A 4-pole d.c. machine has an armature with 90 slots and 6 conductors in each slot. The field current is adjusted to produce a flux of 10 m Wb. The machine is driven at 1500 r.p.m. The armature winding is lap-wound. Calculate the induced e.m.f. in the armature. [Nov 97, K.U.]
   Answer : 135 V

3. A 4-pole shunt generator with lap-connected armature supplies a load of 200 A at 100 volts. Shunt field resistance is 50 ohms. Armature resistance is 0.05 Ω. Calculate,

   i) Total armature current

   ii) Current per parallel path

   iii) E.M.F. generated

   Allow a brush contact drop of 2 V. [Apr 86, K.U.]

   Answer : (i) 202 A (ii) 50.5 A (iii) 112.1 V

4. The e.m.f generated by a 4-pole d.c. generator is 400 V when the armature is driven at 1000 r.p.m. Calculate the flux per pole, if the wave-wound armature has 39 slots, with 16 conductors per slot. Also calculate the e.m.f generated if it is driven at 1500 r.p.m. [Mar 89, K.U.]

   Answer : 19.23 mWb ; 600 Volts

5. A 4-pole lap-wound d.c. motor has 40 slots, each housing 20 conductors, on its armature. It runs at 360 rpm and carries an armature current of 40 A, with a flux per pole of 45 mWb. Calculate the torque and mechanical power output, neglecting stray losses. [Oct 89, K.U.]

   Answer : 228.96 Newton-meters ; 11.7357 H.P.

6. The current drawn from the mains by a 200 V d.c. shunt motor is 3 A on no-load. The armature and field resistances are respectively 0.5 Ω and 200 Ω. If the line current on load is 51 A at a speed of 1200 rpm, find the no-load speed. (Oct 84, K.U.)

   Answer : 1364.57 RPM

7. A 10 H.P., 230 V shunt motor takes an armature current of 6 A from 230 V mains at no-load and runs at 1200 rpm. The armature resistance is 0.25 ohm. Determine the speed and electromagnetic torque when the armature takes 36 A with the same flux. (Apr 84, K.U.)

   Answer : 1160.6 RPM; 65.46 Nw-m

8. A 250 V shunt motor has an armature current of 20 A when running at 1000 RPM against full load torque. The armature resistance is 0.5 Ω. What resistance must be inserted in series with the armature to reduce the speed to 500 RPM at the same torque ? Assume the flux to remain the same.

   Answer : 6 W

MODULE

2

(b) Measuring Instruments

2.23 Essentials of Measuring Instruments

In order to ensure proper & efficient operation of the moving system of measuring instruments, three torques are essential. These are

(a) **Deflecting (or Operating) Torque (T_d)**

(b) **Controlling (or Restraining) Torque**

(c) **Damping Torque**

(a) Deflecting Torque (T_d) :

The deflection of the pointer of a secondary instrument from its position of rest is caused by a torque T_d acting on the spindle. This torque is proportional to the magnitude of the quantity to be measured. It is produced by different mechanisms in different measuring instruments. The deflecting system uses one of the following effects produced by current or voltage, to produce deflecting (or operating) torque:

(i) **Thermal effect :** The current to be measured is passed through a small element which heats it, resulting in a temperature rise, which in turn is converted to an e.m.f by a thermocouple.

When two dissimilar metals are connected end-to-end to form a closed loop and the two junctions are maintained at different temperatures, then e.m.f is induced which results in flow of current through the closed circuit, which is called a *thermocouple*. This effect is used in *ammeters* & *voltmeters*.

(ii) **Magnetic Effect :** When a current-carrying conductor is placed in a uniform magnetic field, it experiences a force which causes it to move. This effect is usually applied in *moving iron attraction & repulsion type of instruments* and in *permanent magnet moving coil instruments*.

(iii) **Electrostatic Effect :** When two plates are charged, there is a force exerted between them, which moves one of the plates. This effect is used in an *electrostatic voltmeter*.

(iv) **Induction Effect :** When a non-magnetic conducting disc is positioned in a magnetic field produced by electromagnets which are excited by alternating currents, an e.m.f is induced in it.

If a closed path is provided, current flows in the disc. The interaction between the induced current and the alternating magnetic field exerts a force on the disc, resulting in its movement. Such an interaction is known as *induction effect*. This effect is used in *energymeters*.

(v) Electrodynamic Effect : Here, the operating field is produced by a fixed coil, causing the motion of a moving coil. This effect is used in a *dynamometer wattmeter*.

(vi) Hall Effect : If a specimen of semiconductor carrying a current I is placed in a *transverse* magnetic field of flux density B, an electric field is developed along a direction perpendicular to both B and I. This phenomenon is known as *Hall Effect*. This affect is mainly used in *flux meters*.

(b) Controlling Torque (T_c) :

In a secondary instrument, the pointer gets deflected from its initial zero position and moves over a graduated scale as a result of the deflecting torque produced by the quantity to be measured. If the pointer moves unopposed, it is obvious that it would continue to move as long as the deflecting torque persists and would swing over to the maximum deflected position irrespective of the magnitude of the quantity being measured. This necessitates the provision of a *controlling* or *restraining torque* (T_c). This torque opposes the deflecting torque and increases with the deflection of the moving system. The pointer will be brought to rest at a position where the two opposing torques are equal *i.e.*, $T_d = T_c$. The controlling torque performs two functions :

(1) It increases with the deflection of the moving system so that the final position of the pointer on the scale will be according to the magnitude of the quantity being measured.

(2) It brings the pointer back to zero position when the deflecting torque is removed. If it were not provided, the pointer once deflected would not return to zero position on removing the deflection torque. The controlling (or restraining) torque in indicating instruments is obtained either by a *spring* or by *gravity* (or weight), as described below.

(i) Spring Control

Two hair springs, each with a large number of turns, are rigidly attached to the spindle, as shown in Fig. 2.51.

These springs are made up of non-magnetic material such as silicon bronze, copper, platinum silver or german silver. However, mostly phosphor bronze spiral springs are used. Practically all instruments use flat spiral springs. The springs should be free from mechanical stress. Further, they should have a small resistance, sufficient cross-sectional area and a low resistance temperature co-efficient.

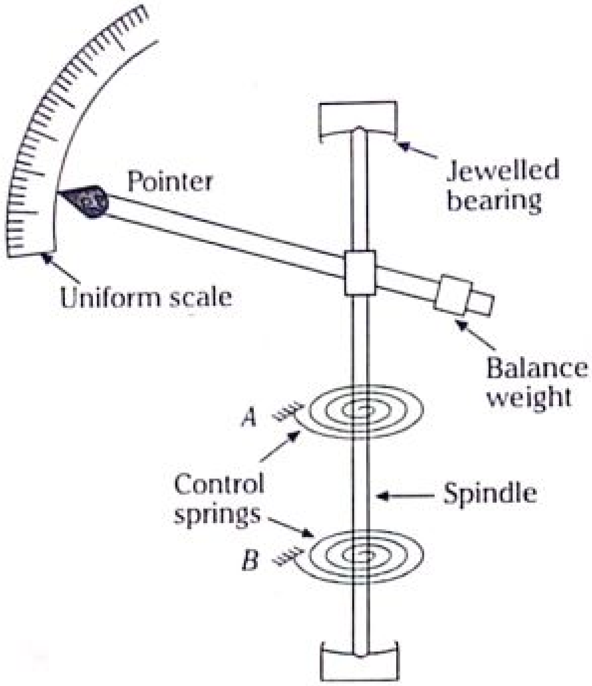


Fig. 2.51 Spring Control

When the spindle rotates, one of the springs gets compressed and the other spring gets extended. This offsets the effect of variations of temperature. The controlling torque produced by the instrument is directly proportional to the angular deflection (θ) of the pointer.

The controlling torque produced by the spiral springs is given by,

$$T_c = \frac{E b t^3}{12 L} \theta = K_S \theta$$

where E = Young's modulus of spring material in N/m^2

t = thickness in metres

b = depth in metres

L = length in metres

$$K_S = \text{spring constant} = \frac{Ebt^3}{12L}$$

$$\therefore \quad T_c \propto \theta$$

Now deflecting torque is proportional to current.

$$\therefore \quad T_d \propto I$$

At equilibrium, $T_d = T_c$

$$\therefore \quad I \propto \theta$$

Thus the deflection is proportional to the current. Hence the scale of the instrument using spring control is uniform. When the current is removed, due to spring force the pointer comes back to initial position. Spring control is used in almost all indicating instruments.

(ii) Gravity Control

In the gravity control method of producing controlling torque, a small control weight is attached to the spindle of the moving system of the instrument, whose position is adjustable.

Fig. 2.52 shows the *gravity control system*. At the zero position of the pointer, the controlling torque is zero.

This position is shown as position A of the control weight in Fig. 2.52.

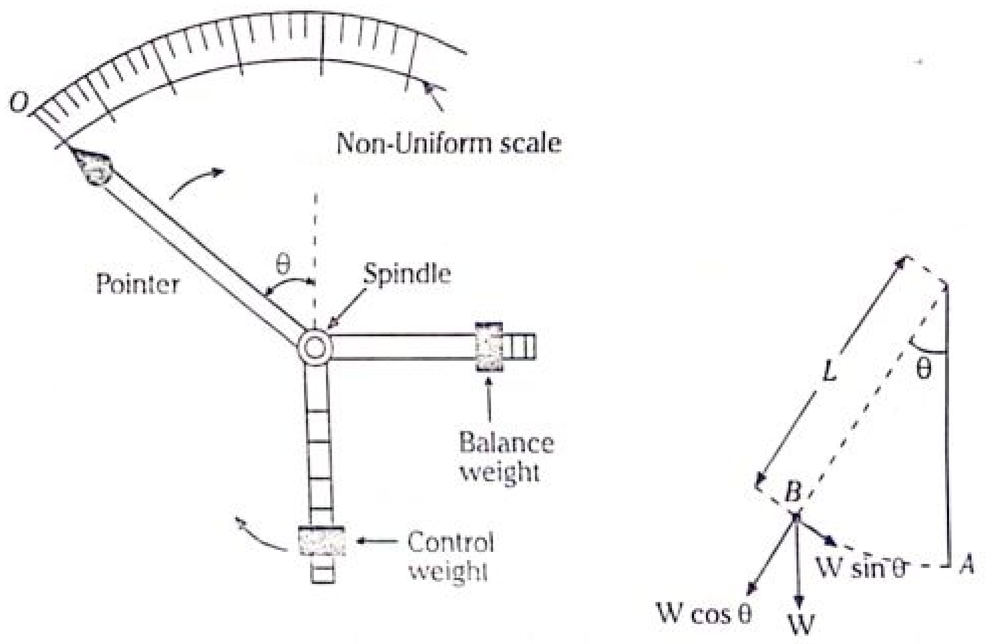


Fig. 2.52

When the spindle rotates under the impact of the *deflecting torque*, the control weight moves up as shown by the curved arrow, resulting in an angular deflection θ as shown in Fig. 4.3. The control weight acts at a distance L from the centre. The component $W \sin \theta$ of this weight tries to *restore* the pointer back to the initial zero position, which is the *controlling torque* T_c.

Thus,

$$\textit{Controlling torque } T_c = W \sin \theta \times L = K \sin \theta$$

$$\text{where } K = WL = \text{gravity constant}$$

Usually, meters are of the current-sensing type.

Hence,

Deflecting torque $T_d = K_t I$

Where K_t is another constant

When equilibrium is achieved, $T_d = T_c$

or $K_t I = K \sin \theta$

Hence $\boldsymbol{I \propto \sin \theta}$

This equation shows that *the deflection is proportional to the current, which is the quantity to be measured*. But as the current I is a function of $\sin \theta$, gravity-controlled instruments have a scale which is *not uniform* but is cramped or crowded at the lower end as shown in Fig. 2.52.

Advantages of Gravity Controlled Instrument

1. It is simple and inexpensive
2. Its performance is unaffected by temperature
3. Its operation is not time dependent
4. Its controlling torque may be altered by adjusting the position of the control weight.
5. This instrument is not subjected to *fatigue*.

Disadvantages

1. The non-uniform nature of the scale causes problems in accurate recording of readings.
2. The instrument has to be kept absolutely vertical and has to be properly levelled, otherwise there could be serious errors in measurement.
3. As the instrument is delicate in nature and is required to be accurately levelled, it is generally not used for indicating purposes and where portability is required.

(c) Damping Torque

The pointer of a measuring instrument moves upscale as the spindle rotates due to the deflecting torque. A controlling torque is brought into play due to the motion of the spindle, and it opposes the deflecting torque. As the angle of deflection increases, the magnitude of the controlling torque also increases (however, the deflecting torque continues to be steady in magnitude). Ultimately the two torques become equal and they cancel out and equilibrium is established; consequently, the pointer should come to rest. However, in practice, the pointer oscillates about the mean deflected position, due to which the final reading cannot be obtained.

So, in order to bring the pointer to rest within a short time, **damping torque** is required. This damping torque should be provided only when the moving system is operating & always *opposes its motion*. Damping torque is proportional to the velocity of the moving system but does not depend on the operating current. It should not affect controlling torque nor should it increase friction.

Fig. 2.53 shows the effect of damping on the variation of position with time of the moving system of an instrument.

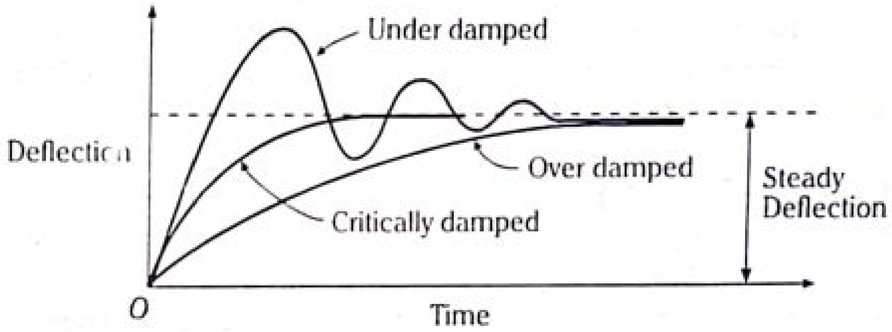


Fig. 2.53

The rapidity with which the moving system settles down to its final steady position depends on relative damping. If the moving system attains its final position quickly but smoothly without oscillations, the instrument is said to be **critically damped**. If the moving system oscillates about its final position with decreasing amplitude and takes some time before coming to rest, then the instrument is said to be **under-damped**. On the other hand, if the moving system moves slowly to its final position, then the instrument is said to be **overdamped**. In the case of an overdamped instrument, the response of the system is slow and sluggish. In practice, slightly underdamped systems are preferred.

Damping can be obtained by the following methods :

(i) Air Friction Damping

(ii) Fluid Friction Damping

(iii) Eddy Current Damping

(i) Air Friction Damping

In this method of damping, a light aluminum piston is attached to the spindle, and it is made to reciprocate inside an air chamber, as shown in Fig. 2.54.

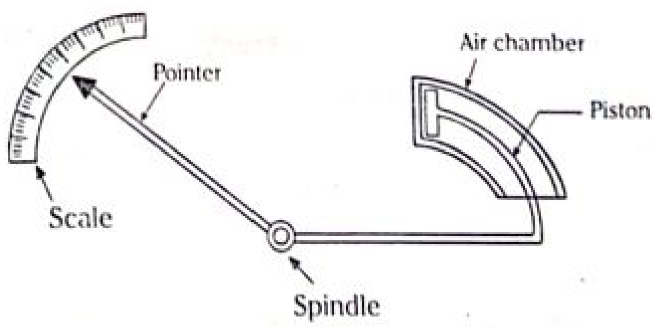


Fig. 2.54 Air friction damping

The piston moves to and fro in the chamber, when the spindle rotates with a small clearance between itself and the walls of the chamber, which is closed at one end. When the pointer moves clockwise, *i.e.*, upscale, the piston moves downwards. The air in the closed portion of the chamber expands and its pressure falls. The pressure in the open portion of the chamber forces the piston upwards. If the pointer moves anticlockwise, *i.e.*, downscale, the piston moves upwards, compressing the air above it. The increased air pressure forces the piston downwards. In this manner, *the motion of the pointer is opposed in either direction (i.e, clockwise or anticlockwise). Thus the oscillations get reduced, providing the necessary damping torque*, and the final steady position of the pointer is achieved very quickly, enabling the deflected position to be noted.

(ii) Fluid Friction Damping

In this method, damping is done by the movement of metallic vanes inside highly viscous liquid. The damping force due to the fluid is greater than that of air due to its viscosity. The arrangement is shown in Fig. 2.55. Thin metallic vanes which are suitably shaped are suspended from the spindle and are kept immersed in the liquid.

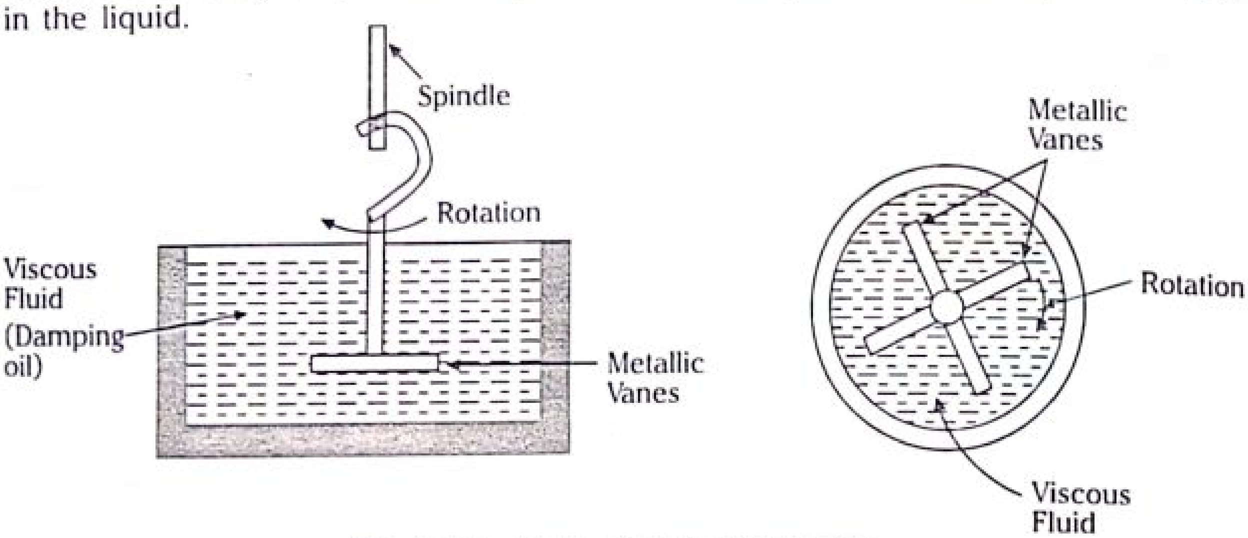


Fig. 2.55 Fluid Friction Damping

When the spindle rotates, the metallic vanes also move through the oil. The frictional force between the oil and the vanes is used to produce the **damping torque**, which opposes the oscillation of the pointer.

The advantages of this method are :

1. As the fluid is viscous more damping is achieved.
2. The oil may be used as an insulator.
3. Due to the upward thrust of the oil, the load on the bearings is reduced, thus reducing frictional errors.

The disadvantages of this method are :

1. It is difficult to keep the instruments clean due to oil leakage.

2. This method of damping can be used only for those instruments which are in a vertical position.

(iii) Eddy Current Damping

Eddy current damping is the most effective and efficient of the three. Damping is produced due to eddy currents induced in an aluminium disc rigidly mounted on the spindle of the moving system carrying the pointer, as shown in Fig. 2.56.

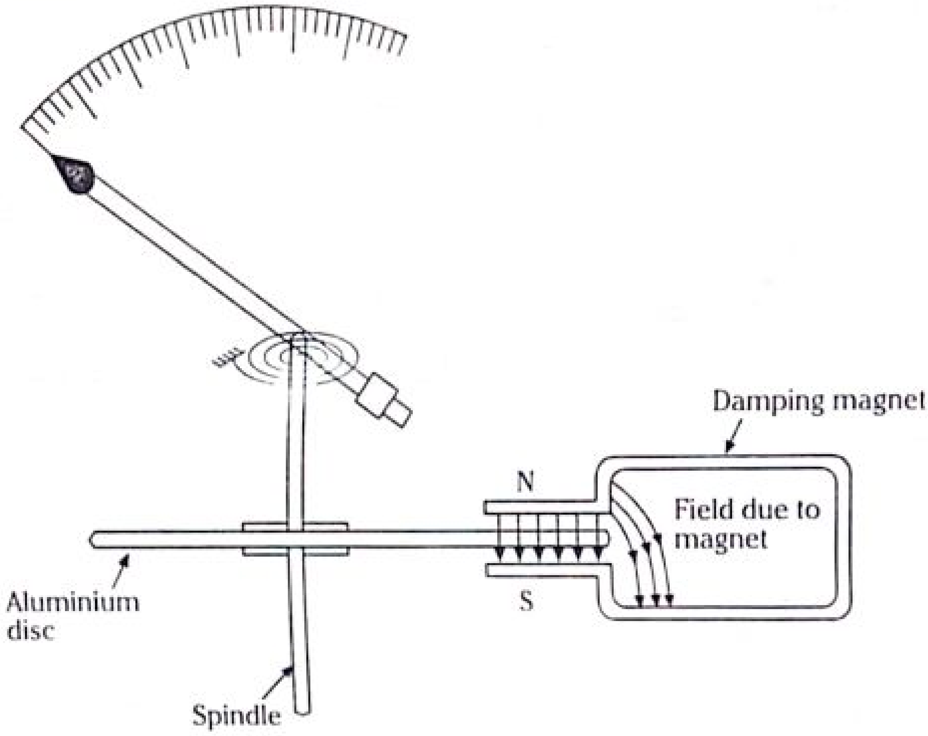
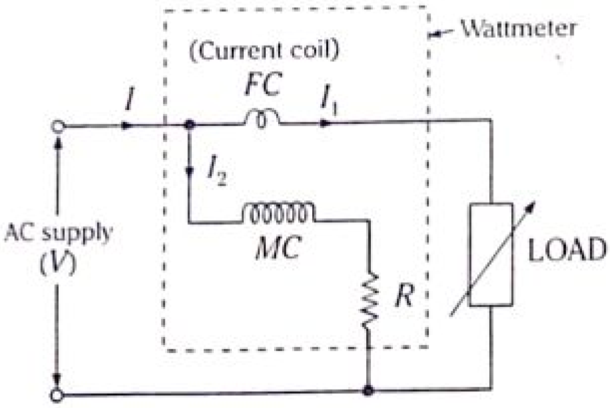


Fig. 2.56 Eddy current damping

The disc is circular and it is so positioned that, when it rotates, its edges cut across the flux due to a small permanent magnet (damping magnet). Due to the flux of the magnet being cut across by the edge of the disc, *eddy currents are induced in the disc*. **As a result of the interaction between these eddy currents and the flux producing them, a torque is set up.** As per Lenz's Law, the direction of this torque is such as to oppose the motion of the spindle. Therefore, damping is brought about.

2.24 Dynamometer Type Wattmeter

This instrument is used to measure power consumption in single-phase AC circuits. It consists of two fixed coils, *FC*-1 and *FC*-2 and a movable coil *MC*, as shown in Fig. 2.57. The fixed coil forms the current coil *CC*, which carries the entire current or a definite fraction of it, and the moving coil *MC* forms the voltage coil which carries a current proportional to the voltage.



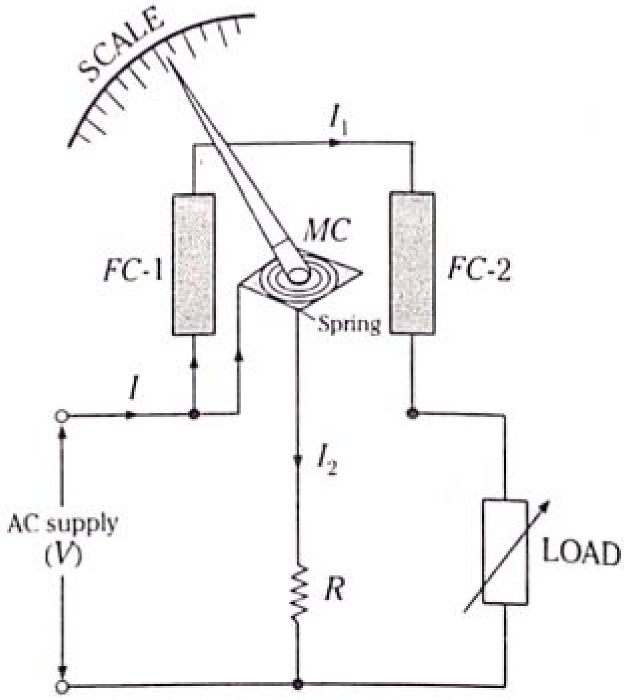


Fig. 2.57 Dynamometer-type Wattmeter
(a) Schematic Diagram (b) Functional Circuit

When currents flow simultaneously through both fixed and moving coils, as during a measurement, a torque is produced, and this, acting on the moving coil, tends to turn it. The moving coil is carried on the spindle and hence the spindle also rotates and causes a pointer fixed to it to move over a graduated scale. The deflecting torque produced must depend on both currents and hence we have $T_d \propto I_1 I_2$.

But $I_2 \propto V$, the supply voltage.

$\therefore$ $T_d \propto VI_1$, *i.e.*, the deflecting torque is proportional to the power in the circuit. The controlling torque T_c, is obtained by means of spring control. Therefore we have $T_c \propto \theta$ (deflection). At equilibrium, $T_c = T_d$, hence **the deflection indicates directly the power consumed**.

The damping is generally obtained by means of air friction damping mechanism. Stray magnetic fields can affect the accuracy of these meters unless they are properly shielded. In the case of measurement of power of a high voltage circuit, instrument transformers could be employed.

2.25 Induction Type Single Phase Energy Meter

A single phase induction-type energy meter is used to measure the quantity of electrical energy supplied to a single phase circuit in a given time (measured in

kilo-watt hours).

Construction : A single-phase induction-type energy meter has the following systems/mechanisms (Fig. 2.58).

a) Moving System

b) Operating Mechanism

c) Recording Mechanism

a) Moving System : The moving system consists of a light aluminium disc mounted on a vertical spindle and supported on a sapphire cup contained in a bottom bearing screw. The bottom pivot, which is usually removable, is of hardened steel, and the end, which is hemispherical in shape, rests in the sapphire cup. The top pivot merely serves to maintain the spindle in a vertical position under working conditions and neither supports any weight nor does it exert appreciable thrust in any direction.

There is no pointer and control spring, so that the disc rotates continuously because of the deflecting torque.

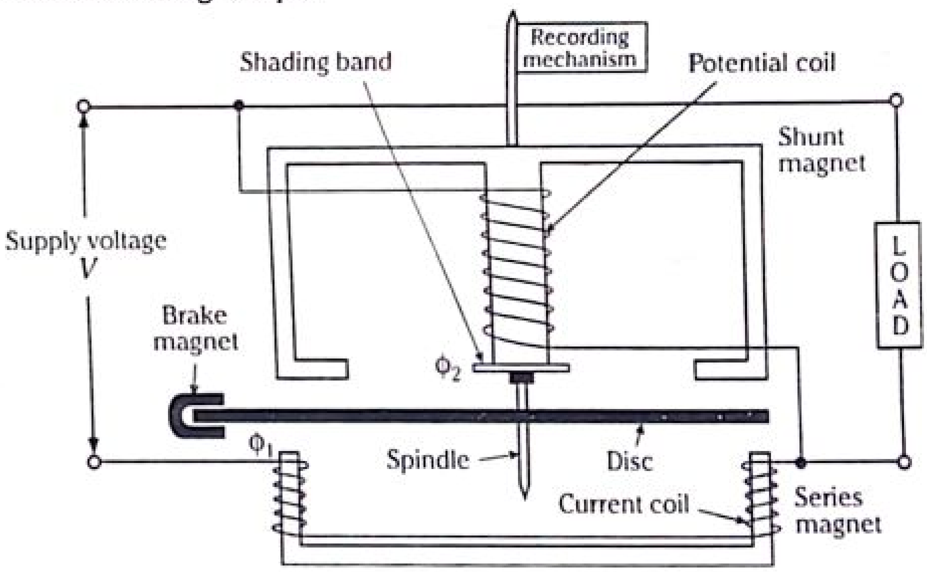


Fig. 2.58

b) Operating Mechanism : The operating mechanism has the following components :

i) Series Magnet

ii) Shunt Magnet

iii) Brake Magnet

i) *Series Magnet* : The series magnet consists of a number of U-shaped iron laminations assembled together to form a core. Each of its two limbs is wound with a few turns of heavy gauge wire. The wound coil is called

current coil, and is connected in series with the load to be metered. The series magnet is placed under the aluminium disc, and produces a magnetic field proportional to and in phase with the current.

ii) *Shunt Magnet* : The shunt magnet consists of a number of M-shaped iron laminations, assembled together to form a core. A coil having a large number of turns of fine wire is fitted on the middle limb of the shunt magnet. This wound coil is called the *potential coil* and is connected across the load, so that it carries current proportional to the supply voltage V. The shunt magnet is situated above the aluminium disc, as shown. A copper shading band (also called 'power factor compensator') is placed over the central limb in order to ensure a 90° phase difference between the flux produced by the supply voltage and the supply voltage itself.

iii) *Brake-Magnet* : The speed of the aluminium disc is controlled by the brake magnet.

c) Recording Mechanism : The number of revolutions of the disc is a measure of the electrical energy passing through the meter and is recorded on dials which are geared to the shaft.

Working : When the line current flows through the current coil, the series magnet is excited. The alternating flux ϕ_1 produced by it is proportional to and in phase with the line current (if effects of hysteresis and iron saturation are ignored). The potential coil of the shunt magnet is connected across the supply line and carries current proportional to the supply voltage V. The flux ϕ_2 produced by it is proportional to the supply voltage V and lags behind it by 90°. This phase displacement of exactly 90° is brought about by adjusting the copper shading band. Most of the flux ϕ_2 crosses the narrow gap between the centre and side-limbs of the shunt magnet.

However, a small quantity, which is the useful flux, passes through the disc. The fluxes ϕ_1 and ϕ_2 induce e.m.f.s in the disc, which give rise to circulatory eddy currents. The reaction between these two fluxes and eddy currents produces the driving torque, which causes the disc to rotate.

The speed of the aluminium disc is controlled to the required value by the C-shaped permanent brake magnet. The magnet is mounted so that the disc revolves in the air-gap between the polar extremities. When the peripheral portion of the rotating disc passes through the air-gap of the brake magnet, eddy currents are induced in it, giving rise to the required torque. The braking torque $T_B \propto \dfrac{\phi^2 N}{R}$, where ϕ is the flux of the brake magnet, N the speed at which the disc rotates and

R is the resistance of the eddy current path. If ϕ and R are constant, $T_B \propto N$. By altering the position of the brake magnet, the desired speed may be obtained. The spindle is geared to the recording mechanism so that the electrical energy consumed in the circuit is directly registered in kWh.

Theory : The current coil carries current proportional to load current, while the potential coil carries current proportional to the voltage.

$$\therefore \quad \text{Average deflecting torque, } T_d \propto \text{Average power}$$

$$\propto VI \cos\phi$$

$$\therefore \quad T_d = k_1 VI \cos\phi$$

But we have seen that the braking torque T_B is proportional to the speed of the disc N, *i.e.*,

$$T_B \propto N$$

$$\therefore \quad T_B = k_2 N$$

The disc rotates at a steady speed N when the braking torque equals the deflecting torque *i.e.*,

$$T_B = T_d$$

$$\therefore \quad k_2 N = k_1 VI \cos\phi$$

Multiplying both sides by time 't', we have

$$k_2 Nt = k_1 VI \cos\phi \cdot t$$

$$\text{or} \quad Nt = \frac{k_1}{k_2} VI \cos\phi \cdot t$$

$$\text{If} \quad \frac{k_1}{k_2} = k_3 \quad \text{and} \quad P = VI \cos\phi, \quad \text{then}$$

$$Nt = k_3 Pt$$

The expression 'Nt' gives the no. of revolutions of the disc in time 't', and the expression 'Pt' represents the energy passing through the meter in time 't'.

Therefore, *the no. of revolutions of the disc $\propto$ electrical energy passing through the meter*.

2.25 Review Questions

Q 1. Explain the principle of operation of a Dynamomeler Type Wattmeter.

(Nov/Dec 84, June/July 89, B.U.)

Q 2. With the help of a neat diagram, describe the constructional features and working of a Dynamometer Wattmeter.

(Mar/Apr 88, M.U.)

Q 3. Describe with a neat sketch, the constructional details and operation of a Single-phase Energy Meter.

(June 81, Feb/Mar 83, Dec 86, Jan 93, B.U.; Feb 96 B.U.; Mar 89, M.U.; Aug/Sep 89, M.U.)

Q 4. Write brief explanatory note on

a) Single-phase energy meter.

b) Dynamometer type wattmeter.

(Aug 95, Feb 96, B.U.)

Equation of Alternating E.M.F.

Let us take up the case of a rectangular coil of N turns rotating in the anticlockwise direction, with an angular velocity of ω radians per second in a uniform magnetic field as shown in Fig. 3.3. Let the time be measured from the instant of coincidence of the plane of the coil with the X-axis. At this instant maximum flux, ϕ_{max}, links with the coil. As the coil rotates, the flux linking with it changes and hence e.m.f. is induced in it. Let the coil turn through an angle θ in time 't' seconds, and let it assume the position as shown in Fig. 3.3. Obviously $\theta = \omega t$.

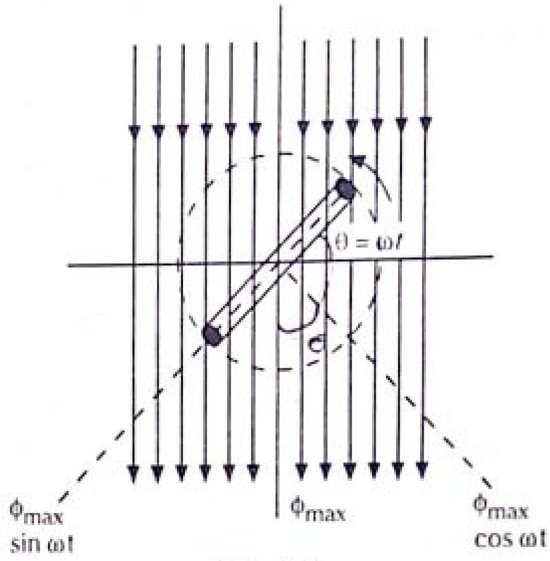


Fig. 3.3

When the coil is in this position, the maximum flux acting vertically downwards can be resolved into two components, each perpendicular to the other, namely :

a) Component $\phi_{max} \sin \omega t$, parallel to the plane of the coil. This component does not induce e.m.f. as it is parallel to the plane of the coil.

b) Component $\phi_{max} \cos \omega t$, perpendicular to the plane of coil. This component induces e.m.f. in the coil.

$\therefore$ Flux linkages of coil at that instant (at $\theta°$) is

$$= \text{No. of turns} \times \text{flux linking}$$

$$= N\phi_{max} \cos \omega t$$

As per Faraday's Laws of Electromagnetic Induction, the e.m.f induced in a coil is equal to the rate of change of flux linkages of the coil. So, instantaneous e.m.f 'e' induced in the coil at this instant is :

$$e = -\frac{d}{dt} \text{(flux linkages)}$$

$$= -\frac{d}{dt} (N\phi_{max} \cos \omega t)$$

$$= -N\phi_{max} \frac{d}{dt} (\cos \omega t)$$

$$= -N\phi_{max} \omega (-\sin \omega t)$$

$$\therefore \quad e = +N\omega\phi_{max} \sin \omega t \text{ volts} \qquad \text{---(1)}$$

It is apparent from eqn.(1) that the value of 'e' will be maximum (E_m), when the coil has rotated through 90° (as sin 90° = 1)

Thus $\quad E_m = N\,\omega\,\phi_{max}$ volts ---(2)

Substituting the value of $N\omega\ \phi_{max}$ from eqn.(2) in eqn.(1), we obtain :

$$e = E_m \sin \omega t \quad \text{---(3)}$$

We know that $\theta = \omega t$

$$\therefore \quad \boldsymbol{e = E_m \sin\theta}$$

It is clear from this expression of alternating e.m.f. induced in the coil that instantaneous e.m.f. varies as the sine of the time angle (θ or ωt).

$\omega = 2\pi f$, where 'f' is the frequency of rotation of the coil. Hence eqn.(3) can be written as

$$e = E_m \sin 2\pi f t \quad \text{---(4)}$$

If T = time period of the alternating voltage $= \frac{1}{f}$, then eqn.(iv) may be re-written as

$$e = E_m \sin\left(\frac{2\pi}{T}\right)t$$

So, the e.m.f. induced varies as the sine function of the time angle, ωt, and if e.m.f. induced is plotted against time, a curve of sine wave shape is obtained as shown in Fig. 3.4. Such an e.m.f. is called *sinusoidal e.m.f.* The sine curve is completed when the coil moves through an angle of 2π radians.

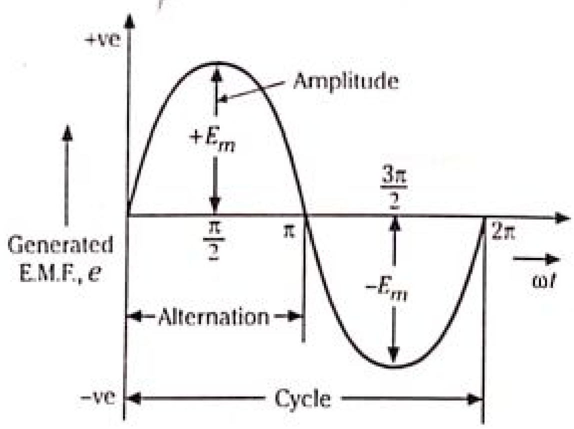


Fig. 3.4

Problem 3.1

An alternating e.m.f. is mathematically expressed as: $e = 200 \sin 314\,t$. Find (i) the amplitude (ii) frequency and (iii) the instantaneous value, when t = 1/200 sec.

Solution :

The given e.m.f equation is $e = 200 \sin 314\,t$. When we compare this with the standard equation $e = E_m \sin 2\pi f t$, we have

(i) Amplitude, E_m = **200 Volts**

(ii) $2\pi f = 314$

or Frequency $f = \frac{314}{2\pi} = \mathbf{50\ Hz}$

(iii) Substituting $t = \frac{1}{200}$ sec, we get

$$e = 200 \sin\left(314 \times \frac{1}{200}\right), \text{ if the angle is in radians.}$$

If the angle is expressed in degrees :

$$e = 200 \sin\left(2 \times 180 \times 50 \times \frac{1}{200}\right), \quad \text{putting } \pi = 180°$$

$$= 200 \sin 90 = \mathbf{200\ Volts.}$$

Equation of Alternating Current

When an alternating voltage $e = E_m \sin \omega t$ is applied across a load, alternating current flows through the circuit which will also have a sinusoidal variation. The expression for the alternating current is given by :

$$i = I_m \sin \omega t$$

In this case the load is resistive (we shall see, later on, that if the load is inductive or capacitive, this current-equation is changed in time angle).

Problem 3.2

An alternating current has a peak value of 141.4 Amps and its frequency is 100 Hz. Write down the mathematical expression for the current.

Solution :

Given : $I_m = 141.4$ Amps and $f = 100$ Hz

The basic equation is $i = I_m \sin 2\pi f t$

or $i = 141.4 \sin(2\pi \times 100 \times t)$

or $\mathbf{i = 141.4 \sin 628\, t}$

3.3 Different Forms of E.M.F. Equation

The standard form of an alternating voltage, as already mentioned in Sec. 3.2 is

$$e = E_m \sin \theta = E_m \sin \omega t = E_m \sin 2\pi f t = E_m \sin \frac{2\pi}{T} t$$

On perusal of the above equations, we find that

a) The amplitude or peak value or maximum value of an alternating voltage is given by the coefficient of the sine of the time angle.

b) The frequency 'f' is given by the coefficient of time divided by 2π.

Taking an example, if the equation of an alternating voltage is given by $e = 20 \sin 314t$, then its maximum value is 20 V and its frequency is

$$f = \frac{314}{2\pi} = 50 \text{ Hz}$$

In a like manner, if the equation is of the form

$$e = I_m \sqrt{(R^2 + 4\omega^2 L^2)} \sin 2\omega t, \text{ then its maximum value is}$$

$$E_m = I_m \sqrt{(R^2 + 4\omega^2 L^2)} \text{ and the frequency is}$$

$$\frac{2\omega}{2\pi} \text{ or } \frac{\omega}{\pi} \text{ Hertz}$$

Problem 3.3

An alternating current i is given by $i = 141.4 \sin 314t$. Find (i) the maximum value (ii) frequency (iii) time period and (iv) the instantaneous value when t is 3 milliseconds.

Solution : An alternating current i is given by the standard equation

$$i = I_m \sin \omega t$$

The given equation is

$$i = 141.4 \sin 314t$$

Comparing the two equations, we have

(i) I_m = **141.4 Amps**

(ii) Frequency, $f = \frac{\omega}{2\pi} = \frac{314}{2\pi} = \mathbf{50 \text{ Hz}}$

(iii) Time period, $T = \frac{1}{f} = \frac{1}{50} = \mathbf{0.02 \text{ second}}$

(iv) When $t = 3 \times 10^{-3}$ seconds, the instantaneous value of the current is

$$i = 141.4 \sin \left[314 \times 3 \times 10^{-3}\right]$$

$$= \mathbf{114.35 \text{ Amps}}$$

3.4 Root-Mean-Square (R.M.S.) Value

The r.m.s. or effective value, of an alternating current is defined as that steady current which when flowing through a given resistance for a given time produces the same amount of heat as produced by the alternating current, when flowing through the same resistance for the same time.

Let us take two circuits with identical resistances, but one is connected to a battery and the other to a sinusoidal voltage source. Wattmeters are employed to measure heat power in each circuit. The voltage applied to each circuit is so adjusted that the heat power produced in each circuit is the same. In this event the direct current I will equal $\frac{I_m}{\sqrt{2}}$, which is termed r.m.s. value of the sinusoidal current.

The following method is used for finding the r.m.s. or effective value of sinusoidal waves.

The equation of an alternating current varying sinusoidally is given by

$$i = I_m \sin\theta$$

Let us consider an elementary strip of thickness $d\theta$ in the first half cycle of the squared wave, as shown in Fig. 3.5. Let i^2 be the mid-ordinate of this strip.

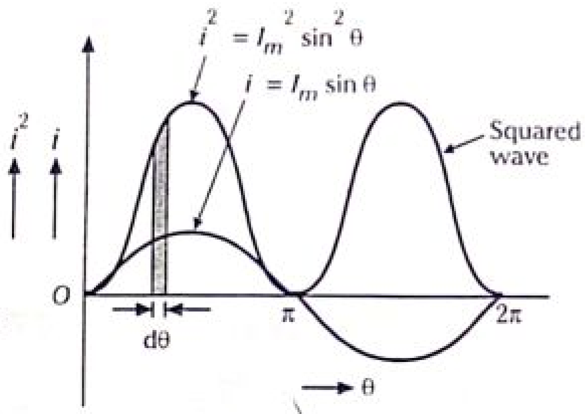


Fig. 3.5

Area of the strip $= i^2\, d\theta$

Area of first half-cycle of squared wave

$$= \int_0^\pi i^2\, d\theta$$

$$= \int_0^\pi (I_m \sin\theta)^2\, d\theta \qquad (\because\ i = I_m \sin\theta)$$

$$= \int_0^\pi I_m^2 \sin^2\theta\, d\theta$$

$$= I_m^2 \int_0^\pi \frac{1-\cos 2\theta}{2} d\theta \qquad \left(\because\ \sin^2\theta = \frac{1-\cos 2\theta}{2}\right)$$

$$= \frac{I_m^2}{2}\int_0^\pi (1-\cos 2\theta)\, d\theta$$

$$= \frac{I_m^2}{2}\left[\theta - \frac{\sin 2\theta}{2}\right]_0^\pi$$

$$= \frac{I_m^2}{2}\left[(\pi - 0) - (0 - 0)\right]$$

$$= \frac{\pi I_m^2}{2}$$

$$\therefore \quad I = \sqrt{\frac{\text{Area of first half cycle of squared wave}}{\text{base}}}$$

$$= \sqrt{\frac{\pi I_m^2}{2} \times \frac{1}{\pi}}$$

$$= \sqrt{\frac{I_m^2}{2}}$$

$$= \frac{I_m}{\sqrt{2}} = 0.707 I_m$$

Hence, for a sinusoidal current,

R.M.S. value of current = 0.707 × *maximum value of current*.

Similarly, $E = 0.707\ E_m$

3.5 Average Value

The arithmetical average of all the values of an alternating quantity over one cycle is called ***average value.***

In the case of a symmetrical wave *e.g.* sinusoidal current or voltage wave, the positive half is exactly equal to the negative half, so that the average value over the entire cycle is zero. Hence, in this case, the average value is obtained by adding or integrating the instantaneous values of current over one alternation (half-cycle) only.

The equation of a sinusoidally varying alternating voltage is given by $e = E_m \sin \theta$.

Let us take an elementary strip of thickness $d\theta$ in the first half-cycle as shown in Fig. 3.6. Let the mid-ordinate of this strip be 'e'.

Area of the strip $= e\ d\theta$

Area of first half-cycle

$$= \int_0^{\pi} e\ d\theta$$

$$= \int_0^{\pi} E_m \sin\theta \, d\theta \qquad (\because e = E_m \sin\theta)$$

$$= E_m \int_0^{\pi} \sin\theta \, d\theta$$

$$= E_m \left[-\cos\theta\right]_0^{\pi} = 2E_m$$

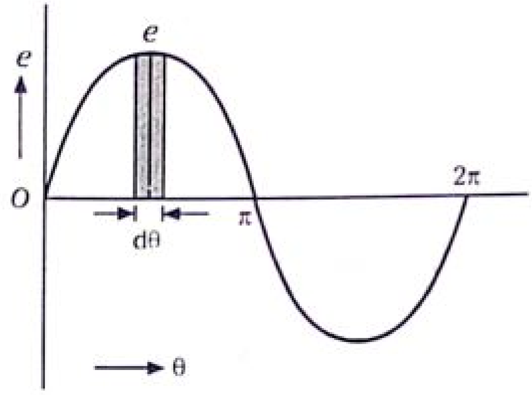


Fig. 3.6

$\therefore$ Average value, $E_{av} = \dfrac{\text{Area of half cycle}}{\text{base}} = \dfrac{2E_m}{\pi}$

or $E_{av} = 0.637\, E_m$

In a similar manner, we can prove that, for alternating current varying sinusoidally,

$$I_{av} = 0.637\, I_m$$

$\therefore$ **Average value of Current = 0.637 × Maximum value**

3.6 Form Factor and Crest or Peak or Amplitude Factor (K_f)

A definite relationship exists between crest value (or peak value), average value and r.m.s. value of an alternating quantity.

1. Form Factor : *The ratio of effective value (or r.m.s. value) to average value of an alternating quantity (voltage or current) is called form factor, i.e.,*

$$\text{Form Factor,} \quad K_f = \frac{\text{r.m.s. value}}{\text{average value}}$$

For sinusoidal alternating current,

$$K_f = \frac{0.707\, I_m}{0.637\, I_m} = 1.11$$

For sinusoidal alternating voltage,

$$K_f = \frac{0.707\, E_m}{0.637\, E_m} = 1.11$$

Hence, the **R.M.S. value (of current or voltage) is 1.11 times its average value.**

2. **Crest or Peak or Amplitude Factor (K_a)** : *It is defined as the ratio of maximum value to the effective value (r.m.s. value) of an alternating quantity. i.e.,*

$$K_a = \frac{\text{maximum value}}{\text{r.m.s. value}}$$

For sinusoidal alternating current,

$$K_a = \frac{I_m}{\frac{I_m}{\sqrt{2}}} = \sqrt{2} = 1.414$$

For sinusoidal alternating voltage,

$$K_a = \frac{E_m}{\frac{E_m}{\sqrt{2}}} = 1.414$$

The knowledge of Crest Factor is particularly important in the testing of dielectric strength of insulating materials ; this is because the breakdown of insulating materials depends upon the maximum value of voltage.

Problem 3.4

An alternating voltage has an amplitude of 100 V. Find its (i) R.M.S. value (ii) Average value

Solution :

(i) R.M.S. value $E = \frac{E_m}{\sqrt{2}} = 0.707\ E_m$

Given $E_m = 100$ V

$\therefore \quad E = 0.707 \times 100$

$= \mathbf{70.7\ volts}$

(ii) Average value $E_{av} = \frac{E_m}{(\pi/2)} = 0.637\ E_m$

$\therefore \quad E_{av} = 0.637 \times 100$

$= \mathbf{63.7\ volts}$

Problem 3.5

An alternating current has an effective value of 200 A. If its frequency is 25 Hz, find its average value and write down the expression for the current.

Solution :

Given : Effective value (R.M.S), I = 200A and f = 25 Hz

Now, Average value $= \dfrac{\text{R.M.S Value}}{1.11}$

or $I_{av} = \dfrac{I}{1.11} = \dfrac{200}{1.11} = \mathbf{180.18\ Amps}$

Also, maximum value $= \sqrt{2} \times$ R.M.S. value

or $I_m = \sqrt{2} \times 200 = 282.84$ A

The expression for current is

$$i = I_m \sin 2\pi ft = 282.84 \sin (2\pi \times 25 \times t)$$

or $\mathbf{i = 282.84 \sin 157\ t}$

3.7 Complex Algebra

3.7.1 Scalar and Vector

A **Scalar** is a quantity which is completely determined by its *magnitude* alone. Examples: Energy, gallons, mass *etc.* These are added algebraically, e.g : 3 glns + 5 glns = 8 glns. A **Vector** quantity has both *direction* and *magnitude*, e.g. Force, current, voltage etc. When a Vector quantity like force is considered, its magnitude as well as its direction is considered. When two forces are added, they are not added algebraically but must be combined in such a way as to consider their *directions* and *magnitudes*. Fig. 3.7 shows two forces acting at a point 'O' & shown by vectors F_1 & F_2. The length of each of these vectors, to scale, is equal to *magnitude* of the forces, and the *direction* of each of these vectors shows the direction in which the force acts, 'θ' being the angle between F_1 and F_2. Their sum F_r would have the same effect at their point of application 'O', as F_1 and F_2 acting in *conjunction* and is known as their *resultant*, F_r being the diagonal of the parallelogram having F_1 and F_2 as adjacent sides.

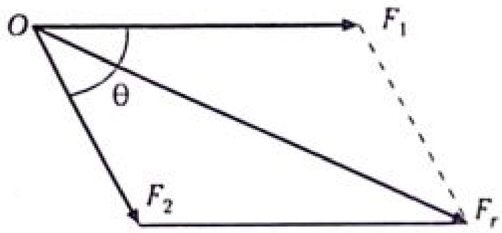


Fig. 3.7 Sum of the two vectors by parallelogram method

3.7.2 Complex Number

A complex number is a number consisting of real and imaginary components. We may analyze any type of a.c. circuit with the help of complex numbers, but they are particularly suitable for solving series - parallel circuits that have both resistance and reactance in one or more branches. We shall soon be dealing extensively with complex numbers in this Chapter.

3.7.3 Positive and Negative Numbers

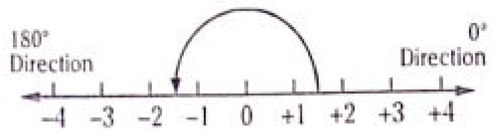


Fig. 3.8

Numbers have both quantity and phase angle. As shown in Fig. 3.8, positive and negative numbers are shown as corresponding to the phase angles of 0° and 180° respectively. For example, 2, 3, 4, represent units along the horizontal or x-axis, extending towards the right along the line of zero phase angle.

In other words, positive numbers represent units having the phase angle of 0°; or this phase angle corresponds to the factor of +1. To indicate 4 with zero phase angle, 4 is multiplied by +1 as a factor for the positive number +4.

In the opposite direction, negative numbers correspond to 180°, so this phase angle corresponds to the factor of −1. In effect, −4 represents the same quantity as 4 but rotated through the phase angle of 180°. The angle of rotation is the *operator* for the number. The operator for −1 is 180" and the operator for +1 is 0°.

3.7.4 The j-Operator

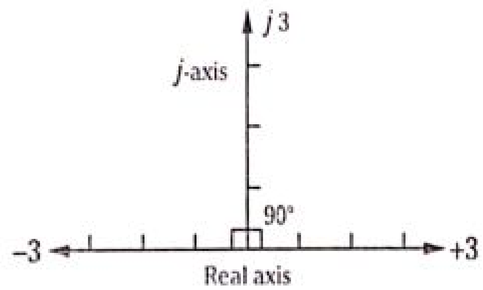


Fig. 3.9 j-axis at right angles to real axis

The operator for a number can be any angle between 0° and 360°. The factor 'j' indicates 90°. Referring to Fig. 3.9, the number 3 means 3 units at 0°, the number −3 is at 180°, while $j3$ shows the 90° angle. The angle of 180° corresponds to the *j operation* of 90° repeated twice; this angular rotation is indicated by the factor j^2.

As j^2 means 180°, which corresponds to the factor of −1, it follows that j^2 is the same as −1. Thus, the operator j^2 for a number means multiply by −1. For example, $j^2\ 4 = -4$. Going further, the angle of 270° is the same as −90°, which corresponds to the operator $-j$. Thus, summarizing the features of the j operator,

$$0^\circ = 1$$

$$90^\circ = j$$

$$180^\circ = j^2 = -1$$

$$\begin{aligned} 270^\circ &= j^3 \\ &= j^2 \times j \\ &= -1 \times j \\ &= -j \end{aligned}$$

$$360^\circ = \text{same as } 0^\circ$$

This is depicted diagrammatically in Fig. 3.10.

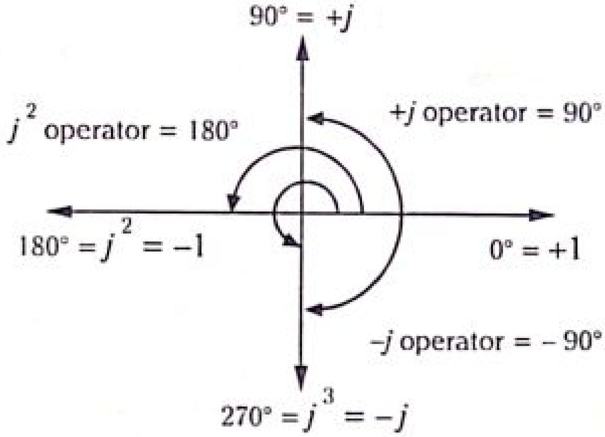


Fig. 3.10 Features of a *j*-operator

A complex number is the combination of a real and an imaginary term. The real number is written first. For example, $2+j3$ is a complex number comprising 2 units on the real axis added to 3 units 90° out of phase on the j axis. The term *complex number* implies that its terms must be added as phasors.

Phasors for complex numbers are shown in Fig. 3.11. The j phasor is up for 90° and the $-j$ phasor is down for −90°. The phasors are shown with one end joined to the start of the next. Graphically, the sum is the hypotenuse of the right angle formed by the two phasors. Since a number like $2+j3$ specifies phasors in rectangular co-ordinates, this system is the *rectangular form* of complex numbers.

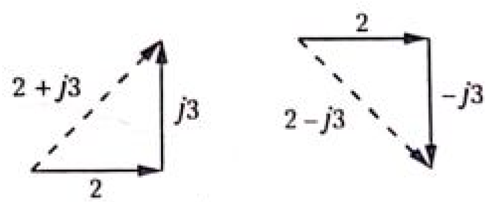


Fig. 3.11

3.7.5 Application of Complex Numbers to AC Circuits

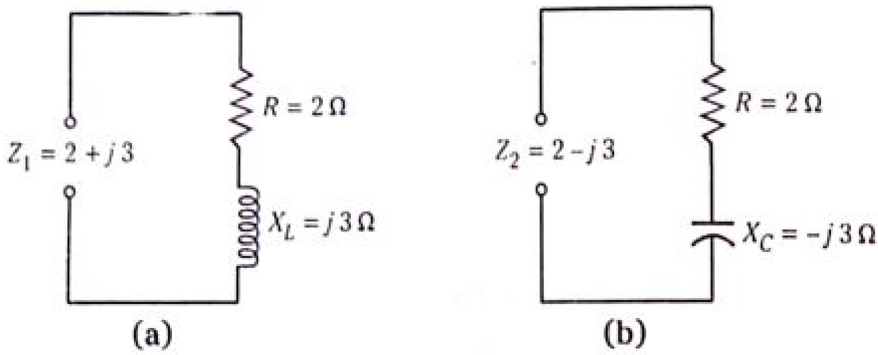


Fig. 3.12

Applications involve using a real term for 0°, $+j$ for 90°, and $-j$ for $-90°$. Fig. 3.12 illustrates the following rules :

a) **An angle of 0° or a real number** without any j operator is used for resistance R. For example, $2\,\Omega$ of R is expressed as just $2\,\Omega$.

b) **An angle of 90° or $+j$** is used for inductive reactance X_L. For example, a $3\,\Omega$ X_L is $j3\,\Omega$. This rule is always applicable to X_L, whether it is in series or in parallel with R. The reason for this is the fact that V_L represents voltage across an inductance, which always leads the current through the inductance by 90°. The $+j$ is also used for V_L.

c) ***An angle of* $-90°$ or $-j$** is used for capacitive reactance X_C. For example, a $3\,\Omega$ X_C is $-j3\,\Omega$. This rule always applies to X_C, whether it is in series or in parallel with R. The reason is the fact that V_C represents voltage across a capacitor, which always lags the charge and discharge current of the capacitor by $-90°$. The $-j$ is also used for V_C.

With reactive branch currents, the sign for j is reversed, compared with reactive ohms, because of the opposite phase angle. As shown in Fig. 3.13 (a) and (b), $-j$ is used for inductance branch current I_L and $+j$ for capacitive branch current I_C.

3.7.6 Impedance in Complex Form

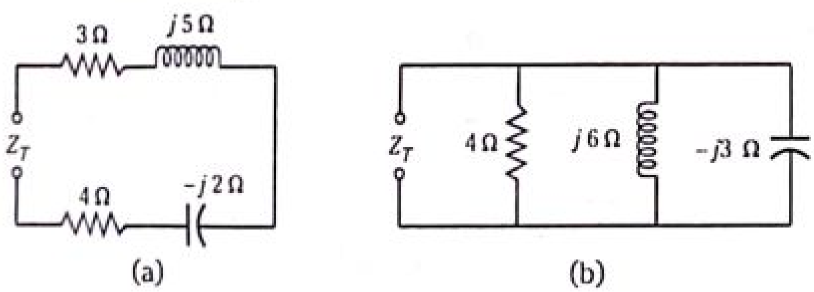


Fig. 3.13

The impedance of a resistance in series with a reactance is expressed in the rectangular form of complex numbers. In Fig. 3.12 (a), the impedance is $Z_1 = 2 + j3$, which is the phasor sum of a 2 Ω resistance in series with an inductive reactance X_L of $j3\ \Omega$. Similarly, impedance $Z_2 = 2 - j3$ is the phasor sum of a 2 Ω resistance in series with a capacitive reactance X_c of $-j3\ \Omega$ [Fig. 3.12 (b)].

Multiple impedances can be written as complex numbers and then calculated thus

$$Z_T = Z_1 + Z_2 + Z_3 + \ldots\ldots \quad \text{for series impedances}$$

$$\frac{1}{Z_T} = \frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3} + \ldots\ldots\ldots \quad \text{for parallel impedances}$$

$$Z_T = \frac{Z_1 \times Z_2}{Z_1 + Z_2} \ldots\ldots\ldots \quad \text{for two parallel impedances}$$

Fig. 3.13 and 3.14 show some typical examples. Fig. 3.13 (a) shows a series combination of resistances and reactances. Here, by combining the real terms and j terms separately,

$$Z_T = (3+4) + (j5 - j2) = 7 + j3$$

Taking the parallel combination of a resistance, inductive reactance and capacitive reactance, as in Fig. 3.13 (b),

$$\frac{1}{Z_T} = \frac{1}{4} + \frac{1}{j6} + \frac{1}{-j3}$$

Next, dealing with complex branch impedances Z_1 and Z_2 in parallel, as in Fig. 3.14.

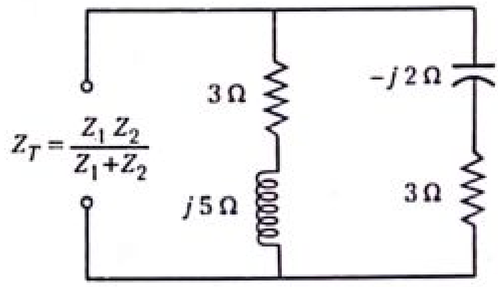


Fig. 3.14

$$Z_T = \frac{(3+j5) \times (3-j2)}{(3+j5) + (3-j2)}$$

The impedance Z_T stated here is expressed as a complex impedance. Calculation of Z_T can be performed by using some of the rules in the next Section.

3.7.7 Operations with Complex Numbers

Real numbers and j numbers cannot be directly combined as they are 90° out of phase. The following rules are applied :

a) Addition or Subtraction

Real and j numbers are added or subtracted separately:

$$(2 + j3) + (4 + j5) = 2 + 4 + j3 + j5 = 6 + j8$$

$$(2 + j5) + (4 - j2) = 2 + 4 + j5 - j2 = 6 + j3$$

$$(2+j5)+(3-j7) = 2+3+j5-j7 = 5-j2$$

The result should be in the form $R \pm jX$, where R is the algebraic sum of all the real or resistive terms and X is the algebraic sum of all the imaginary or reactive terms.

b) Multiplication or Division of a *j* Term by a Real Number

The numbers are simply multiplied or divided and the result will be still a *j* term. Where both the factors have the same sign, either + or −, the result is +; if one factor is negative, the result is negative.

$$2 \times j3 = j6 \qquad j8 \div 4 = j2$$

$$j3 \times (-4) = -j12 \qquad -j8 \div (-2) = j4$$

$$-j5 \times 6 = -j30 \qquad -j8 \div 2 = -j4$$

$$-j5 \times (-3) = j15 \qquad j12 \div (-3) = -j4$$

c) Multiplication or Division of a Real Number by a Real Number

This is done as in normal arithmetic.

d) Multiplication of a *j* Term by a *j* Term

The result of multiplying one *j* term by another is a j^2 term; $j^2 = -1$, so it is a real term, on the real axis. Multiplication of two *j* terms shift the number 90° from the *j*-axis to the real axis of 180°. For example,

$$j3 \times j2 = j^2 6 = (-1)(6) = -6$$

$$j4 \times (-j6) = -j^2 24 = -(-1)(24) = 24$$

e) Division of a *j* Term by a *j* Term

Division of one *j* term by another results in a real number, as the *j* coefficients cancel out *e.g.*

$$j6 \div j3 = 2 \qquad -j6 \div (-j3) = 2$$

$$j15 \div (-j5) = -3 \qquad -j15 \div j3 = -2$$

f) Multiplication of Complex Numbers

The rules of normal algebra are followed for multiplying two factors at a time.

$$\begin{aligned}(2+j3) \times (4-j5) &= 8 - j10 + j12 - j^2 15 \\ &= 8 + j2 - (-1)15 \\ &= 8 + j2 + 15 \\ &= 23 + j2\end{aligned}$$

g) Division of Complex Numbers

Division of a real number by an imaginary number is not possible. So, it is necessary to convert the denominator to a real number without any j term. The process of converting the denominator to a real number devoid of any j term is called **rationalization** of the fraction. To do this, both numerator and denominator are multiplied by the conjugate of the denominator. Conjugate complex numbers have equal terms but opposite signs for the j term. For example, the conjugate of $(2 + j3)$ is $(2 - j3)$. The following example illustrates this process :

$$\frac{(4-j2)}{2+j3} = \frac{(4-j2)}{(2+j3)} \times \frac{(2-j3)}{(2-j3)}$$

$$= \frac{8-j12-j4+j^2 6}{4-j^2 9}$$

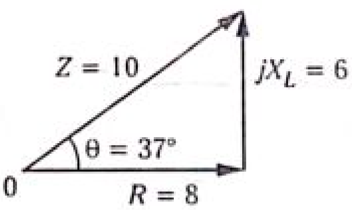
$$= \frac{8-j16-6}{4+9}$$

$$= \frac{2-j16}{13}$$

$$= 0.153 - j1.23$$

3.7.8 Magnitude and Angle of a Complex Number

A complex impedance $(8 + j6)$ means an $8\,\Omega$ resistance and a $6\,\Omega$ inductive reactance with a leading angle of 90°. Referring to Fig. 3.15, the magnitude of the resultant $Z = \sqrt{64+36} = \sqrt{100} = 10\,\Omega$. The phase angle of the resultant is the angle whose tangent is $\frac{6}{8}$ or 0.75. This angle (as seen from the tables) is 37°.



$$Z = \sqrt{R^2 + X_L^2}$$

$$\theta = \text{arc tan}\left(\frac{X_L}{R}\right) \quad \text{or} \quad \tan^{-1}\left(\frac{X_L}{R}\right)$$

Fig. 3.15

Thus, $8 + j6 = 10\ \angle 37°$

We should keep in mind that the complex number $(8+j6)$ is in rectangular co-ordinates, where the real term is 8 and the imaginary term is $j6$. The resultant 10 is the magnitude, absolute value, or *modulus* of the complex number, and its phase angle or *argument* is 37°. We may express the resultant value as $|10|$,

with vertical lines which show that it is only the magnitude, without the phase angle. The meter will only read the magnitude. For example, with a current of $10\angle 37°$ A in a circuit, an ammeter will read 10 A. Some other typical examples are:

$$4 + j3 = \sqrt{16+9}\left(\tan^{-1} 0.75\right) = 5\angle 37°$$

$$8 - j6 = \sqrt{64+36}\left(\tan^{-1} -0.75\right) = 10\angle -37°$$

$$4 - j4 = \sqrt{16+16}\left(\tan^{-1} -1\right) = 5.66\angle -45°$$

3.7.9 Polar Form of Complex Numbers

In the previous section we calculated the magnitude and phase angle of a complex number, which is nothing other than an angular form in polar co-ordinates. As shown in Fig. 3.15, the rectangular form $8 + j6$ is equal to $10\angle 37°$ in polar form. In polar co-ordinates, the distance out from the centre is the magnitude of the vector Z. Its phase angle θ is measured counterclockwise from the 0° axis (Fig. 3.16).

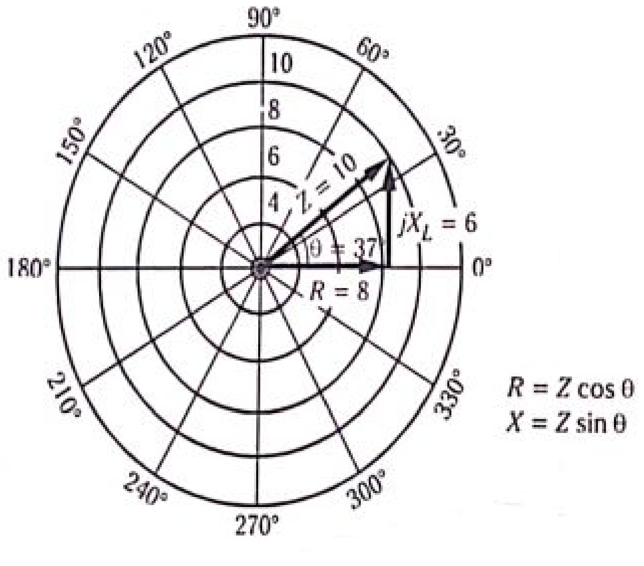


Fig. 3.16

To convert any number to polar form,

a) Find the magnitude by phasor addition of the j term and the real term.

b) Find the angle whose tangent is the j term divided by the real term.

Examples of polar form are the same as given at the end of the previous Section.

Note the following cases where either resistance or reactance is zero :

$$0 + j4 = 4\angle 90°$$

$$0 - j4 = 4\angle -90°$$

$$4 + j0 = 4\angle 0°$$

Multiplication : The magnitudes are multiplied, whereas the angles are added algebraically :

$$12 \angle 30° \times 2 \angle 40° = 24 \angle +70°$$

$$12 \angle 30° \times (-2 \angle 40°) = -24 \angle +70°$$

$$12 \angle -20° \times 3 \angle -40° = 36 \angle -60°$$

$$12 \angle -20° \times 4 \angle 10° = 48 \angle -10°$$

While multiplying by a real number, the magnitudes are multiplied (the angles remaining the same) :

$$6 \times 4 \angle 30° = 24 \angle 30°$$

$$6 \times 4 \angle -30° = 24 \angle -30°$$

$$-6 \times 4 \angle -30° = -24 \angle -30°$$

$$-6 \times (-4 \angle 30°) = 24 \angle 30°$$

Division : The magnitudes are divided, while the angles are subtracted algebraically, *e.g.*,

$$24 \angle 40 \div 3 \angle 30° = 8 (\angle 40° - \angle 30°)$$

$$= 8 \angle 10°$$

$$12 \angle 30° \div 3 \angle 50° = 4 (\angle 30° - \angle 50°)$$

$$= 4 \angle -20°$$

$$12 \angle -30° \div 4 \angle 50° = 3 (-\angle 30° - \angle 50°)$$

$$= 3 \angle -80°$$

To divide by a real number, we only divide the magnitudes

$$12 \angle 30° \div 3 = 4 \angle 30°$$

$$12 \angle -30° \div 3 = 4 \angle -30°$$

For dividing a real number by a complex number, we still follow the procedure of subtracting angles for division, as a real number has a phase angle of 0°.

e.g.

$$\frac{15}{3 \angle 30°} = \frac{15 \angle 0°}{3 \angle 30°}$$

$$= 5 (\angle 0° - \angle 30°) = 5 \angle -30°$$

$$\frac{15}{3 \angle -30°} = \frac{15 \angle 0°}{3 \angle -30°}$$

$$= 5 (\angle 0° - \angle -30°) = 5 \angle 30°$$

3.7.10 Converting Polar to Rectangular Form & Vice Versa

Complex numbers in polar form *cannot be added or subtracted*, as changing of angles is done only in the case of multiplication and division, as discussed in the previous Section. So, *for adding or subtracting complex numbers in polar form, they must be converted back to rectangular form.*

Let us take the impedance $Z \angle\theta$ in polar form, whose value is the hypotenuse of a right-angled triangle, with the horizontal side representing the real term and the vertical side the imaginary term (Fig. 3.17). Thus, we perform conversion from the polar form to the rectangular form. Going further into specific detail :

Real term: $R = Z \cos\theta$

j-term : $X = Z \sin\theta$

Referring the Fig. 3.17(a), let $Z \angle\theta$ in polar form be $10 \angle 37°$.

So, $\sin 37° = 0.6$ and $\cos 37° = 0.8$

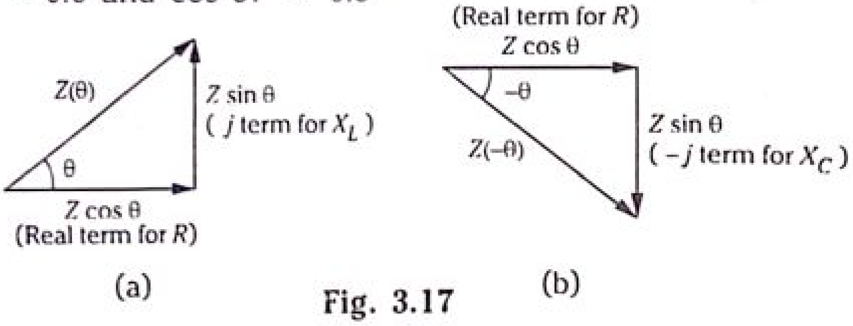


Fig. 3.17

To convert to rectangular form.

$$R = Z\cos\theta = 10 \times 0.8 = 8$$

$$X = Z\sin\theta = 10 \times 0.6 = 6$$

Thus, $10 \angle 37° = 8 + j6$

The + sign for the j term indicates inductive reactance X_L. This example is the same as given in Fig. 3.15.

In Fig. 3.17 (b), the magnitudes of the real and j terms are the same, but as θ is negative the j term is also negative; the real term, however, remains positive. The − sign for the j term indicates capacitive reactance X_C. We apply these rules for angles in the first or fourth quadrant.

For conversion from Rectangular to Polar Form refer to Sec. 3.7.8 and Problem 3.6 below.

Examples :

1) 14.14 ∠45°

$$Z\angle\theta = 14.14 \angle 45°$$

$$R = Z\cos\theta = 14.14 \cos 45° = 10$$

$$X_L = Z\sin\theta = 14.14 \sin 45° = 10$$

$$\therefore \quad 14.14 \angle 45° = 10 + j10$$

Here, the sign for j is +, hence indicating inductive reactance X_L.

2) 14.14 $\angle -45°$

$$Z(\theta) = 14.14\ \angle -45°$$

$$R = Z\cos\theta = 14.14\cos 45° = 10$$

$$X_C = Z\sin\theta = 14.14\sin 45° = 10$$

$$\therefore \quad 14.14\ \angle -45° = 10 - j10$$

The – sign for j indicates capacitive reactance X_c.

3) 100 $\angle 30°$

$$R = 100\ \cos 30° = 100 \times 0.866 = 86.6$$

$$X_L = 100\ \sin 30° = 100 \times 0.5 = 50$$

$$\therefore \quad 100\angle 30° = 86.6 + j50$$

4) 100 $\angle -30°$ = 86.6 – j50

5) 10 $\angle 90°$ = 0 + j10

6) 10 $\angle -90°$ = 0 – j10

Problem 3.6

a) Perform the following operation and express the final result in the polar form : 5 $\angle 30°$ + 8 $\angle -30°$. (Oct 85, B.U.)

Solution :

a) 5 $\angle$ 30°

$$Z\angle\theta = 5\angle 30°$$

$$R = Z\cos\theta = 5\cos 30° = 5 \times 0.866 = 4.33\ \Omega$$

$$X_L = Z\sin\theta = 5\sin 30° = 5 \times 0.5 = 2.5\ \Omega$$

$$\therefore \quad 5\ \angle 30° = 4.33 + j2.5$$

b) 8 $\angle -30°$

$$Z\angle\theta = 8\ \angle -30°$$

$$R = Z\cos\theta = 8\cos 30° = 8 \times 0.866 = 6.93\ \Omega$$

$$X_C = Z\sin\theta = 8\sin 30° = 8 \times 0.5 = 4\ \Omega$$

$$\therefore \quad 8\ \angle -30° = 6.93 - j4$$

c) 5 $\angle$ 30° + 8 $\angle -30°$

$$= 4.33 + j2.5 + 6.93 - j4$$

$$= 11.26 - j1.5$$

$$= \sqrt{(11.26)^2 + (1.5)^2} \angle \tan^{-1}\left(\frac{-1.5}{11.26}\right)$$

$$= \sqrt{126.78 + 2.25} \angle \tan^{-1}(-0.1332)$$

$$= \sqrt{129.03} \angle -7°36$$

$$= \mathbf{11.35 \angle -7.6°}$$

b) Perform the following operation and the final result may be given in the polar form : $(8 + j\,6) \times (-10 - j\,7.5)$. (Oct 85, Dec 86, B.U.)

Solution :

Method 1

$$\begin{array}{r} 8 + j6 \\ \times -10 - j7.5 \\ \hline -80 - j60 \\ \\ -j60 - j^2 45 \\ \hline -80 - j120 + 45 \end{array}$$

or $$-35 - j120 = \sqrt{(-35)^2 + (-120)^2} \angle \tan^{-1} 3.42$$

$$= 125 \angle \tan^{-1} 3.42 \qquad \text{---(i)}$$

As both the components are negative, the vector lies in the third quadrant. Hence, the angle as measured from the +ve direction of X-axis and in the CCW direction is

$$= \left(180° + \tan^{-1} 3.42\right)$$

$$= 180° + 73.8° = 253.8°$$

The same vector, measured in CW direction from the +ve direction of X-axis will be

.2°

$\angle$ **− 106.2°**

---(ii)

to polar form and then multiply follows :

.9°

b) $-10-j7.5=\sqrt{(-10)^2+(-7.5)^2}\ \angle\tan^{-1}0.75$

$$=12.5\ \angle\ \tan^{-1}0.75$$

This vector also falls in the third quadrant, so, following the same reasoning as mentioned in Method 1, the angle when measured in CCW direction is

$$=\left(180° + \tan^{-1}0.75\right)$$

$$=180°+36.9° = 216.9°$$

Measured in CCW direct from +ve co-ordinate of X-axis, the angle is

$$-(360°-216.9°)=-143.1°$$

So this expression is written as $12.5\ \angle{-143.1°}$

So, expression (ii) is rewritten as

$$10\ \angle\ 36.9° \times 12.5\ \angle-143.1°$$

$\mathbf{125\ \angle-106.2°}$ which is the same as before.

3.8 Phase

An alternating voltage or current changes in magnitude and direction at every instant. So, it is necessary to know the condition of the alternating quantity at a particular instant. The location of the condition of the alternating quantity at any particular instant is called its *phase*.

We may define the phase of an alternating quantity at any particular instant as the fractional part of a period or cycle through which the quantity has advanced from the selected origin.

Taking an example, the phase of current at point A ($+ve$ maximum value) is $T/4$ second, where T is the time period, or expressed in terms of angle, it is $\pi/2$ radians (Fig. 3.18).

In other words, it means that the condition of the wave, after having advanced through $\pi/2$ radians (90°) from the selected origin (*i.e.*, O) is that it is at maximum value (in the positive direction). Similarly, $-ve$ maximum value is reached after $3\pi/2$ radians (270°) from the origin, and the phase of the current at point B is $3T/4$ second.

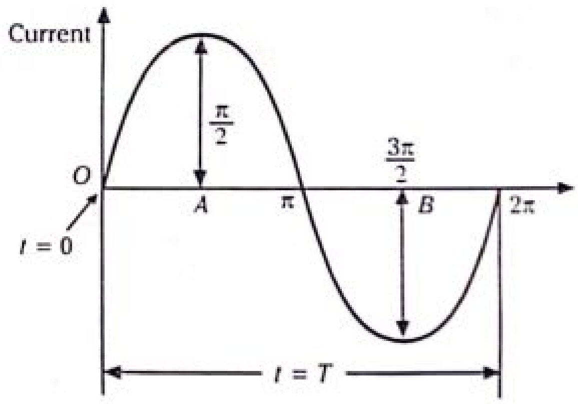


Fig. 3.18

3.9 Phase Difference (Lagging or Leading of Sinusoidal Wave)

When two alternating quantities, say, two voltages or two currents or one voltage and one current are considered simultaneously, the frequency being the

same, they may not pass through a particular point at the same instant.

One may pass through its maximum value at the instant when the other passes through a value other than its maximum one. These two quantities are said to have a *phase difference*. Phase difference is specified either in degrees or in radians.

The phase difference is measured by the angular difference between the points where the two curves cross the base or reference line in the same direction.

The quantity ahead in phase is said to *lead* the other quantity, whereas the second quantity is said to *lag* behind the first one. In Fig. 3.19, current I_1, represented by vector OA, leads the current I_2, represented by vector OB, by ϕ, or current I_2 lags behind the current I_1 by ϕ.

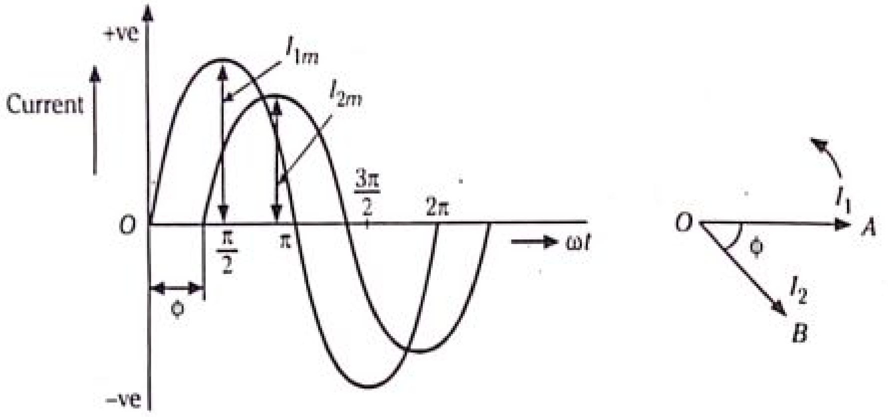


Fig. 3.19

The leading current I_1 goes through its zero and maximum values first and the current I_2 goes through its zero and maximum values after time angle ϕ. The two waves representing these two currents are shown in Fig. 3.19. If I_1 is taken as reference vector, the two currents are expressed as

$$i_1 = I_{1m} \sin \omega t \qquad \text{and} \qquad i_2 = I_{2m} \sin (\omega t - \phi)$$

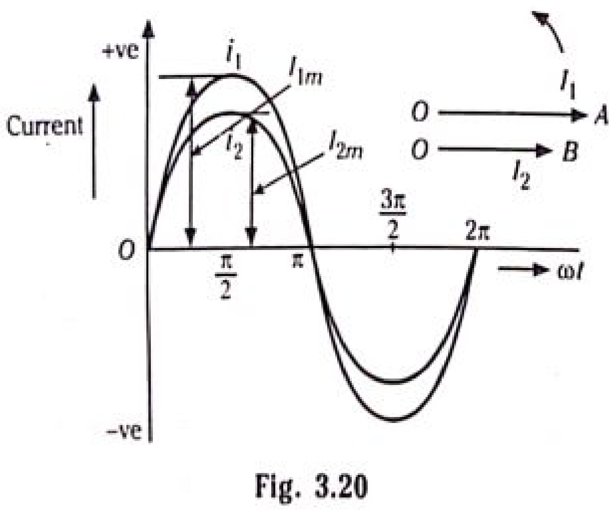


Fig. 3.20

The two quantities are said to be in phase with each other if they pass through zero values at the same instant and rise in the same direction, as shown in Fig. 3.20. However, if the two quantities pass through zero values at the same instant but rise in opposite directions, as shown in Fig. 3.21, they are said to be in phase opposition *i.e.*, the phase difference is 180°. When the two alternating quantities have a phase difference of 90° or $\pi/2$ radians they are said to be in quadrature.

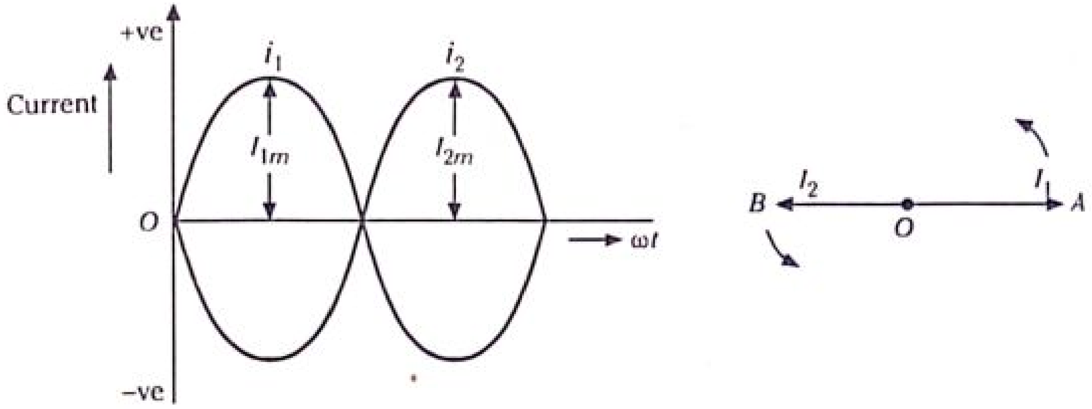


Fig. 3.21

Problem 3.7

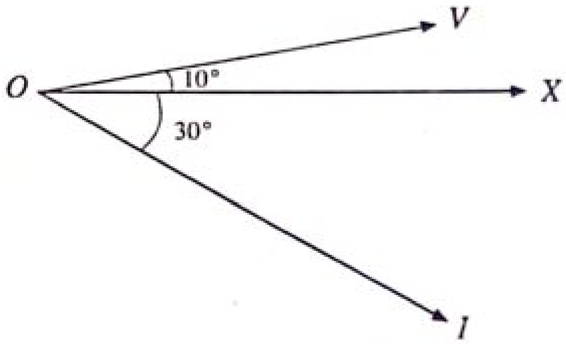
The voltage applied to a series circuit is 100 sin (ωt + 10°) and the current is 10 sin (ωt − 30°). Find the power. (B'lore Univ. July 1993)

Solution :

Given : V_{max} = 100 V and

I_{max} = 10 A

∴ The r.m.s values are :



$$V = \frac{100}{\sqrt{2}} \text{ volts and}$$

$$I = \frac{10}{\sqrt{2}} \text{ A}$$

Representing the voltage and current vectors, the voltage vector is at +10° with the X-axis and the current vector is at −30° with it, as shown.

Thus, the phase angle between V and I is = 10° + 30° = 40°, *i.e.*, θ = 40°.

∴ Power $P = V I \cos \theta$

$$= \left(\frac{100}{\sqrt{2}}\right)\left(\frac{10}{\sqrt{2}}\right) \cos 40°$$

= **383 watts**

3.10 Phasor Representation of Alternating Quantities

We know that an alternating voltage or current has sine waveform, and generators are designed to give e.m.f.s with the sine waveforms. The method of representing alternating quantities continuously by equations giving instantaneous values (like $e = E_m \sin \omega t$) is quite tedious. So, it is more convenient to represent a sinusoidal quantity by a phasor rotating in an anticlockwise direction (Fig. 3.22).

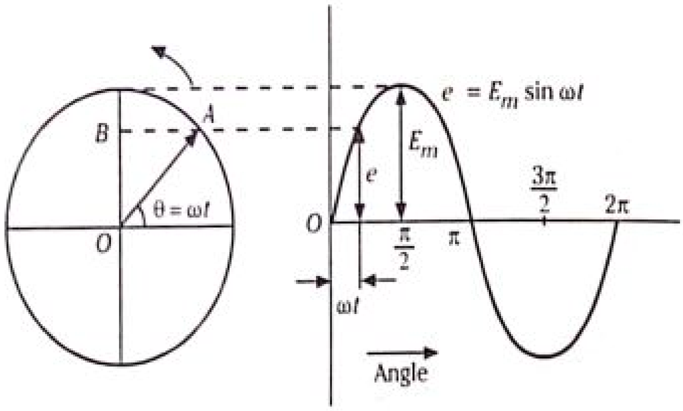


Fig. 3.22

While representing an alternating quantity by a phasor, the following points are to be kept in mind :

i) The length of the phasor should be equal to the maximum value of the alternating quantity.

ii) The phasor should be in the horizontal position at the instant the alternating quantity is zero and is increasing in the positive direction.

iii) The inclination of the line with respect to some axis of reference gives the direction of that quantity and an arrow-head placed at one end indicates the direction in which that quantity acts.

iv) The angular velocity in an anti-clockwise direction of the phasor should be such that it completes one revolution in the same time as taken by the alternating quantity to complete one cycle.

Consider phasor OA, which represents the maximum value of the alternating e.m.f. and its angle with the horizontal axis gives its phase (Fig. 3.22). Now, it will be seen that the projection of this phasor OA on the vertical axis will give the instantaneous value of e.m.f.

$$\therefore \quad OB = OA \sin \omega t$$

$$\text{or} \quad e = OA \sin \text{wt}$$

$$= E_m \sin \omega t$$

Note : The term 'Phasor' is also known as 'Vector'.

3.11 Phasor Diagrams of Sine Waves of Same Frequency

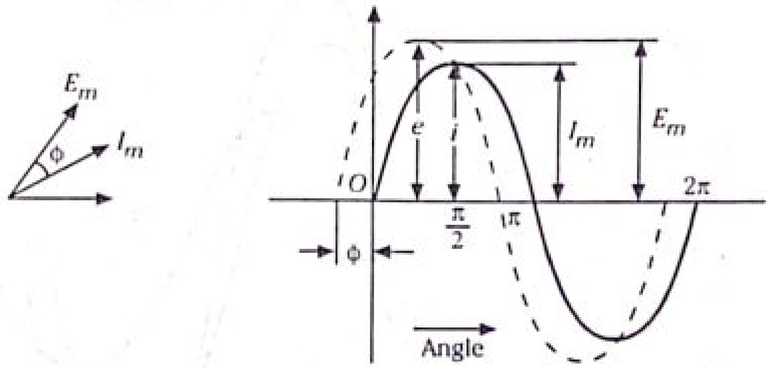


Fig. 3.23

If two or more than two sine waves have the same frequency, they can be represented on the same phasor diagram. This is because the various phasors representing the sine waves rotate in an anticlockwise direction at the same angular velocity 'ω' ($= 2\pi f$) and hence maintain a fixed position relative to one another. Fig. 3.23 shows the voltage and current sine waves of the same frequency; the voltage wave leads the current wave by $\phi°$. The two waves can be represented on the same phasor diagram with voltage phasor leading the current vector by $\phi°$. The current wave passes upwards through zero at the instant when $t = 0$, while at the same time the voltage wave has already advanced an angle ϕ from its zero value. As the vectors rotate with the same angular velocity, the phase difference between them remains fixed *i.e.*, $\phi°$ in this case. The equations of the two waves are written as :

$$i = I_m \sin \omega t$$
$$v = E_m \sin (\omega t + \phi)$$

3.12 Addition of Two Sinusoidal Quantities

Often it is required to consider the combined action of several e.m.f.'*s* or voltages acting in a series circuit and action of several currents flowing through the branches of a parallel circuit.

Let our objective be to add two currents given by the expressions

$$i_1 = I_{1m} \sin \omega t \quad \text{and} \quad i_2 = I_{2m} \sin (\omega t + \phi)$$

We will follow the convenient method of adding the sinusoidal quantities as phasors.

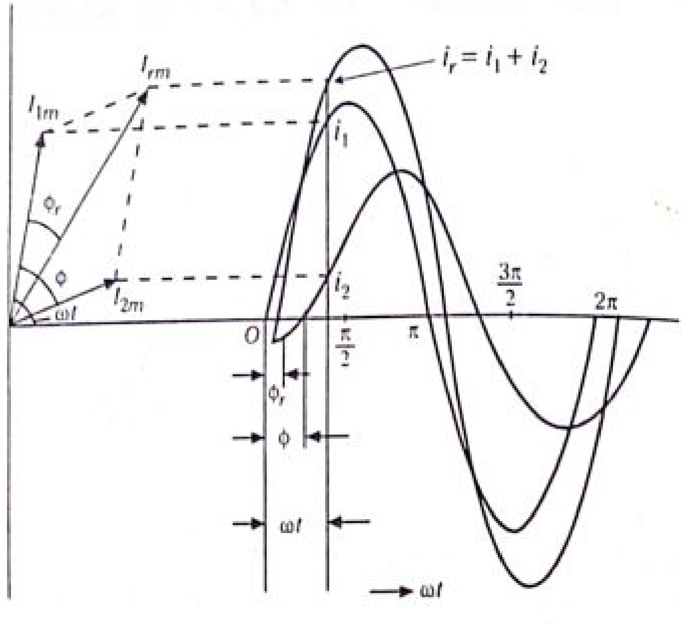


Fig. 3.24

Let us take phasors I_{1m} and I_{2m} that will generate the two curves i_1 and i_2, and let the situation be as depicted in Fig. 3.24, at one particular instant of time. When we add I_{1m} and I_{2m} vectorially by completing the parallelogram as shown, the diagonal I_{rm} will, when rotated, generate a third sine curve. The vertical component of I_{rm} is the sum of the vertical components of I_{1m} and I_{2m}, and the waveform of i_r could be obtained by adding corresponding points on the waveforms of i_1 and i_2, point by point. Thus, the waveform of i_r is the graph generated by rotating I_{rm} in a counter-clockwise direction.

Thus, two alternating quantities could be added by constructing parallelograms and either measuring or calculating the length of the diagonal and magnitude of the phase angles.

3.13 Addition of Phasors

In problems concerning a.c. circuits, we could deal with a number of alternating voltages or currents of the same frequency but of different phases and we may wish to obtain the resultant voltage or current. This can be done in any of the following three ways, which we shall illustrate by means of an example:

Let the following three alternating e.m.f.s be given, and we are required to find the equation of the resultant e.m.f.:

$$e_1 = 10 \sin\left(\omega t + \frac{\pi}{3}\right)$$

$$e_2 = 15 \sin\left(\omega t + \frac{3\pi}{4}\right)$$

$$e_3 = 20 \sin\left(\omega t + \frac{4\pi}{3}\right)$$

a) By compounding according to the Parallelogram Law

We have seen earlier how the resultant of two phasors can be obtained by means of a parallelogram. If a third phasor is also to be considered, the final resultant phasor can be obtained by a second parallelogram comprising the first resultant and the third phasor.

This will be more clear when we consider the above example (See Fig. 3.25).

Phasor OA is drawn to measure the maximum value of the alternating voltage e_1 (E_{m1} = 10 V), at an angle of $\frac{\pi}{3}$ radians (60°) with reference axis XX'.

Then vector OB is drawn to measure the maximum value of the alternating voltage e_2 (E_{m2} = 15 V), at an angle of $\frac{3\pi}{4}$ radians (135°) with XX'. A parallelogram is completed as shown in the figure and the resultant OD of these two voltages (E_{m1} and E_{m2}) is drawn.

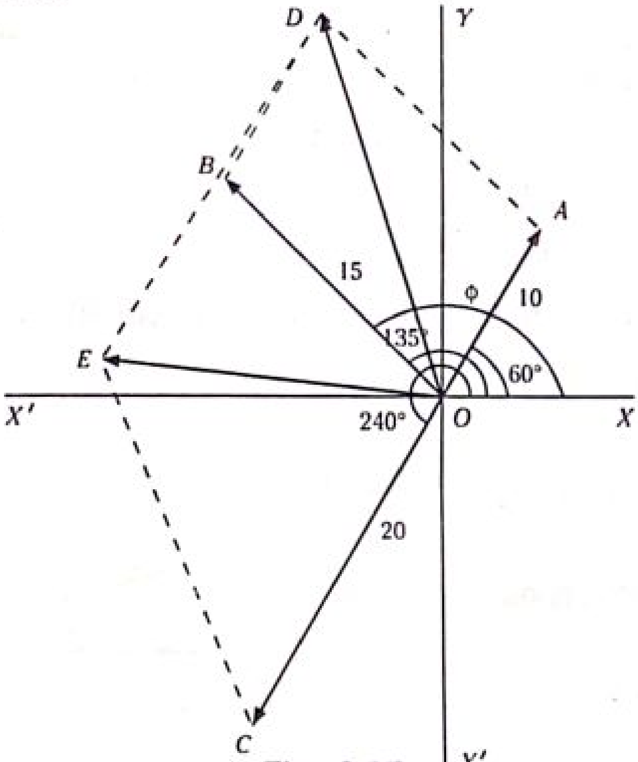


Fig. 3.25

Finally, vector OC is drawn to measure the maximum value of the alternating voltage e_3 (E_{m3} = 20 V). Another parallelogram is constructed, and the final resultant OE will be the phasor sum of OC and OD. When all the phasors are drawn to scale, the length of resultant OE and its angle ϕ with XX' axis can be measured.

b) By resolving the phasors into their X and Y components as in Fig. 3.26.

Let us first draw the three phasors representing the maximum values of the given alternating voltages.

$$e_1 = 10 \sin\left(\omega t + \frac{\pi}{3}\right)$$

: its phasor will be 60° above OX

$$e_2 = 15 \sin\left(\omega t + \frac{3\pi}{4}\right)$$

: its phasor will be 135° with OX in CCW direction

$$e_3 = 20 \sin\left(\omega t + \frac{4\pi}{3}\right)$$

: its phasor will be 240° with OX in CCW direction.

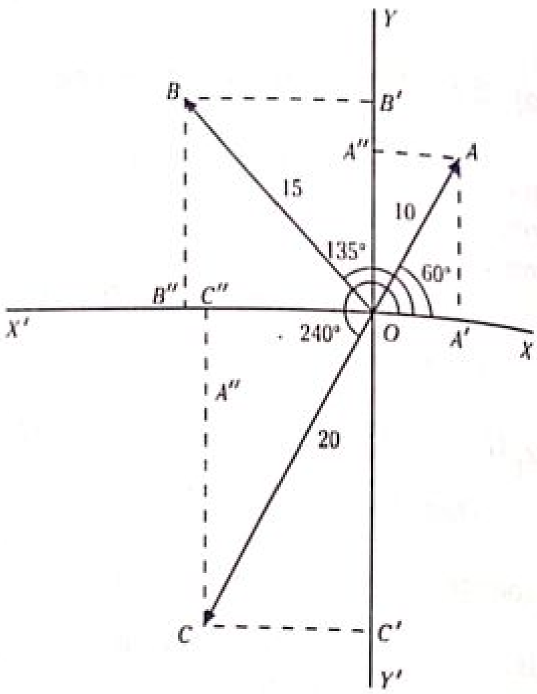


Fig. 3.26

These three phasors, OA, OB and OC are shown in Fig. 3.26. Resolving them into X- and Y-components, we have

$$X\text{-component} = 10 \cos 60° - 15 \cos 45° - 20 \cos 60°$$
$$= 5 - 10.6 - 10$$
$$= -15.6$$

$$Y\text{-component} = 10 \sin 60° + 15 \sin 45° - 20 \sin 60°$$
$$= 8.66 + 10.6 - 17.32$$
$$= 1.94$$

From Fig. 3.27, the maximum value of the resultant voltage is

$$OD = \sqrt{(-15.6)^2 + (1.94)^2} = 15.7 \text{ V}$$

$$\tan \alpha = \frac{1.94}{15.6} = 0.124$$

$$\alpha = 7° \text{ (approx.)}$$

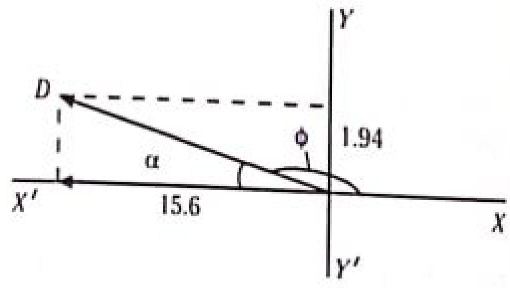


Fig. 3.27

$\therefore$ Phase angle $\phi = 180° - 7° = 173°$

c) By situating the various phasors, end-to-end, at their proper phase angles and then measuring the closing phasor as shown in Fig. 3.28.

The first phasor OA is drawn at an angle of 60°.

From the end of OA, AB is drawn at 135°, and from the end of AB, BC is drawn at 240°, as shown in the diagram. OC, the closing phasor, gives the resultant and ϕ is the phase angle.

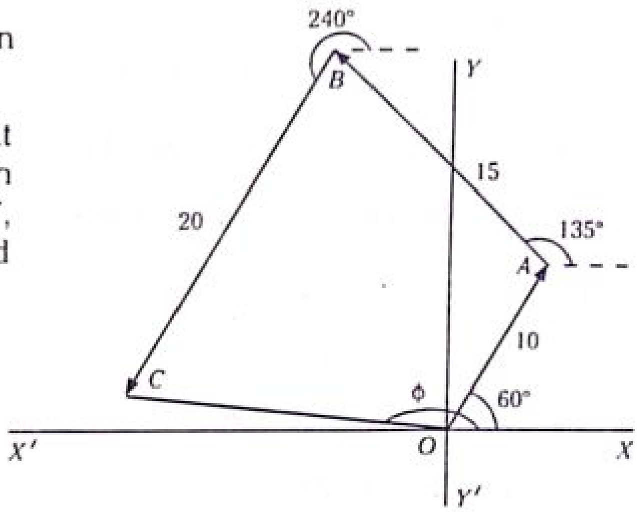


Fig. 3.28

Problem 3.8

Two electric devices A and B are connected in parallel. The R.M.S. current in A is 15 A. If the current in B lags behind the current in A by 60° and the total current in A is 23.4 A, find the current in B. (B'lore Univ; Feb 1988)

Solution :

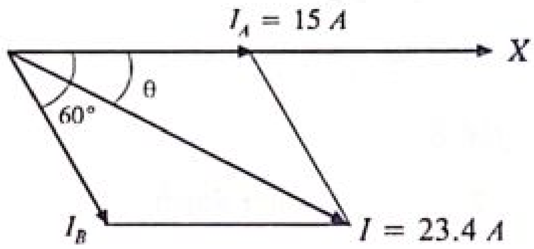


Fig. 3.29

Given current in A,

$$I_A = 15 \text{ A}$$

Current in B, I_B lags behind I_A by 60°

Total current, $I = I_A + I_B = 23.4$ A

The phasor diagram is as given

The X-component of $I = 23.4 \cos\theta$

$$= I_A + I_B \cos 60°$$

$$= 15 + 0.5\, I_B \qquad \text{---(i)}$$

The Y-components of $I = 23.4 \sin\theta$

$$= 0 + I_B \sin 60°$$

$$= 0.866\, I_B \qquad \text{---(ii)}$$

or $23.4^2\left(\sin^2\theta+\cos^2\theta\right)=\left(15+0.5I_B\right)^2+\left(0.866\,I_B\right)^2$

Simplifying, we have

$$I_B^2+15\,I_B-322.56=0$$

The roots of the above quadratic equation are 11.96 and −26.96

Ignoring the negative value, we have current $\boldsymbol{I_B=11.96}$ **A (magnitude)**

Substituting this value of I_B in eqn (ii), we get

$$23.4\sin\theta = (0.866\times 11.96)$$

or $\sin\theta = (0.866\times 11.96)/23.4$

or $\theta = 26.28°$

Thus, the total current lags behind I_A by 26.28°

Verification :

$I_A = 15\,A$ and $I_B = 11.96\ \angle -60°$

$\therefore\ I_A+I_B = 15\ \angle 0° + 11.96\ \angle -60°$

$= (15+j0)+(5.98-j10.37)$

$= 20.98-j10.37$

$= 23.4\ \angle -26.28°$ in polar form.

Problem 3.9

Find the sum of the five e.m.f's

$$e_1 = 20\sin\omega t$$

$$e_2 = 10\sin(\omega t+30°)$$

$$e_3 = 15\cos\omega t$$

$$e_4 = 10\sin(\omega t-60°)\ \text{and}$$

$$e_5 = 25\cos(\omega t+120°)$$

Express the resultant in the form $e = \sin(\omega t - \phi)$

Solution :

The e.m.f.'s are represented by phasors. These phasors are then resolved into X- and Y components and then the resultant e.m.f is determined.

Given : $e_1 = 20 \sin \omega t$

$$e_2 = 10 \sin(\omega t + 30°)$$

$$e_3 = 15 \cos \omega t$$

$$= 15 \sin(\omega t + 90°)$$

$$e_4 = 10 \sin(\omega t - 60°)$$

$$e_5 = 25 \cos(\omega t + 120°)$$

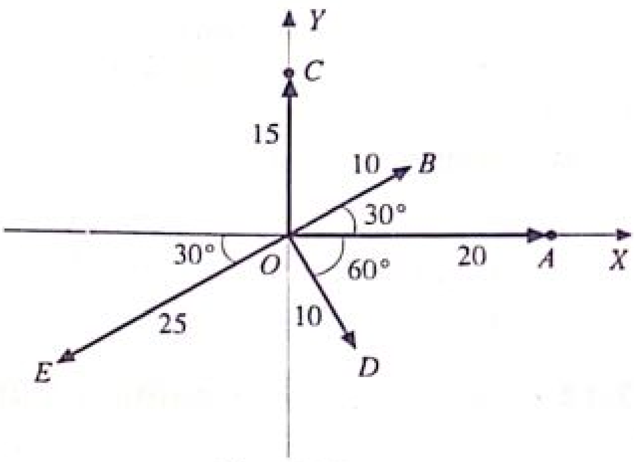


Fig. 3.30

It is seen that the e.m.f.'s are all uniformly shown in terms of the sine quantitites and are accordingly represented by the phasors OA, OB, OC, OD and OE, taking the maximum value E_1, as reference.

Resolving the phasors into their X-component,

$$= 20\cos 0° + 10\cos 30° + 15\sin 90° + 10\cos 60° - 25\cos 30°$$

$$= 20 + (10 \times 0.866) + 0 + (10 \times 0.5) - (25 \times 0.866) = 12.01$$

Resolving the same phasors into their Y-component,

$$= 20\sin 0° + 10\sin 30 + 15 - 10\sin 60 - 25\sin 30$$

$$= 0 + 5 + 15 - (10 \times 0.866) - (25 \times 0.5)$$

$$= -1.16$$

The resultant phasor has a magnitude

$$= \sqrt{12.01^2 + 1.16^2}$$

$$= 12.07$$

The angle the resultant phasor makes with OX

$$= \tan^{-1}\left[\frac{-1.16}{12.01}\right]$$

$$= -5.52°$$

Hence the resultant phasor is expressed as

$12.07 \sin(\omega t - 5.52°)$

3.14 A.C. Circuit

The path for the flow of alternating current is called an a.c. circuit.

In a d.c. circuit, the current I flowing through the circuit is given by the simple relation $I = \dfrac{V}{R}$. However, in an a.c. circuit, voltage and current change from instant

to instant and so give rise to *magnetic* (*inductive*) and *electrostatic* (*capacitive*) effects. So, in an a.c. circuit, *inductance* and *capacitance* must be considered in addition to resistance.

We shall now deal with the following a.c. circuits :

i) AC circuit containing pure ohmic resistance only.
ii) AC circuit containing pure inductance only.
iii) AC circuit containing pure capacitance only.

3.14.1 AC Circuit containing Pure Ohmic Resistance

When an alternating voltage is applied across a pure ohmic resistance, electrons (current) flow in one direction during the first half-cycle and in the opposite direction during the next half-cycle, thus constituting alternating current in the circuit.

Let us consider an a.c. circuit with just a pure resistance R only, as shown in Fig. 3.31.

Let the applied voltage be given by the equation

$$v = V_m \sin\theta = V_m \sin\omega t \qquad \text{---(i)}$$

As a result of this alternating voltage, alternating current 'i' will flow through the circuit.

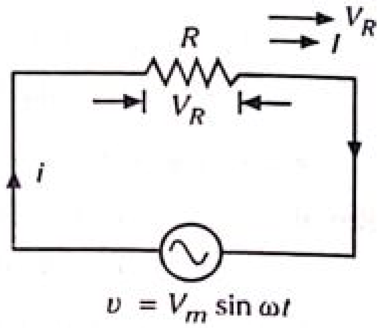


Fig. 3.31

The applied voltage has to supply the drop in the resistance, *i.e.*,

$$v = iR$$

Substituting the value of 'v' from eqn.(i), we get

$$V_m \sin\omega t = iR \quad \text{or} \quad i = \frac{V_m}{R}\sin\omega t \qquad \text{---(ii)}$$

The value of the alternating current 'i' is maximum when $\sin\omega t = 1$,

$$i.e., \quad I_m = \frac{V_m}{R}$$

∴ Eqn.(ii) becomes,

$$i = I_m \sin\omega t \qquad \text{---(iii)}$$

From eqns.(i) and (ii), it is apparent clear that voltage and current are *in phase with each other*. This is also indicated by the wave and vector diagram shown in Fig. 3.32.

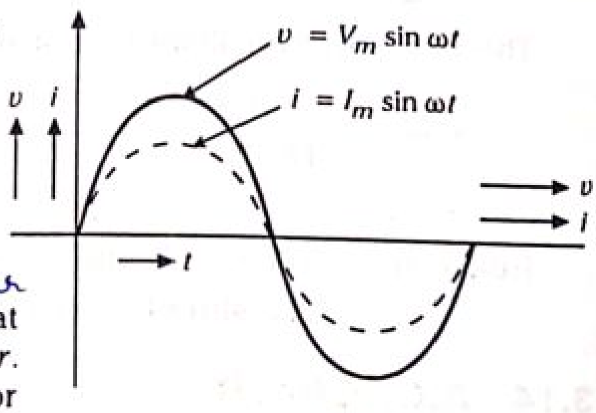


Fig. 3.32

Power : The voltage and current are changing at every instant.

$\therefore$ Instantaneous power, $p = V_m \sin \omega t \, . \, I_m \sin \omega t$

$$= V_m I_m \sin^2 \omega t$$

$$= V_m I_m \frac{(1-\cos 2\omega t)}{2}$$

$$= \frac{V_m I_m}{2} - \frac{V_m I_m}{2} \, . \cos 2\omega t$$

Thus instantaneous power consists of a constant part $\frac{V_m I_m}{2}$ and a fluctuating part $\frac{V_m I_m}{2} \cos 2\omega t$ of frequency double that of voltage and current waves. The average value of $\frac{V_m I_m}{2} \cos 2\omega t$ over a complete cycle is zero.

So, power for the complete cycle is

$$P = \frac{V_m I_m}{2} = \frac{V_m}{\sqrt{2}} \times \frac{I_m}{\sqrt{2}}$$

or $\quad P = VI$ watts

where V = r.m.s. value of applied voltage

I = r.m.s. value of the current

Power Curve

The power curve for a purely resistive circuit is shown in Fig. 3.33. It is apparent that power in such a circuit is zero only at the instants a, b and c, when both voltage and current are zero, but is positive at all other instants. In other words, power is never negative, so that power is always lost in a resistive a.c. circuit. This power is dissipated as heat.

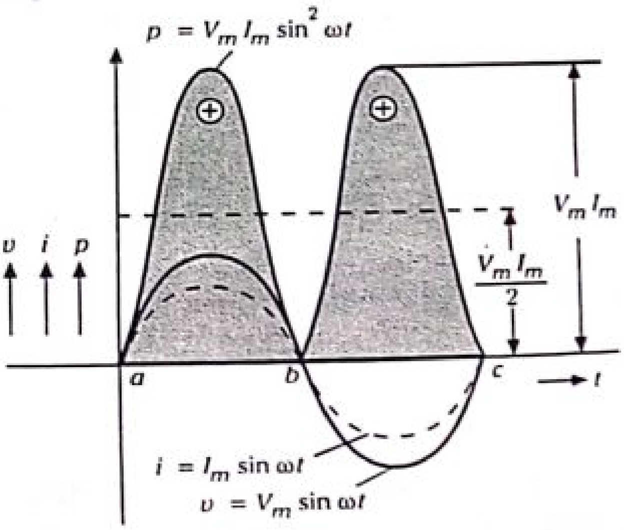


Fig. 3.33

Problem 3.10

An e.m.f. given by 400 sin 418 t is applied to a certain circuit. The current taken is 2.4 sin (418 t – 1.37). Find :

i) Frequency

ii) The phase angle between voltage and current

iii) The resistance of the circuit. **(Nov/Dec 84, B.U.)**

Solution :

Peak value of applied voltage, V_m = 400 V

Peak value of circuit current, I_m = 2.4 A

Impedance of the circuit, $Z = \dfrac{V_{r.m.s.}}{I_{r.m.s.}} = \dfrac{\frac{V_m}{\sqrt{2}}}{\frac{I_m}{\sqrt{2}}}$

$$= \frac{V_m}{I_m} = \frac{400}{2.4} = 166.6\ \Omega$$

i) Frequency,

$$f = \frac{\text{Co-efficient of time in expression of voltage or current}}{2\pi}$$

$$= \frac{418}{2\pi} = \mathbf{66.5\ Hz}$$

ii) Phase angle between voltage and current,

ϕ = 1.37 radians (lag)

$$= \frac{1.37}{\pi} \times 180° = \mathbf{78.5°(lag)}$$

iii) The resistance of the circuit,

$$R = Z \cos\phi = 166.6 \cos 78.5°$$

$$= 166.6 \times 0.1994$$

$$= \mathbf{33.22\ \Omega}$$

3.14.2 A.C. Circuit Containing Pure Inductance

An inductive coil is a coil with or without an iron core and has negligible resistance. In practice, pure inductance can never be had as the inductive coil has always a small resistance. However, a coil of thick copper wire wound on a laminated iron core has negligible resistance, so, for the purpose of our study, we will consider a purely inductive coil.

On the application of an alternating voltage (Fig. 3.34) to a circuit containing a pure inductance, a back e.m.f. is produced due to the self-inductance of the coil. This back e.m.f. opposes the rise or fall of current, at every stage. Because of the absence of voltage drop, the applied voltage has to overcome this self-induced e.m.f. only.

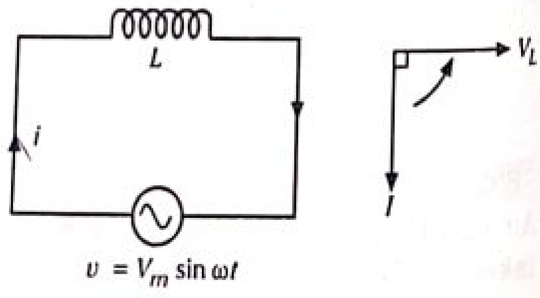


Fig. 3.34

Let the applied voltage be $v = V_m \sin \omega t$, and the self-inductance of the coil $= L$ henry.

Self-induced e.m.f. in the coil, $e_L = -L\dfrac{di}{dt}$

Since applied voltage at every instant is equal and opposite to the self-induced e.m.f., *i.e.* $v = -e_L$

$$\therefore \qquad V_m \sin \omega t = -\left(-L\frac{di}{dt}\right)$$

$$\text{or} \qquad di = \frac{V_m}{L} \sin \omega t \, dt$$

Integrating both sides, we get

$$i = \frac{V_m}{L} \int \sin \omega t \, dt$$

$$\text{or} \qquad i = \frac{V_m}{\omega L}(-\cos \omega t) + A$$

where A is a constant of integration which is found to be zero from initial conditions.

$$\text{So,} \qquad i = \frac{-V_m}{\omega L} \cos \omega t$$

$$\text{or} \qquad i = \frac{V_m}{\omega L} \sin\left(\omega t - \frac{\pi}{2}\right)$$

Current will be maximum when $\sin\left(\omega t - \dfrac{\pi}{2}\right) = 1$, hence, the value of maximum current, $I_m = \dfrac{V_m}{\omega L}$, and instantaneous current may be expressed as $i = I_m \sin\left(\omega t - \dfrac{\pi}{2}\right)$.

From the expressions of instantaneous applied voltage ($v = V_m \sin \omega t$) and the instantaneous current flowing through a purely inductive coil, it is clear that the current lags behind the voltage by $\dfrac{\pi}{2}$ as shown in Fig. 3.35.

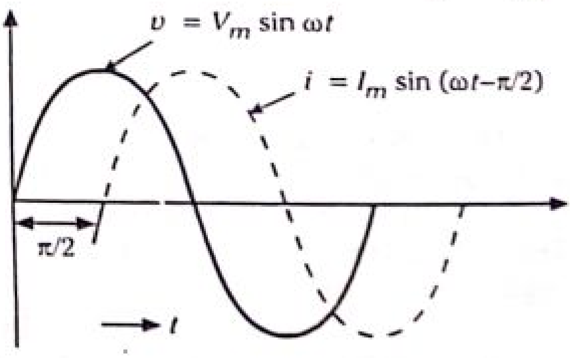


Fig. 3.35

Inductive Reactance : ωL in the expression $I_m = \dfrac{V_m}{\omega L}$ is known as *inductive reactance* and is denoted by X_L, *i.e.*, $X_L = \omega L$. If 'L' is in henry and 'ω' is in radians per second, then X_L will be in ohms. So, inductive reactance plays the part of resistance.

Power : Instantaneous power,

$$p = v \times i = V_m \sin \omega t \,.\, I_m \sin\left(\omega t - \frac{\pi}{2}\right)$$

$$= -V_m I_m \sin \omega t \cos \omega t$$

$$= \frac{-V_m I_m}{2} \sin 2\omega t$$

The power measured by a wattmeter is the average value of 'p', which is zero since average of a sinusoidal quantity of double frequency over a complete cycle is zero. Put in mathematical terms,

Power for the whole cycle, $p = -\dfrac{V_m I_m}{2} \displaystyle\int_0^{2\pi} \sin 2\omega t \, dt = 0$

Hence, power absorbed in a pure inductive circuit is zero.

Power Curve

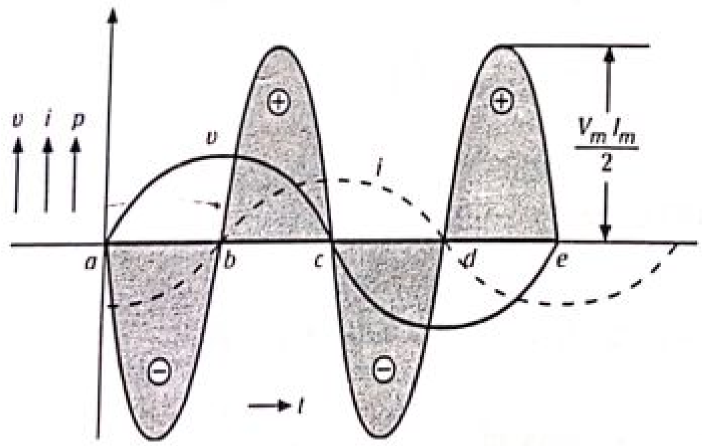


Fig. 3.36

The power curve for a pure inductive circuit is shown in Fig. 3.36. This indicates that power absorbed in the circuit is zero. At the instants a, c and e, voltage is zero, so that power is zero ; it is also zero at points b and d when the current is zero. Between a and b voltage and current are in opposite directions, so that power is negative and energy is taken from the circuit. Between b and c voltage and current are in the same direction, so that power is positive and is put back into the circuit. Similarly, between c and d, power is taken from the circuit and between d and e it is put into the circuit. Hence, *net power is zero*.

Problem 3.11

A pure inductive coil allows a current of 10 amperes to flow from a 230 volts, 50 Hz supply. Find (i) inductive reactance (ii) inductance of the coil. Also write down the equations for voltage and current.

Solution :

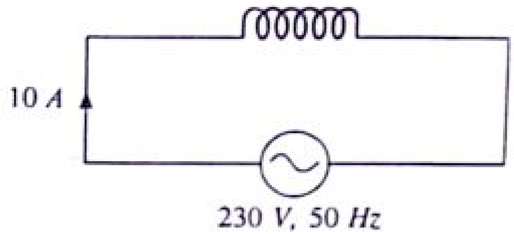


Fig. 3.37

(i) Now, $I = \dfrac{V}{X_L}$

or $X_L = \dfrac{V}{I} = \dfrac{230}{10} = \mathbf{23\ ohms}$

Thus, inductive reactance, $X_L = \mathbf{23\ ohms}$

(ii) $L = \dfrac{X_L}{2\pi f} = \dfrac{23}{2\pi \times 50} = \mathbf{0.073\ H}$

(iii) $V_m = \sqrt{2} \times V = \sqrt{2} \times 230 = 325.27$ volts

$I_m = \sqrt{2} \times I = \sqrt{2} \times 10 = 14.14$ Amps

$\omega = 2\pi f = 2\pi \times 50 = 314$ rad/sec

$\therefore \quad v = V_m \sin \omega t$

$v = 325.27 \sin 314t$ volts

$i = I_m \sin\left(\omega t - \dfrac{\pi}{2}\right)$

$= \mathbf{14.14 \sin\left(314t - \dfrac{\pi}{2}\right)}$ **Amps**

3.14.3 AC Circuit Containing Pure Capacitance

When an alternating voltage is applied across the plates of a capacitor, the capacitor is charged in one direction and then in the opposite direction as the voltage reverses. With reference to Fig. 3.38,

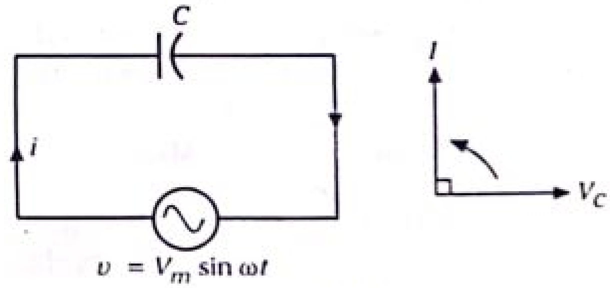


Fig. 2.38

Let alternating voltage represented by $v = V_m \sin \omega t$ be applied across a capacitor of capacitance C Farads.

Instantaneous charge, $q = Cv = CV_m \sin \omega t$

Capacitor current is equal to the rate of change of charge, or

$$i = \frac{dq}{dt} = \frac{d}{dt}(CV_m \sin \omega t)$$

$$= \omega C V_m \cos \omega t$$

or $$i = \frac{V_m}{\frac{1}{\omega C}} \cos \omega t$$

or $$i = \frac{V_m}{\frac{1}{\omega C}} \sin\left(\omega t + \frac{\pi}{2}\right)$$

The current is maximum when $t = 0$

$$\therefore \quad I_m = \frac{V_m}{\frac{1}{\omega C}}$$

Substituting $\frac{V_m}{\frac{1}{\omega C}} = I_m$ in the above expression for instantaneous current, we get

$$i = I_m \sin\left(\omega t + \frac{\pi}{2}\right)$$

Capacitive Reactance : $\frac{1}{\omega C}$ in the expression $I_m = \frac{V_m}{\frac{1}{\omega C}}$ is known as *capacitive reactance* and is denoted by X_c.

i.e., $$X_c = \frac{1}{\omega C}$$

If C is in farads and 'ω' is in radians, then X_c will be in ohms.

It is seen that if the applied voltage is given by $v = V_m \sin \omega t$, then the current is given by $i = I_m \sin\left(\omega t + \frac{\pi}{2}\right)$; this shows that the current in a pure capacitor leads its voltage by a quarter cycle as shown in Fig. 3.39, or phase difference between its voltage and current is $\pi/2$ with the current leading.

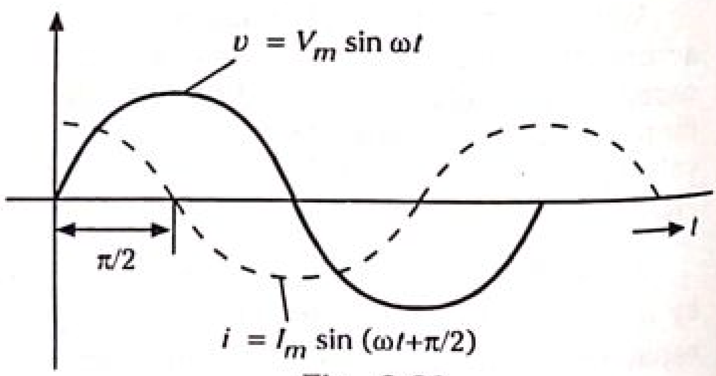


Fig. 3.39

Power : Instantaneous Power,

$$p = vi$$

$$= V_m \sin \omega t \,.\, I_m \sin\left(\omega t + \frac{\pi}{2}\right)$$

$$= V_m I_m \sin \omega t \cos \omega t$$

$$= \frac{1}{2} V_m I_m \sin 2\omega t$$

Power for the complete cycle

$$= \frac{1}{2} V_m I_m \int_0^{2\pi} \sin 2\omega t \, dt = 0$$

Thus the average power consumed by a pure capacitor is zero.

Power Curves (Fig. 3.40)

At the instants b, d, the current is zero, so that power is zero ; it is also zero at the instants a, c and e, when the voltage is zero. Between a and b, voltage and current are in the same direction, so that power is positive and is being put back in the circuit. Between b and c, voltage and current are in the opposite directions, so that power is negative and energy is taken from the circuit. Similarly, between c and d, power is put back into the circuit, and between d and e it is taken from the circuit.

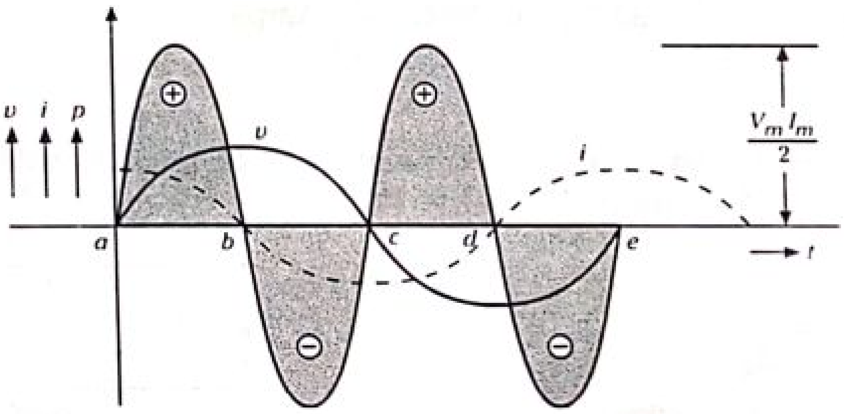


Fig. 3.40

Therefore, power absorbed in a pure capacitive circuit is zero.

Problem 3.12

A 318 μF capacitor is connected across a 230 volts, 50 Hz system. Determine (i) the capacitive reactance (ii) R.M.S value of current and (iii) equations for voltage and current.

Solution :

(i) Capactive reactance,

$$X_C = \frac{1}{2\pi fC}$$

$$\therefore \quad X_C = \frac{1}{2\pi \times 50 \times (318 \times 10^{-6})} = \mathbf{10\ ohms}$$

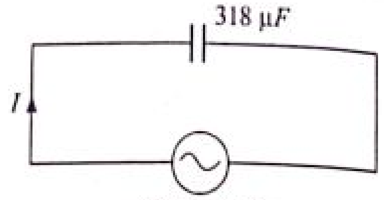


Fig. 3.41

(ii) R.M.S value of current, $I = \frac{V}{X_C}$

$$I = \frac{230}{10} = \mathbf{23\ Amps}$$

(iii) $V_m = 230 \times \sqrt{2} = \mathbf{325.27\ volts}$

$I_m = 23 \times \sqrt{2} = \mathbf{32.53\ Amps}$

$\omega = 2\pi f = 2\pi \times 50 = \mathbf{314\ rad/sec}$

$v = V_m \sin \omega t$

or $\quad v = 325.27 \sin 314t$ volts

$$i = I_m \sin\left(\omega t + \frac{\pi}{2}\right) = \mathbf{32.53\ sin\left(314t + \frac{\pi}{2}\right)\ Amps}$$

Problem 3.13

The current drawn by a pure capacitor of 20 microfarads is 1.382 A from a 220 volts A.C. supply. What is the supply frequency ?

Solution :

We have $I = \frac{V}{X_C}$

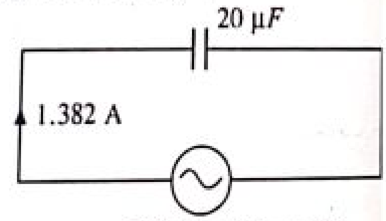


Fig. 3.42

or $\quad X_C = \frac{V}{I} = \frac{220}{1.382} = \mathbf{159.19}$

Also, $\quad X_C = \frac{1}{2\pi fC}$

or, $\quad f = \frac{1}{2\pi C X_C}$

$$= \frac{1}{2\pi \times (20 \times 10^{-6}) \times 159.19} = \mathbf{50\ Hz}$$

Problem 3.14

Two similar capacitors are connected in series and a voltage with an instantaneous value of e = 100 sin 314 t is applied. Calculate the capacitance of each capacitor if the r.m.s. current taken by the combination is 0.6 A.

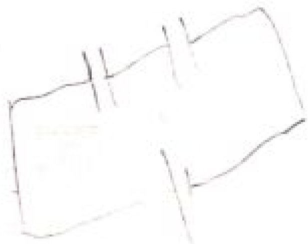
Solution :

Let C be the capacitance of each capacitor; hence the combined capacitance is $\frac{C}{2}$ farads.

Comparing the equation with the standard equation,

$$V_m = 100\,\text{V} \quad \text{and} \quad \omega = 314 \text{ radians.}$$

Then $$V = \frac{100}{\sqrt{2}} \quad \text{and} \quad f = \frac{314}{\pi} = 50\text{ Hz}$$



Hence the capacitive reactance,

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C} = \frac{1}{100\pi C}\text{ Ohms}$$

Given $$I = 0.6\text{ A} = \frac{V}{X_C} = \frac{100/\sqrt{2}}{1/(100\pi C)}$$

$$\therefore \quad C = \frac{0.6\times\sqrt{2}}{100\times 100\pi}$$

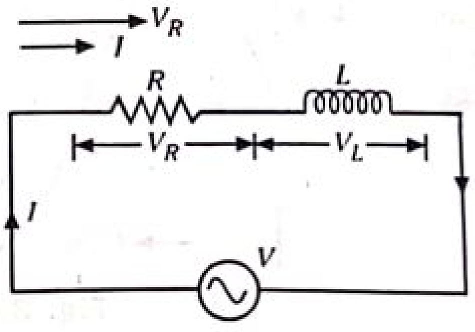
$$= \frac{0.848}{31400}$$

$$= 0.000027\text{ F}$$

$$= \mathbf{27\ \mu F}$$

3.15 Series *R-L* Circuit

Let us consider an a.c. circuit containing a pure resistance R ohms and a pure inductance of L henrys, as shown in Fig. 3.43.



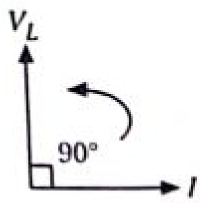


Fig. 3.43

Let V = r.m.s. value of the applied voltage

I = r.m.s. value of the current

Voltage drop across R, $V_R = IR$ (in phase with I)

Voltage drop across L, $V_L = IX_L$ (leading I by 90°)

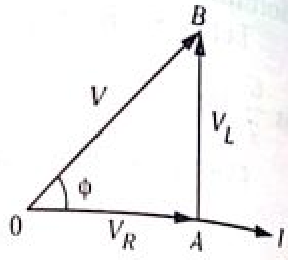


Fig. 3.44

The voltage drops across these two circuit components are shown in Fig. 3.44, where vector OA indicates V_R and AB indicates V_L. The applied voltage V is the vector sum of the two, *i.e.* OB.

$$\therefore \quad V = \sqrt{V_R^2 + V_L^2} = \sqrt{(IR)^2 + (IX_L)^2}$$

$$= I\sqrt{R^2 + X_L^2}$$

$$\therefore \quad I = \frac{V}{\sqrt{R^2 + X_L^2}}$$

The term $\sqrt{R^2 + X_L^2}$ offers opposition to current flow and is called the impedance (Z) of the circuit. It is measured in ohms.

$$\therefore \quad I = \frac{V}{Z}$$

Referring to the impedance triangle ABC, (Fig. 3.45)

$$Z^2 = R^2 + X_L^2$$

or (impedance)^2 = (resistance)^2 + (reactance)^2

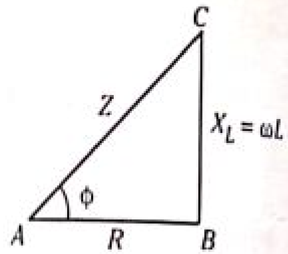


Fig. 3.45

Referring back to Fig. 3.44, we observe that the applied voltage V leads the current I by an angle ϕ.

$$\tan\phi = \frac{V_L}{V_R} = \frac{I.X_L}{I.R} = \frac{X_L}{R} = \frac{\omega L}{R} = \frac{\text{reactance}}{\text{resistance}}$$

$$\therefore \quad \phi = \tan^{-1}\frac{X_L}{R}$$

The same feature is shown by means of waveforms (Fig. 3.46). We observe that circuit current lags behind applied voltage by an angle ϕ.

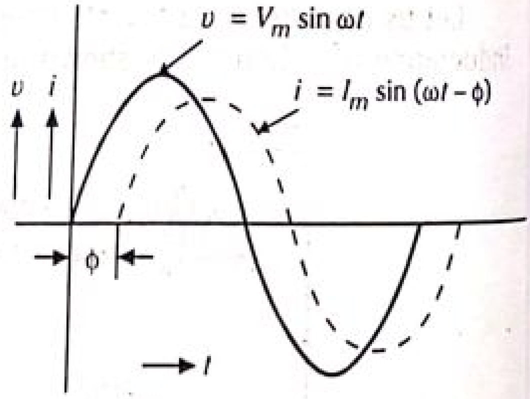


Fig. 3.46

So, if applied voltage is expressed as $v = V_m \sin \omega t$, the current is given by $i = I_m(\sin \omega t - \phi)$, where $I_m = \frac{V_m}{Z}$.

Definition of Real Power, Reactive Power, Apparent Power and Power Factor

Let a series *R-L* circuit draw a current *I* (r.m.s. value) when an alternating voltage of r.m.s. value *V* is applied to it. Suppose the current lags behind the applied voltage by an angle ϕ as shown in Fig. 3.47.

Power Factor and its significance

Power Factor may be defined as the cosine of the angle of lead or lag. In Fig. 3.47, the angle of lag is shown. Thus Power Factor = $\cos \phi$.

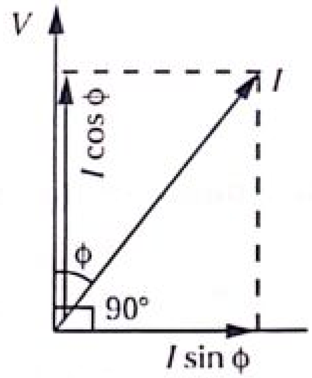


Fig. 3.47

In addition to having a numerical value, the power factor of a circuit carries a notation that signifies the nature of the circuit, i.e., whether the equivalent circuit is resistive, inductive or capacitive. Thus, the p.f. might be expressed as 0.8 lagging. *The lagging and leading refers to the phase of the current vector with respect to the voltage vector*. Thus, a lagging power factor means that the current lags the voltage and the circuit is inductive in nature. However, in the case of leading power factor, the current leads the voltage and the circuit is capacitive.

Apparent Power : The product of r.m.s. values of current and voltage, *VI*, is called the *apparent power* and is measured in volt-amperes (VA) or in kilo-volt amperes (kVA).

Real Power : The real power in an a.c. circuit is obtained by multiplying the apparent power by the power factor and is expressed in watts or kilo-watts (kW).

Real Power (W) = volt-amperes (VA) × power factor $\cos \phi$

or **Watts = *VA* cos ϕ**

Here, it should be noted that power consumed is due to ohmic resistance only as a pure inductance does not consume any power.

Thus, $P = VI \cos \phi$

$$\cos \phi = \frac{R}{Z} \quad \text{(refer to the impedance triangle of Fig. 3.45)}$$

$$\therefore \quad P = VI \times \left(\frac{R}{Z}\right)$$

$$= \left(\frac{V}{Z}\right) \times IR = I^2 R$$

or $P = I^2 R$ watts

Reactive Power : It is the power developed in the inductive reactance of the circuit. The quantity $VI \sin\phi$ is called the *reactive power*; it is measured in *reactive volt-amperes or vars (VAr).*

The power consumed can be represented by means of waveform in Fig. 3.48.

We will now calculate power in terms of instantaneous values.

Instantaneous power, $p = vi = V_m \sin\omega t \times I_m \sin(\omega t - \phi)$

$$= V_m I_m \sin\omega t \sin(\omega t - \phi)$$

$$= \frac{1}{2} V_m I_m [\cos\phi - \cos(2\omega t - \phi)]$$

This power consists of two parts :

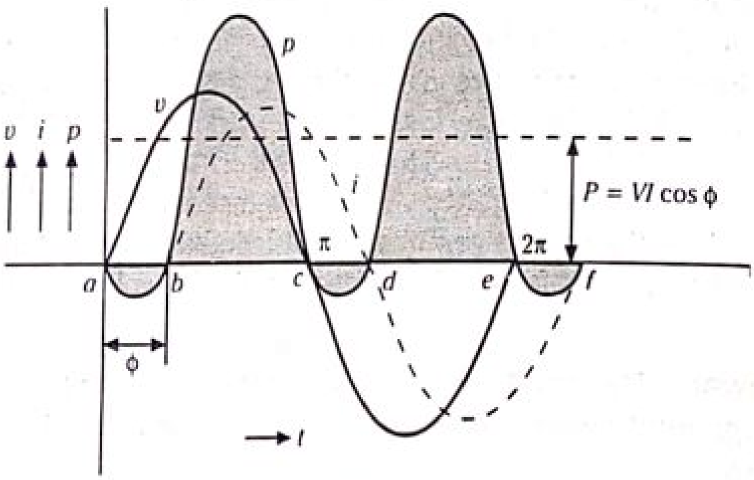


Fig. 3.48

i) Constant part $\frac{1}{2} V_m I_m \cos\phi$ which contributes to real power.

ii) Sinusoidally varying part $\frac{1}{2} V_m I_m \cos(2\omega t - \phi)$, whose frequency is twice that of the voltage and the current, and whose average value over a complete cycle is zero (so it does not contribute to any power).

So, average power consumed, $P = \frac{1}{2} V_m I_m \cos\phi$

$$= \frac{V_m}{\sqrt{2}} \cdot \frac{I_m}{\sqrt{2}} \cos\phi$$

$$= \boldsymbol{VI \cos\phi}$$

where V and I are r.m.s. values

Power Curves : The power curve for R-L series circuit is shown in Fig. 3.48. The curve indicates that the greater part is positive and the smaller part is negative, so that the net power over the cycle is positive.

During the time interval a to b, applied voltage and current are in opposite directions, so that power is negative. Under such conditions, the inductance L returns power to the circuit. During the period b to c, the applied voltage and current are in the same direction so that power is positive, and therefore, power is put into the circuit. In a similar way, during the period c to d, inductance L returns power to the circuit while between d and e, power is put into the circuit. The power absorbed by resistance R is converted into heat and not returned.

Problem 3.15

A current of $i = 2.8 \sin\left(94.3t + \frac{\pi}{6}\right)$ is flowing through a resistance of 10 Ω connected in series with an inductance of 1 H. Obtain an expression for voltage across the R and L combination in the form $v = V_{max} \sin(\omega t + \theta)$.

(83-84., B.U.)

Solution :

$$\omega = 2\pi f = 94.3$$

$$\therefore \quad f = \frac{94.3}{2\pi} = 15 \text{ Hz}$$

$$L = 1 \text{ H}; \quad R = 10\ \Omega$$

Inductive Reactance $X_L = 2\pi f L$

$$= 2\pi \times 15 \times 1$$

$$= 94.26\ \Omega$$

$$\therefore \quad Z = \sqrt{R^2 + X_L{}^2} = \sqrt{10^2 + 94.26^2}$$

$$= 94.78\ \Omega$$

From the equation, $I_m = 2.8$ A

$$V_m = I_m Z = 2.8 \times 94.78$$

$$= 265.4 \text{ V}$$

$$\tan\phi = \frac{X_L}{R} = \frac{94.26}{10} = 9.426 \qquad \therefore \quad \phi = 83.25°$$

$\therefore$ Phase angle between voltage and current = 83.25°

Now, as per equation, current leads the reference by $\frac{\pi}{6}$ radians or 30° and as per voltage eqn, voltage leads reference by ϕ.

As this is an inductive circuit, current lags behind the voltage by $\phi = 83.25°$

$\therefore$ $\theta = 83.25° + 30° = 113.25°$ or 0.627π rad

$\therefore$ Substituting the values of V_m, $\omega(= 94.3)$, and θ in the voltage equation,

$$v = \mathbf{265.4 \sin(94.3t + 0.627\pi)}$$

Problem 3.16

A voltage of 100 V at 50 Hz is applied to a *R–L* series circuit. The current in the circuit is 5 A lagging behind the voltage by 35°. Write the expression for the current and the voltage. (B'lore Univ. July 1993)

Solution :

R.M.S. value of applied voltage $V = 100$ volts.

$\therefore$ Maximum value of applied voltage $V_m = \sqrt{2}V = 141.4$ volts

R.M.S. value of current in the circuit $I = 5$ A

$\therefore$ Maximum value of current in circuit, $I_{max} = \sqrt{2}\, I = 7.1$ amps

Frequency $\phi = 50$ Hz

$\therefore$ $\omega = 2\pi f = 2\pi \times 50 = 314$

Expression for voltage : $v = V_m \sin \omega t$

$$= \mathbf{141.4 \sin 314\, t}$$

Current lags behind the voltage by and angle $\phi = 35°$

$\therefore$ Expression for current : $i = I_m \sin(\omega t - \phi)$

$$= \mathbf{7.1 \sin(314\,t - 35°)}$$

Problem 3.17

A series *R–L* circuit takes 384 watts at a power factor of 0.8 from a 120 V, 60 Hz supply. What are the values of *R* and *L* ? (Dec 86, B.U.)

Solution :

Given : $V = 120$ V, $f = 60$ Hz, $P = 384$ W and $\cos\phi = 0.8$

$P = VI\cos\phi = 384$

or $120\, I \times 0.8 = 384$

or $I = 4$ A

Impedance, $Z = \dfrac{V}{I} = \dfrac{120}{4} = 30\ \Omega$

Resistance, $R = Z\cos\phi = 30 \times 0.8 = \mathbf{24\ \Omega}$

Inductive Reactance, $X_L = Z\sin\phi = 30 \times 0.6 = 18\ \Omega$

Now, $X_L = 2\pi f L = 18$

$$\therefore \quad L = \frac{18}{2\pi \times 60} = \mathbf{0.047\ H}$$

Problem 3.18

Find an expression for the current and calculate power when a voltage v = 283 sin 100 πt is applied to a coil having R = 50 ohms and L = 0.159 H. **(Aug 94, B.U.)**

Solution :

$$v = 283 \sin 100\,\pi t$$

Frequency of applied voltage, $f = \dfrac{100\pi}{2\pi} = 50$ Hz

Inductive Reactance $X_L = 2\pi f L$

$$= 2\pi \times 50 \times 0.159$$
$$= 50\ \Omega$$

Given $R = 50\ \Omega$

$\therefore$ Coil impedance $Z = \sqrt{R^2 + X_L^2}$

$$= \sqrt{50^2 + 50^2} = 70.7\ \Omega$$

$$\phi = \angle \tan^{-1} \frac{50}{50} = \angle \tan^{-1} 1 = 45°$$

It means that circuit current lags behind the applied voltage by 45°. The expression for the current becomes

$$i = \frac{283}{Z} \sin\,(100\pi t - 45°)$$

$$= \frac{283}{70.7} \sin\left(100\pi t - \frac{\pi}{4}\right)$$

$$= \mathbf{4 \sin\left(100\pi t - \frac{\pi}{4}\right)}$$

Now max. value of voltage $V_m = 283$ volts

$\therefore$ RMS value of voltage $V = \dfrac{283}{\sqrt{2}}$ volts

Max. value of current $I_m = 4$ A

$\therefore$ RMS value of current $I = \dfrac{4}{\sqrt{2}}$ A

Power $= VI \cos\phi$

$$= \frac{283}{\sqrt{2}} \times \frac{4}{\sqrt{2}} \cos 45°$$

$$= \frac{283}{\sqrt{2}} \times \frac{4}{\sqrt{2}} \times 0.707$$

$$= \mathbf{400\ W}$$

Problem 3.19

A non-inductive resistance is connected in series with a coil across a 230 volts, 50 Hz supply. The current is 1.8 amperes and the potential difference across the resistance and the coil are 80 and 170 volts respectively. Calculate the resistance and inductance of the coil, and the phase difference between the current and the supply voltage and the power dissipated in the coil. Draw the phasor diagram. **(Mar 95, B.U.)**

Solution :

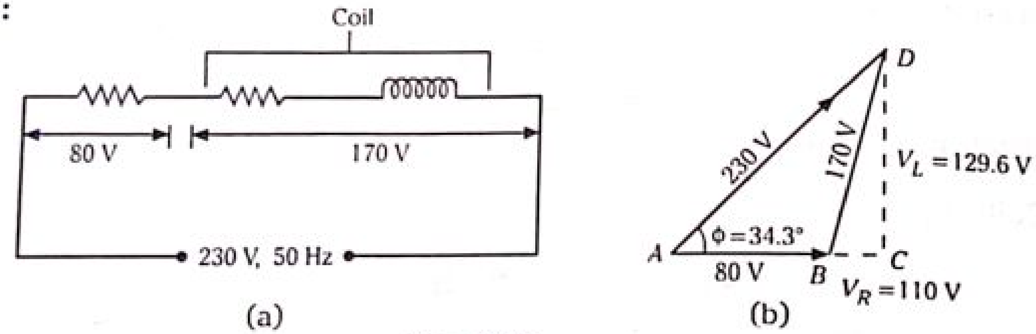


Fig. 3.49

As seen from the phasor diagram of Fig. 3.49(b).

$$BC^2 + CD^2 = 170^2 \quad \text{---(i)}$$

$$(80 + BC)^2 + CD^2 = 230^2 \quad \text{---(ii)}$$

Subtracting eqn.(i) from eqn.(ii),

$$(80 + BC)^2 - BC^2 = 230^2 - 170^2$$

$$BC = 110 \text{ V}$$

$$\therefore \quad CD = \sqrt{170^2 - 110^2} = 129.6 \text{ V}$$

$$\text{Coil impedance} = \frac{170}{1.8} = 94.44\ \Omega$$

$$V_R = IR = BC$$

or $\quad 1.8\,R = 110$

or $\quad R = \mathbf{61.1\ \Omega}$

$$V_L = IX_L = 129.6$$

$$\therefore \quad X_L = \frac{129.6}{1.8} = 72\ \Omega$$

or $$2\pi f L = 72$$

or $$L = \frac{72}{2\pi \times 50} = \mathbf{0.23\ H}$$

$$\cos\phi = \frac{190}{230} = 0.826$$

$$\therefore \quad \phi = 34.3°$$

Power dissipated in the coil

$$= I^2 R$$

$$= 1.8^2 \times 61.1$$

$$= \mathbf{198\ W}$$ **(Ans)**

The Phasor Diagram is drawn as shown.

Problem 3.20

A coil when connected to 200 V, 50 Hz supply takes a current of 10 A and dissipates 1200 W. Find the Resistance and Inductance of the coil.

Solution :

Power dissipated = 1200 W and I = 10 A

or $$I^2 R = 1200$$

or $$R = \mathbf{12\ \Omega}$$

Impedance $$Z = \frac{V}{I} = \frac{200}{10} = 20\ \Omega$$

Now, $$Z = \sqrt{R^2 + X_L^2}$$

where X_L is the inductive reactance

or $$20 = \sqrt{144 + X_L^2}$$

or $$400 = 144 + X_L^2$$

$$\therefore \quad X_L = 16$$

or $$2\pi f L = 16$$

$$\therefore \quad L = \frac{16}{2\pi \times 50} = \mathbf{0.051\ Henry}$$

Problem 3.21

A current i = sin (31 t – 10°) produces a potential drop v = 220 sin (31 t+20°) in a circuit. Find the values of circuit parameters, assuming a series combination. **(May/June 86, B.U.)**

Solution :

We notice that the voltage leads by 20° and the current lags by 10°, with regard to the reference quantity.

The phase difference between the voltage and current is

$$20^\circ - (-10^\circ) = 30^\circ, \text{ with the current lagging.}$$

The angular frequency is ω = 31 rad/sec.

As current lags, the circuit is inductive.

$$V_m = 220 \text{ V} \quad \text{and} \quad I_m = 1 \text{A}$$

$$\therefore \quad Z = \frac{V_m}{I_m} = \frac{200}{1} = 220\ \Omega$$

$$R = Z\cos\phi = 220\ \cos 30^\circ = 220 \times 0.866 = \mathbf{190.5\ \Omega}$$

$$X_L = Z\sin\phi = 220 \sin 30^\circ = 220 \times 0.5 = 110\ \Omega$$

$$\omega = 2\pi f$$

$$\text{or} \quad 31 = 2\pi f \qquad \therefore \quad f = \frac{31}{2\pi} = 4.9 \text{ Hz}$$

$$X_L = 2\pi f L = 110\ \Omega$$

$$\therefore \quad L = \frac{110}{2\pi \times 4.9} = \mathbf{3.57\ H}$$

Problem 3.22

An e.m.f. given by 100 sin $\left(314\,t - \frac{\pi}{4}\right)$ is applied to a circuit and the curre[nt] is 20 sin (314 t – 1.5708) Amps. Find the (i) frequency (ii) circuit elements. **(Sep/Oct 83, B.U**

Solution :

$$v(t) = 100 \sin\left(314\,t - \frac{\pi}{4}\right)$$

$$i(t) = 20\ \sin(314t - 1.5708)$$

$$\text{Phase angle of voltage} = -\frac{\pi}{4} \text{ radians (lag)}$$

$$= -\frac{180^\circ}{4} = -45^\circ \text{(lag)}$$

Phase angle of current = 1.5708 rad (lag)

$$= -\frac{1.5708}{\pi} \times 180° = -90° (\text{lag})$$

So, phase angle between voltage and current, ϕ = 45°, the current lagging; so the circuit is inductive.

Peak value of voltage, $V_m = 100$ V

Peak value of current, $I_m = 20$ A

Impedance of circuit, $Z = \frac{V_m}{I_m} = 5\ \Omega$

Circuit resistance, $R = Z \cos 45°$

$$= 5 \times 0.707 = \mathbf{3.53\ \Omega}$$

Circuit inductive reactance, $X_L = Z \sin 45° = 3.53\ \Omega$

The co-efficient of 't' in the voltage and current equation is 'ω'.

Thus $\omega = 314$.

or $\quad 2\pi f = 314$

or $\quad f = \frac{314}{2\pi} = \mathbf{50\ Hz}$

Inductive reactance, $X_L = 3.53\ \Omega$

or $\quad 2\pi f L = 3.53$

$$\therefore \quad L = \frac{3.53}{2\pi \times 50} = \mathbf{0.011\ H}$$

Problem 3.23

A current of 10 A flows in a circuit with a 30° angle of lag when applied voltage is 100 V. Find i) the resistance, reactance and impedance.
ii) conductance, susceptance and admittance.

Solution :

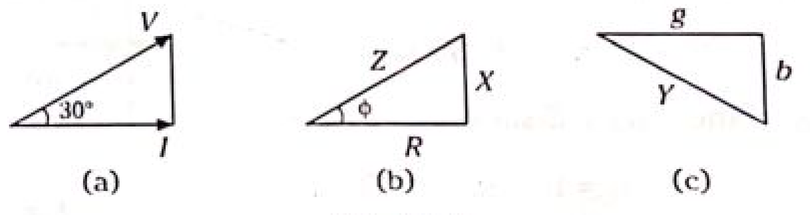


Fig. 3.50

Current I = 10 A, lagging the voltage V by 30° as shown in Fig. 3.50(a). Thus, it is an inductive circuit.

The impedance triangle is given at (b), where 'Z' is the impedance, 'R' the resistance and 'X' the reactance.

The admittance triangle is given at (c), where 'Y' is the admittance. 'g' the conductance and 'b' the susceptance.

Taking the Impedance triangle,

$$\text{Impedance } Z = \frac{V}{I} = \frac{100}{10} = \mathbf{10\ ohms}$$

$$\text{Resistance } R = Z \cos\phi = 10 \cos 30^\circ = \mathbf{8.66\ ohms}$$

$$\text{Reactance } X = Z \sin\phi = 10 \sin 30^\circ = \mathbf{5\ ohms}$$

Taking the admittance triangle,

$$\text{Admittance } Y = \frac{1}{Z} = \frac{1}{10} = \mathbf{0.1\ Siemen}$$

$$\text{Conductance } g = Y \cos\phi = 0.1 \times \cos 30^\circ$$

$$= 0.1 \times 0.866 = \mathbf{0.0866\ Siemen}$$

$$\text{Susceptance } b = Y \sin\phi$$

$$= 0.1 \times \sin 30^\circ = 0.1 \times 0.5$$

$$= \mathbf{0.05\ Siemen.}$$

Problem 3.24

A 100 V, 50 Hz inductive circuit takes a current of 10 Amps, lagging the voltage by 30°. Calculate the resistance and inductance of the circuit. Draw the waveforms of current and voltage. (B.U. Feb/Mar 83)

Solution :

The current lags behind the applied voltage by 30°, hence the circuit is inductive

∴ The p.f., cos 30° = 0.866 (lagging).

$$\text{Impedance } Z = \frac{V}{I} = \frac{100}{10} = \mathbf{10\ ohms}$$

$$\text{Resistance } R = Z \cos\phi$$

$$= 10 \times 0.866 = \mathbf{8.66\ \Omega}$$

$$\text{Inductive reactance } X_L = Z \sin\phi$$

$$= 10 \times 0.5 = 5\ \Omega$$

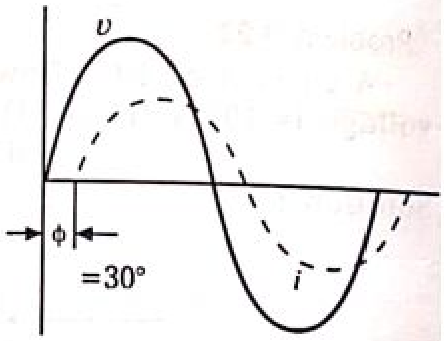


Fig. 3.51

$$2\pi f L = 5$$

$$\therefore \quad L = \frac{5}{2\pi \times 50} = \mathbf{0.016\ H}$$

Waveforms of current and voltage are given in Fig. 3.51.

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Problem 3.25

A choke coil takes a current of 2 A, lagging 60° behind the applied voltage of 200 V at 50 Hz. Calculate the inductance, resistance and impedance of the coil. Also determine the power consumed when it is connected across 100 V, 25 Hz supply. (June/July 89, B.U.)

Solution :

a) Applied voltage, $V = 200$ volts

Current, $I = 2$ amperes

Impedance of the coil, $Z = \frac{V}{I} = \frac{200}{20} = \mathbf{100\ \Omega}$

Resistance of the coil, $R = Z\cos\phi = 100\cos 60° = \mathbf{50\ \Omega}$

Inductive Reactance of coil, $X_L = Z\sin\phi = 100\sin 60°$

$= 100 \times 0.866 = 86.6\ \Omega$

$\therefore\quad L = \frac{86.6}{2\pi \times f} = \frac{86.6}{2\pi \times 50} = \mathbf{0.275\ H}$

b) The same coil is connected across a 100 V, 25 Hz, supply. However its impedance will now be different as the supply frequency is changed.

$R = 50\ \Omega;\quad V_1 = 100\ V$

Inductive Reactance, $X_{L1} = 2\pi \times f_1 \times L$

$= 2\pi \times 25 \times 0.275$

$= 43.3\ \Omega$

$\therefore$ Impedance $Z_1 = \sqrt{50^2 + 43.3^2}$

$= \sqrt{4374.9} = 66.1\ \Omega$

Current $I_1 = \frac{V_1}{Z_1} = \frac{100}{66.1} = 1.5$ A

Power factor, $\cos\phi_1 = \frac{R}{Z_1} = \frac{50}{66.1} = 0.75$

Power consumed $P = V_1 I_1 \cos\phi_1$

$= 100 \times 1.5 \times 0.75$

$= \mathbf{112.5\ Watts}$

3.16 Series *R-C* Circuit

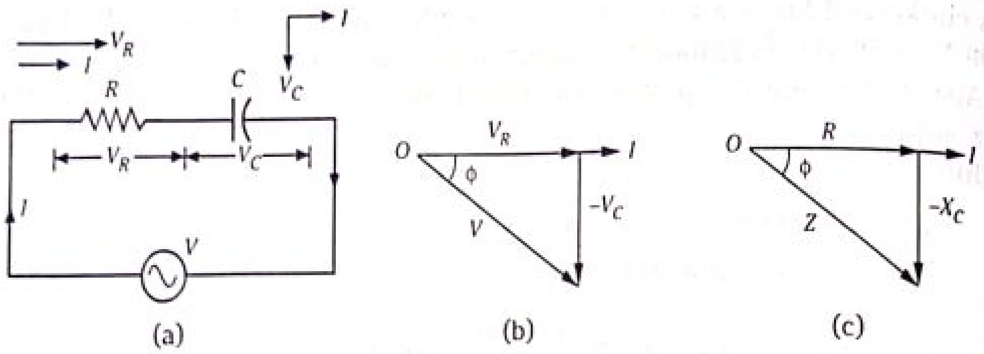


Fig. 3.52

Consider an a.c. circuit containing resistance R ohms and capacitance C farads as shown in Fig. 3.52(a).

Let V = r.m.s. value of voltage

I = r.m.s. value of current

$\therefore$ Voltage drop across R, $V_R = IR$ - in phase with I

Voltage drop across C, $V_C = IX_C$ - lagging I by $\frac{\pi}{2}$

The capacitive reactance is negative, so V_C is in the negative direction of Y-axis, as shown in Fig. 3.52(b).

We have $$V = \sqrt{V_R^2 + (-V_C)^2} = \sqrt{(IR)^2 + (-IX_C)^2}$$

$$= I\sqrt{R^2 + X_C^2}$$

or $$I = \frac{V}{\sqrt{R^2 + X_C^2}} = \frac{V}{Z}$$

The denominator, Z is the *impedance* of the circuit, *i.e.*, $Z = \sqrt{R^2 + X_C^2}$. Fig. 3.52(c) depicts the impedance triangle.

Power factor, $\cos\phi = \frac{R}{Z}$

Fig. 3.52(b) shows that I leads V by an angle ϕ, so that $\tan\phi = -\frac{X_C}{R}$.

This implies that if the alternating voltage is $v = V_m \sin \omega t$, the resultant current in the R-C circuit is given by $i = I_m \sin(\omega t + \phi)$, such that current *leads* the applied voltage

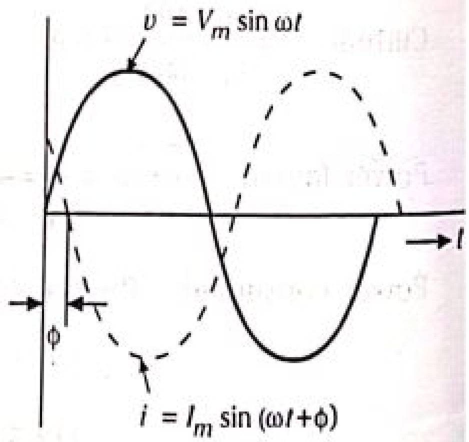


Fig. 3.53

by angle ϕ. The waveforms of Fig. 3.53 depict this fact.

Power : Average Power, $P = v \times i = VI \cos \phi$ (as in Sec. 3.17).

Power Curves : The power curve for R-C series circuit is shown in Fig. 3.54. The curve indicates that the greater part is positive and the smaller part is negative, so that the net power is positive.

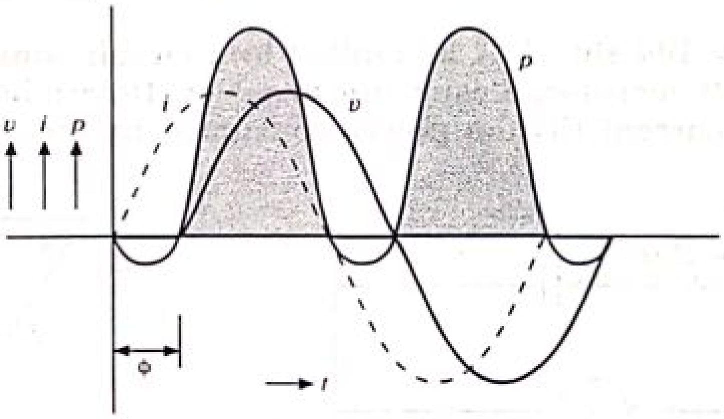


Fig. 3.54

Problem 3.26

A pure resistance of 50 ohms is in series with a pure capacitance of 100 microfarads. The series combination is connected across a 100 V, 50 Hz supply. Find (i) the impedance (ii) current (iii) power factor.

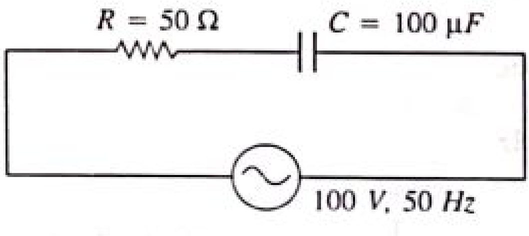


Fig. 3.55

Solution :

Given : $R = 50$ ohms ; $C = 100\ \mu F$

$V = 100$ volts ; $f = 50$ Hz

Now, $X_C = \dfrac{1}{2\pi f C}$

$$= \frac{1}{2\pi \times 50 \times \left(100 \times 10^{-6}\right)} = \mathbf{32\ ohms}$$

(i) $Z = \sqrt{R^2 + X_C^2} = \sqrt{50^2 + 32^2} = \mathbf{59.4\ ohms}$

(ii) $I = \frac{V}{Z} = \frac{100}{59.4} = \mathbf{1.684}$ **Amps**

(iii) Power factor, $\cos\phi = \frac{R}{Z} = \frac{50}{59.4} = \mathbf{0.842}$ **(leading)**

Problem 3.27

A voltage v = 100 sin 314 t is applied to a circuit consisting of a 25 ohm resistor and an 80 microfarad capacitor in series. Determine (i) an expression for the value of current (ii) the power consumed by the circuit.

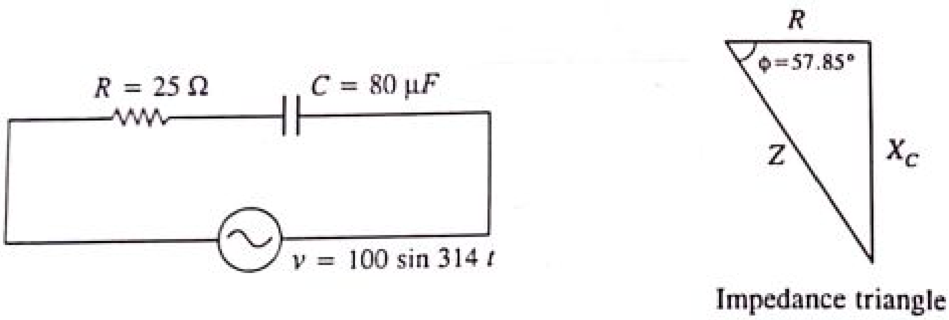


Fig. 3.56

Solution : $V_m = 100$

$$V = \frac{100}{\sqrt{2}} = 70.72 \text{ volts}$$

$$\omega = 314 \text{ rad/sec}$$

$$\therefore \quad f = \frac{314}{2\pi} = 50 \text{ Hz}$$

$$X_C = \frac{1}{2\pi fC} = \frac{1}{314 \times \left(80 \times 10^{-6}\right)} = \mathbf{39.8 \text{ ohms}}$$

$$Z = \sqrt{R^2 + X_C^2} = \sqrt{25^2 + 39.8^2} = \mathbf{47 \text{ ohms}}$$

$$I_m = \frac{V_m}{Z} = \frac{100}{47} = 2.13 \text{ Amps}$$

$$\therefore \quad I = \frac{I_m}{\sqrt{2}} = \frac{2.13}{\sqrt{2}} = 1.51$$

$$\cos\phi = \frac{R}{Z} = \frac{25}{47} = 0.532; \quad \therefore \phi = 57.85°$$

$$\therefore \quad i = 2.13 \sin(314\,t + 57.85°)$$

Power consumed by the circuit $= VI \cos \phi$

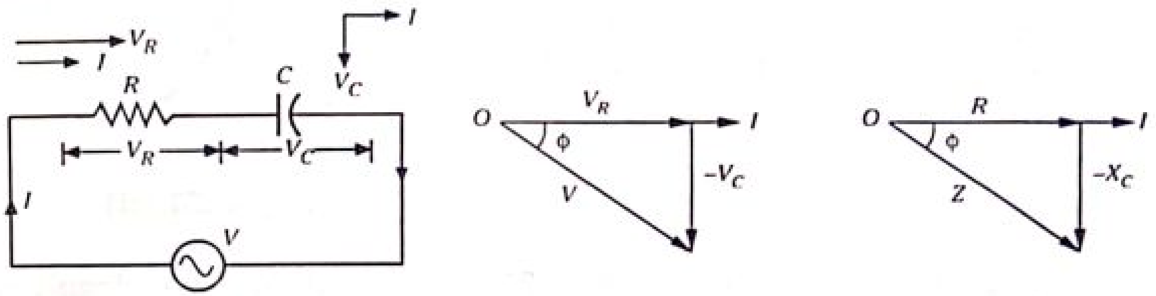
or $\quad P = 70.72 \times 1.51 \times 0.532$

or $\quad P = \mathbf{56.8}$ **watts**

Problem 3.28

A current $i = I_m \sin \omega t$ is flowing through a R–C series circuit. Obtain an expression of voltage across the R and C combination in the form $v = V_{max} \sin (\omega t - \phi)$.

Solution :



(a) (b) (c)

Fig. 3.57

Let us take an a.c. circuit containing resistance R and capacitance C farads, as shown in Fig. 3.57 (a).

Let V = r.m.s. value of voltage

I = r.m.s. value of current.

$\therefore$ Voltage drop across R, $V_R = IR$ - in phase with I

Voltage drop across C, $V_C = IX_C$ - lagging I by $\frac{\pi}{2}$

The capacitive reactance is negative, so V_C is in the negative direction of Y-axis, as shown in Fig. 3.57 (b).

$$\text{Thus,} \quad V = \sqrt{V_R^2 + (-V_C)^2} = \sqrt{(IR)^2 + (-IX_C)^2}$$

$$= I\sqrt{R^2 + X_C^2}$$

$$\text{or} \quad I = \frac{V}{\sqrt{R^2 + X_C^2}} = \frac{V}{Z}$$

where Z is the impedance to the circuit.

The Impedance Triangle is given in Fig. 3.57(c).

Power factor $\cos\phi = \dfrac{R}{Z}$

Fig. 3.57 (b) shows that V lags behind I by an angle ϕ, so that $\tan\phi = -\dfrac{X_C}{R}$ [Fig. 3.57 (c)].

This implies that if the alternating voltage is $v = V_{max} \sin(\omega t - \phi)$, then the resultant current in the R–C circuit is given by $i = I_m \sin \omega t$, such that the applied voltage lags the current by angle ϕ. The waveforms of Fig. 3.57 (d) depicts this fact.

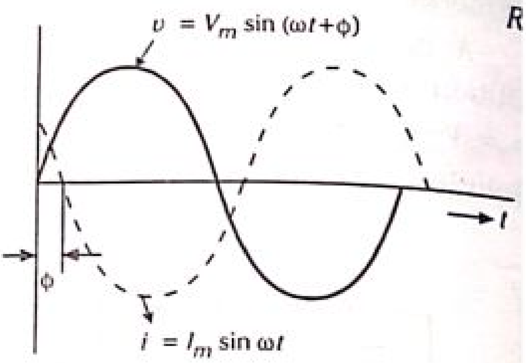


Fig. 3.57 (d)

Problem 3.29

An alternating voltage of $(160 + j\,120)$ V is applied to a circuit and th current in the circuit is given by $(6 + j8)$ A. Find

i) the values of the elements of the circuit,
ii) the power factor of the circuit,
iii) power consumed. **(Jan 93, B.U.**

Solution :

$$V = (160 + j120) = \sqrt{160^2 + 120^2}\,\angle \tan^{-1} 0.75$$
$$= 200 \angle 36.9^\circ$$

$$I = (6 + j8) = \sqrt{6^2 + 8^2}\,\angle \tan^{-1} 1.33$$
$$= 10 \angle 53.1^\circ$$

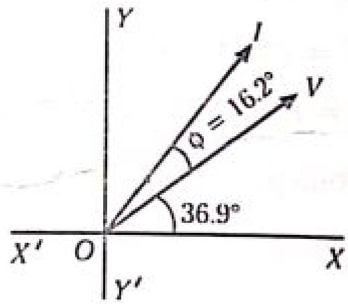


Fig. 3.58

$$Z = \frac{V}{I} = \frac{200\angle 36.9^\circ}{10\angle 53.1^\circ}$$
$$= 20\angle -16.2^\circ = 20\,(\cos 16.2^\circ - j\sin 16.2^\circ)$$
$$= 20\,(0.96 - j0.28)$$
$$= 19.2 - j5.6$$

Because of the negative sign before the 'j' term, circuit is capacitive, and th phase angle ϕ between voltage and current is $= 16.2^\circ$, with current leading th voltage, as shown in Fig. 3.58.

From the above expression for impedance Z, we have

R = **19.2** ohms and

Capacitive Reactance X_C = 5.6 ohms

Now, $\frac{1}{2\pi f C} = 5.6$

$\therefore \quad C = \frac{1}{2\pi \times 50 \times 5.6}$ F

$= \frac{10^6}{100\pi \times 5.6}$ μF

= **570 μF**

Power consumed, $P = I^2 R = 10^2 \times 19.2 = 1920$ **Watts**

Power Factor, $\cos\phi = \cos 16.2° =$ **0.96**

3.17 Resistance, Inductance and Capacitance in Series

Consider an a.c. series circuit containing resistance R ohms, inductance L henries and capacitance C farads as shown in Fig. 3.59.

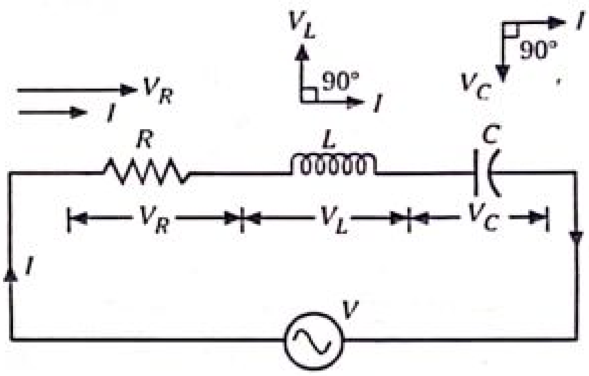


Fig. 3.59

Let V = r.m.s. value of applied voltage
I = r.ms. value of current

$\therefore$ Voltage drop across R, $V_R = IR$ - in phase with I

Voltage drop across L, $V_L = I.X_L$ - leading I by 90°

Voltage drop across C, $V_C = I.X_C$ - lagging I by 90°

Referring to the voltage triangle of Fig. 3.60, OA represents V_R, AB and AC represent inductive and capacitive drops respectively. We observe that V_L and V_C are 180° out of phase.

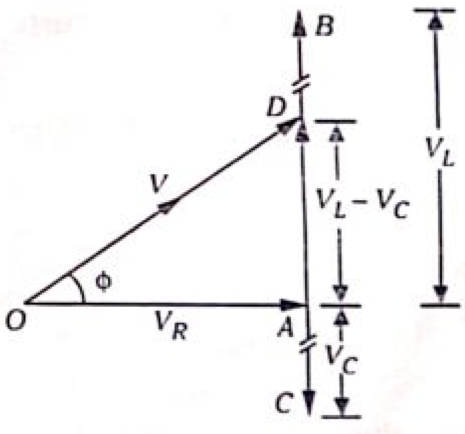


Fig. 3.60

Thus, the net reactive drop across the combination is

$$AD = AB - AC$$
$$= AB - BD \qquad (\because BD = AC)$$
$$= V_L - V_C$$
$$= I(X_L - X_C)$$

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OD, which represents the applied voltage V, is the vector sum of OA and AD

$$\therefore \quad OD = \sqrt{OA^2 + AD^2} \quad \text{or} \quad V = \sqrt{(IR)^2 + (IX_L - IX_C)^2}$$

$$= I\sqrt{R^2 + (X_L - X_C)^2}$$

$$\text{or} \quad I = \frac{V}{\sqrt{R^2 + (X_L - X_C)^2}} = \frac{V}{\sqrt{R^2 + X^2}} = \frac{V}{Z}$$

The denominator $\sqrt{R^2 + (X_L - X_C)^2}$ is the impedance of the circuit.

So (impedance)^2 = (resistance)^2 + (net reactance)^2

or $Z^2 = R^2 + (X_L - X_C)^2 = R^2 + X^2$

where the net reactance = X (Fig. 3.61)

Phase angle ϕ is given by

$$\tan\phi = \frac{(X_L - X_C)}{R} = \frac{X}{R}$$

Power Factor,

$$\cos\phi = \frac{R}{Z} = \frac{R}{\sqrt{R^2 + (X_L - X_C)^2}} = \frac{R}{\sqrt{R^2 + X^2}}$$

Power = $VI \cos\phi$

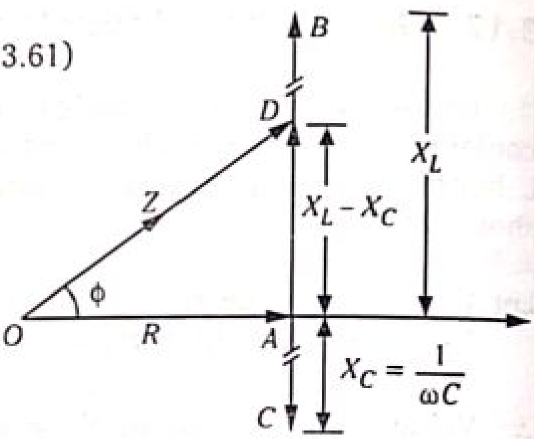


Fig. 3.61

If applied voltage is represented by the equation $v = V_m \sin \omega t$, then the resulting current in an R-L-C circuit is given by the equation

$$i = I_m \sin(\omega t \pm \phi)$$

If $X_C > X_L$, then the current leads and the $+ve$ sign is to be used in the above equation.

If $X_L > X_C$, then the current lags and the $-ve$ sign is to be used.

In any case, the current leads or lags the supply voltage by an angle ϕ, so that $\tan\phi = \frac{X}{R}$.

If we employ the j operator (Fig. 3.62), then we have

$$Z = R + j(X_L - X_C)$$

The value of the impedance is

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

The phase angle $\phi = \tan^{-1}\frac{(X_L - X_C)}{R}$

$$Z\angle\phi = Z\angle\tan^{-1}\left[\frac{X_L - X_C}{R}\right]$$

$$= Z\angle\tan^{-1}\left[\frac{X}{R}\right]$$

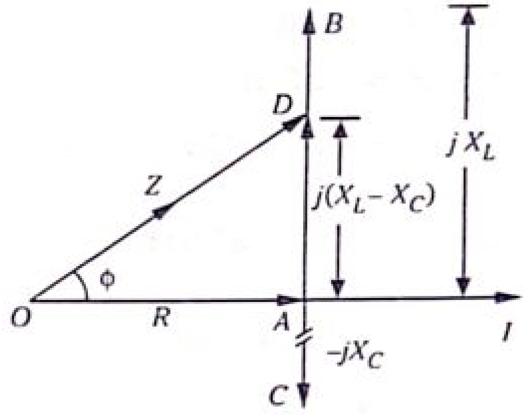


Fig. 3.62

Problem 3.30

A series circuit with resistance of 10 Ω, inductance of 0.2 H and capacitance of 40 μF is supplied with a 100 V supply at variable frequency. Find the frequency at which resonance takes place. At resonance, find the current, power and power factor. Also find the voltage across resistance, inductance and capacitance at that time.

Solution :

i) $f_o = \frac{1}{2\pi\sqrt{LC}} = \frac{10^3}{2\pi\sqrt{0.2\times 40}} = \mathbf{56.3\ Hz}$

ii) Current is maximum and its value is $I_o = \frac{100}{10} = \mathbf{10\ A}$

iii) Power $= I_o^2 R = 10^2 \times 10 = \mathbf{1000\ W}$

iv) P.F = 1

v) $V_R = 10 \times 10 = 100\ V;\quad V_L = IX_L = 10\,(2\pi \times 56.3 \times 0.2)$

$= \mathbf{707.5\ V}$

$$V_C = IX_C = 10 \times \frac{1}{2\pi \times 56.3 \times 40 \times 10^{-6}}$$

$$= \mathbf{707.5\ V}$$

Problem 3.31

A voltage of 200 V is applied to a series circuit consisting of a resistor, an inductor and a capacitor. The respective voltages across these components are 170, 150 and 100 V and the current is 4 A. Find the power factor of the circuit. **(Aug. 94, B.U.)**

Solution :

Current $I = 4$ A

$V_R = IR = 170$

or $4R = 170$

$\therefore \quad R = 42.5\ \Omega$

$V_L = IX_L = 150$

$\therefore \quad X_L = \dfrac{150}{4} = 37.5\ \Omega$

$V_C = IX_C = 100$

$\therefore \quad X_C = 25\ \Omega$

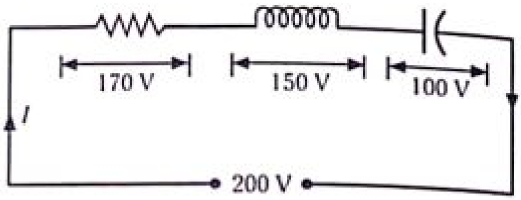


Fig. 3.63

Impedance $Z = \sqrt{R^2 + (X_L - X_C)^2}$

$= \sqrt{42.5^2 + 12.5^2}$

$= 44.3$

Power Factor, $\cos\phi = \dfrac{R}{Z}$

$= \dfrac{42.5}{44.3}$

$= \mathbf{0.96}$

Problem 3.32

A series circuit with $R = 10\ \Omega$, $L = 50$ mH and $C = 100\ \mu$F is supplied with 200 V, 50 Hz. Find (i) the impedance, (ii) current, (iii) power and (iv) power factor. (May/June 86, B.U.)

Solution :

Circuit resistance, $R = 10\ \Omega$

Inductive reactance of the circuit, $X_L = 2\pi f L$

$= 2\pi \times 50 \times 50 \times 10^{-3}$

$= 15.71\ \Omega$

Capacitive reactance of the circuit, $X_C = \dfrac{1}{2\pi f C}$

$= \dfrac{1}{2\pi \times 50 \times 100 \times 10^{-6}}$

$= 31.8\ \Omega$

Impedance of the circuit $Z = \sqrt{R^2 + (X_L - X_C)^2}$

$$= \sqrt{10^2 + (15.71 - 31.8)^2}$$

$$= \sqrt{100 + 258.8}$$

$$= \mathbf{18.94\ \Omega}$$

$X_C = \frac{1}{WC}$

Circuit current, $I = \frac{V}{Z} = \frac{200}{18.94} = \mathbf{10.55\ A}$

Circuit power factor, $\cos\phi = \frac{R}{Z} = \frac{10}{10.55}$

$$= \mathbf{0.947\ (leading).} \qquad (\because X_C > X_L)$$

Power consumed, $P = VI\cos\phi$

$$= 200 \times 10.55 \times 0.947$$

$$= \mathbf{1998\ Watts}$$

3.18 Parallel AC circuits

In a parallel a.c. circuit, the voltage across each branch of the circuit is the same whereas current in each branch depends upon the branch impedance. Since alternating currents are vector quantities, total line current is the vector sum of branch currents.

The following are the three methods of solving parallel a.c. circuits :

a) Vector method.
b) Admittance method.
c) Symbolic or *j*-method.

3.18.1 Vector Method

In this method the total line current is found by drawing the vector diagram of the circuit. As voltage is common, it is taken as the reference vector and the various branch currents are represented vectorially. The total line current can be determined from the vector diagram either by the *parallelogram method* or by the *method of components*.

Let us take a parallel a.c. circuit, consisting of two branch impedances Z_1 (R_1, L) and Z_2 (R_2, C) connected in parallel across an alternating voltage of V volts (r.m.s.) as shown in Fig. 3.64. The total line current I is the vector sum of branch currents I_1 and I_2.

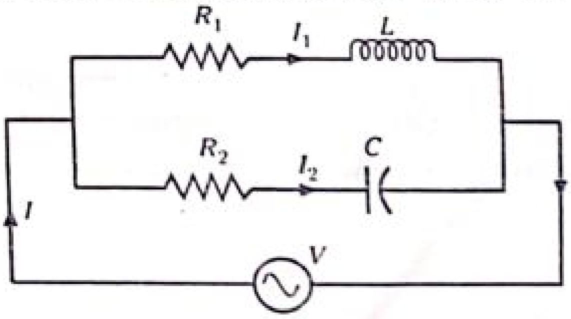


Fig. 3.64

Branch 1

Impedance $Z_1 = \sqrt{R_1^2 + X_L^2}$

Current $I_1 = \dfrac{V}{Z_1}$

$\cos\phi_1 = \dfrac{R_1}{Z_1}$ or $\phi_1 = \cos^{-1}\left[\dfrac{R_1}{Z_1}\right]$

Current I_1 lags behind the applied voltage by ϕ_1 (Fig. 3.65).

Branch 2

Impedance $Z_2 = \sqrt{R_2^2 + X_C^2}$

Current $I_2 = \dfrac{V}{Z_2}$

$\cos\phi_2 = \dfrac{R_2}{Z_2}$ or $\phi_2 = \cos^{-1}\left(\dfrac{R_2}{Z_2}\right)$

Current I_2 leads V by ϕ_2 (Fig. 3.65).

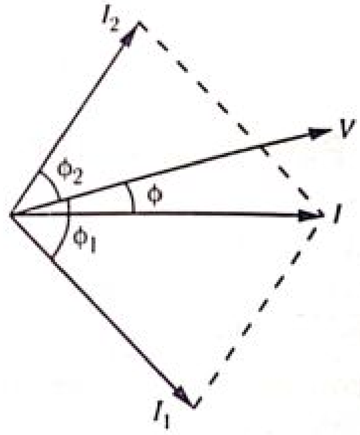


Fig. 3.65

Resultant Current : The total line current I is the vector sum of the branch currents I_1 and I_2 and is found by using the *Parallelogram Law of Vectors*, as shown in Fig. 3.65.

The second method is the *Method of Components i.e.*, resolving the branch currents I_1 and I_2 along the X-axis and Y-axis and then finding the resultant of these components (Fig. 3.66).

Let the resultant current be I and ϕ be its phase angle, as shown in Fig. 3.66 (b). Then the component of I along X-axis is equal to the algebraic sum of the components of branch currents I_1 and I_2 along the X-axis (active components).

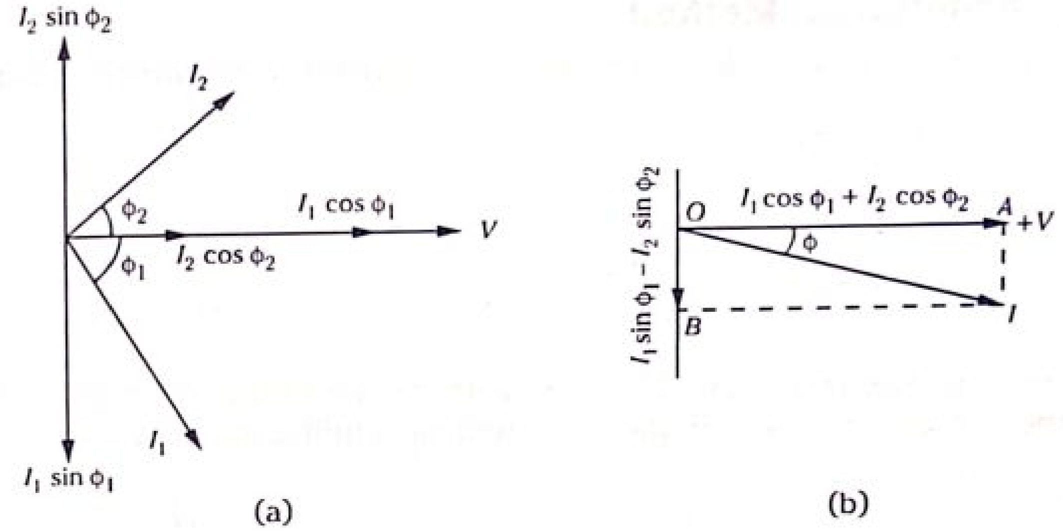


Fig. 3.66

Similarly, the component of I along Y-axis is equal to the algebraic sum of the components of I_1 and I_2 along Y-axis *i.e*,

Component of resultant current along X-axis

= algebraic sum of I_1 and I_2 along X-axis

or $\quad I \cos \phi = I_1 \cos \phi_1 + I_2 \cos \phi_2$

Component of resultant current along Y-axis

= algebraic sum of I_1 and I_2 along Y-axis

or $\quad I \sin \phi = I_1 \sin \phi_1 - I_2 \sin \phi_2$

$$\therefore \quad I = \sqrt{(I \cos \phi)^2 + (I \sin \phi)^2}$$

$$= \sqrt{(I_1 \cos \phi_1 + I_2 \cos \phi_2)^2 + (I_1 \sin \phi_1 - I_2 \sin \phi_2)^2}$$

$$\text{and} \quad \tan \phi = \frac{I_1 \sin \phi_1 - I_2 \sin \phi_2}{I_1 \cos \phi_1 + I_2 \cos \phi_2}$$

$$\therefore \quad \text{Phase angle } \phi = \tan^{-1}\left(\frac{I_1 \sin \phi_1 - I_2 \sin \phi_2}{I_1 \cos \phi_1 + I_2 \cos \phi_2}\right)$$

If $\tan \phi$ is positive, current leads and if $\tan \phi$ is negative, then the current lags behind applied voltage V. Power Factor for the entire circuit

$$\cos \phi = \frac{I_1 \cos \phi_1 + I_2 \cos \phi_2}{I}$$

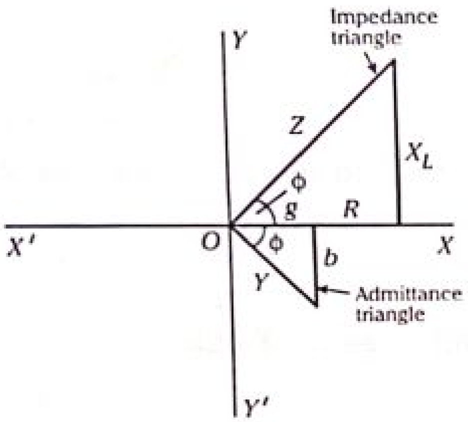
3.18.2 Admittance Method

The reciprocal of impedance of a circuit is called its admittance. It is represented by Y.

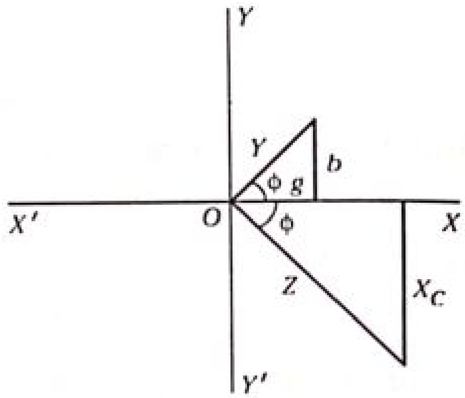
$$Y = \frac{1}{Z} = \frac{I}{V}$$

$$\text{So, } Y = \frac{\text{r.m.s. amperes}}{\text{r.m.s. volts}}$$

Its unit is *Siemens* (S). A circuit with an impedance of one ohm has an admittance of one Siemen. Earlier, the unit of admittance was *mho*.



(a) Inductive Circuit



(b) Capacitive circuit

Fig. 3.67

Just as impedance Z of a circuit has two rectangular components, resistance R and reactance X, admittance Y also has two rectangular components known as conductance g and susceptance b. Fig. 3.67 shows the impedance triangle and the admittance triangle. It is clear the admittance has two components g and b. The component g along the X-axis is the *conductance* which is the reciprocal of resistance. The component b is called *susceptance*, which is the reciprocal of reactance.

In Fig. 3.67 (a), the impedance and admittance triangles for an inductive circuit are shown. It is apparent that susceptance b is negative, being below X-axis. Hence *inductive susceptance is negative*. In Fig. 3.67 (b), the impedance and admittance triangles for capacitive circuit is shown. It is evident that susceptance is positive, being above the X-axis; Hence, *capacitive susceptance is positive*.

Relations

Conductance $g = Y \cos \phi$

$$\text{or} \quad g = \frac{1}{Z} \cdot \frac{R}{Z} = \frac{R}{Z^2} = \frac{R}{R^2 + X^2}$$

Conductance is always positive.

Susceptance $b = Y\sin\phi = \frac{1}{Z}\cdot\frac{X}{Z} = \frac{X}{Z^2} = \frac{X}{R^2+X^2}$

Susceptance b is positive if reactance X is capacitive and negative if reactance is inductive.

$$\text{Admittance } Y = \sqrt{g^2+b^2}$$

The units of g, b and Y are in Siemens.

3.18.3 Application of Admittance Method

Let us consider a parallel circuit with three branches, as given in Fig. 3.68. We can determine the conductance by just adding the conductances of the three branches. In a like manner, susceptance is determined by the algebraic addition of the susceptances of the different branches.

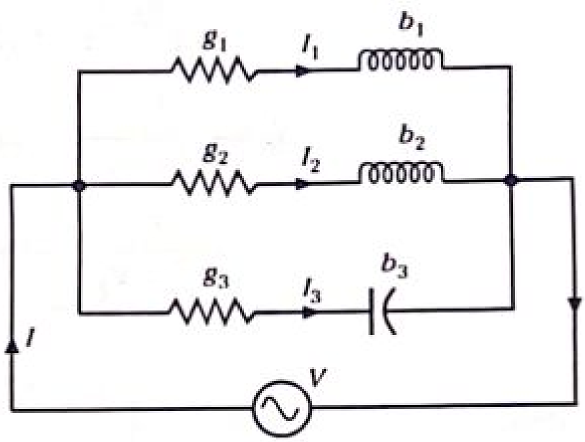


Fig. 3.68

Total conductance,

$$G = g_1 + g_2 + g_3$$

Total susceptance

$$B = (-b_1)+(-b_2)+b_3$$

∴ Total admittance $Y = \sqrt{G^2+B^2}$

Total current $I = VY$

Power Factor, $\cos\phi = \frac{G}{Y}$

3.18.4 Symbolic or j - Method

Let us take the parallel two-branch circuit of Fig. 3.69, with the same p.d. across the two impedances Z_1 and Z_2.

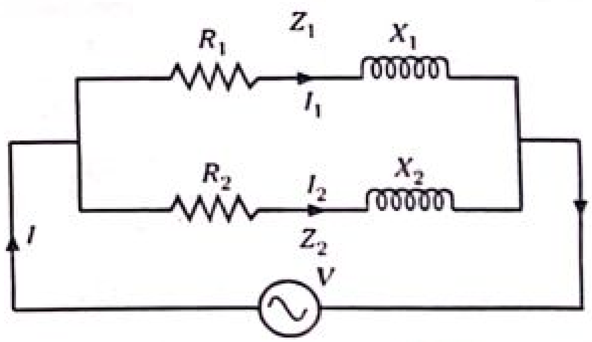


Fig. 3.69

$$I_1 = \frac{V}{Z_1} \quad \text{and} \quad I_2 = \frac{V}{Z_2}$$

$$\text{Total current } I = I_1 + I_2 = \frac{V}{Z_1} + \frac{V}{Z_2} = V\left(\frac{1}{Z_1}+\frac{1}{Z_2}\right)$$

$$= V\,(Y_1 + Y_2)$$

$$= V\,Y$$

where the total admittance $Y = Y_1 + Y_2$

We should note that *admittances are added for parallel* branches, where impedances are added for series branches. Both admittances and impedances a complex quantities, so all additions have to be performed in complex form.

In the case of the two parallel branches of Fig. 3.70,

$$Y_1 = \frac{1}{Z_1} = \frac{1}{R_1 + jX_L} = \frac{(R_1 - jX_L)}{(R_1 + jX_L)(R_1 - jX_L)} = \frac{R_1 - jX_L}{R_1^2 + X_L^2}$$

$$= \frac{R_1}{R_1^2 + X_L^2} - j\frac{X_L}{R_1^2 + X_L^2} = g_1 - jb_1$$

where $g_1 = \dfrac{R_1}{R_1^2 + X_L^2}$ – conductance of top branch

$b_1 = \dfrac{X_L}{R_1^2 + X_L^2}$ – susceptance of top branch

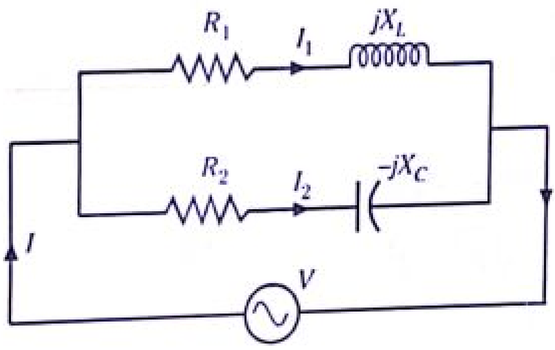


Fig. 3.70

In a similar manner,

$$Y_2 = \frac{1}{Z_2} = \frac{1}{R_2 - jX_C} = \frac{(R_2 + jX_C)}{(R_2 - jX_C)(R_2 + jX_C)}$$

$$= \frac{R_2 + jX_C}{R_2^2 + X_C^2} = \frac{R_2}{R_2^2 + X_C^2} + j\frac{X_C}{R_2^2 + X_C^2}$$

$$= g_2 + jb_2$$

Total admittance $Y = Y_1 + Y_2$

$$= (g_1 - jb_1) + (g_2 + jb_2)$$

$$= (g_1 + g_2) - j(b_1 - b_2)$$

$$= G - jB$$

$$Y = \sqrt{(g_1 + g_2)^2 + (b_1 - b_2)^2}$$

$$\phi = \tan^{-1}\left[\frac{b_1 - b_2}{g_1 + g_2}\right]$$

In polar form, admittance $Y = Y \angle \phi°$

$$Y = \sqrt{G^2 + B^2} \angle \tan^{-1}\left(\frac{B}{G}\right)$$

Total current $I = VY$; $I_1 = VY_1$ and $I_2 = VY_2$

$V = V \angle 0°$ and $Y = Y \angle \phi$

So $I = VY = V \angle 0° \times Y \angle \phi = VY \angle \phi$

Taking a general case,

if $V = V \angle \alpha$ and $Y = Y \angle \beta$, then

$I = VY = V \angle \alpha \times Y \angle \beta = VY \angle \alpha + \beta$

Problem 3.33

A circuit consisting of branches A and B connected in parallel, is connected across a 220 V, 50 Hz supply.

Branch A : A resistance of 7 Ω in series with 0.0125 H inductor.

Branch B : A resistance of 8 Ω in series with 1000 μF capacitor. Find the branch currents and the total current.

Draw the phasor diagram.

(B.U. – Jan 1993)

Solution : The circuit is as shown.

Let the total current be I, and the branch currents be I_A and I_B.

Branch A :

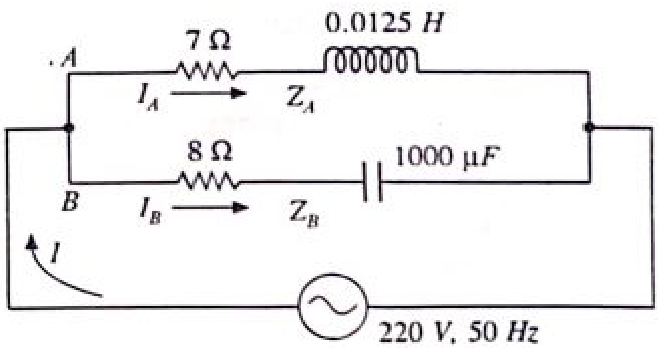


Fig. 3.71(a)

Resistance $R_A = 7\ \Omega$ and

Inductance $L_A = 0.0125$ H

Inductive reactance

$X_A = 2\pi f L_A$

$= 2\pi \times 50 \times 0.0125$

$= 3.929\ \Omega$

Impedance $Z_A = (7 + j3.929)\ \Omega$

$= 8.027 \angle 29.29°\ \Omega$

Branch current $I_A = \dfrac{V}{Z_A}$

where the total admittance $Y = Y_1 + Y_2$

We should note that *admittances are added for parallel* branches, whe impedances are added for series branches. Both admittances and impedance complex quantities, so all additions have to be performed in complex form.

In the case of the two parallel branches of Fig. 3.70,

$$Y_1 = \frac{1}{Z_1} = \frac{1}{R_1 + jX_L} = \frac{(R_1 - jX_L)}{(R_1 + jX_L)(R_1 - jX_L)} = \frac{R_1 - jX_L}{R_1^2 + X_L^2}$$

$$= \frac{R_1}{R_1^2 + X_L^2} - j\frac{X_L}{R_1^2 + X_L^2} = g_1 - jb_1$$

where $g_1 = \dfrac{R_1}{R_1^2 + X_L^2}$ – conductance of top branch

$b_1 = \dfrac{X_L}{R_1^2 + X_L^2}$ – susceptance of top branch

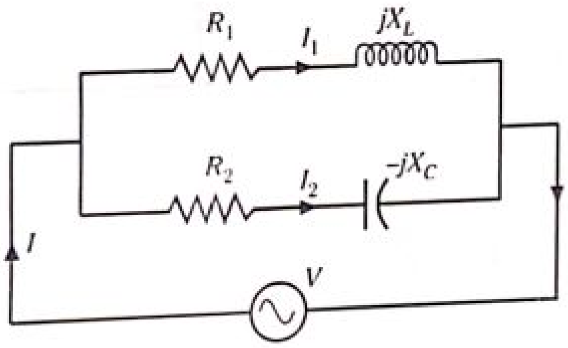


Fig. 3.70

In a similar manner,

$$Y_2 = \frac{1}{Z_2} = \frac{1}{R_2 - jX_C} = \frac{(R_2 + jX_C)}{(R_2 - jX_C)(R_2 + jX_C)}$$

$$= \frac{R_2 + jX_C}{R_2^2 + X_C^2} = \frac{R_2}{R_2^2 + X_C^2} + j\frac{X_C}{R_2^2 + X_C^2}$$

$$= g_2 + jb_2$$

Total admittance $Y = Y_1 + Y_2$

$$= (g_1 - jb_1) + (g_2 + jb_2)$$

$$= (g_1 + g_2) - j(b_1 - b_2)$$

$$= G - jB$$

where the total admittance $Y = Y_1 + Y_2$

We should note that *admittances are added for parallel* branches, whe impedances are added for series branches. Both admittances and impedance complex quantities, so all additions have to be performed in complex form.

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$$= \frac{R_1}{R_1^2 + X_L^2} - j\frac{X_L}{R_1^2 + X_L^2} = g_1 - jb_1$$

where $g_1 = \dfrac{R_1}{R_1^2 + X_L^2}$ – conductance of top branch

$b_1 = \dfrac{X_L}{R_1^2 + X_L^2}$ – susceptance of top branch

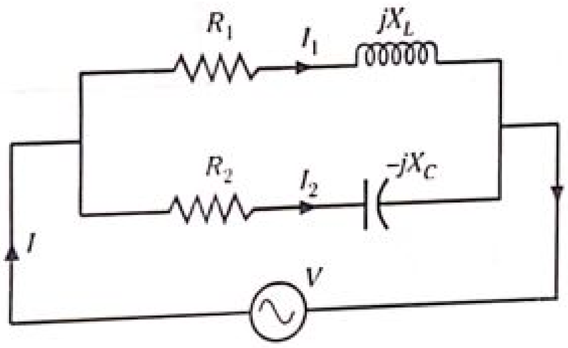


Fig. 3.70

In a similar manner,

$$Y_2 = \frac{1}{Z_2} = \frac{1}{R_2 - jX_C} = \frac{(R_2 + jX_C)}{(R_2 - jX_C)(R_2 + jX_C)}$$

$$= \frac{R_2 + jX_C}{R_2^2 + X_C^2} = \frac{R_2}{R_2^2 + X_C^2} + j\frac{X_C}{R_2^2 + X_C^2}$$

$$= g_2 + jb_2$$

Total admittance $Y = Y_1 + Y_2$

$$= (g_1 - jb_1) + (g_2 + jb_2)$$

$$= (g_1 + g_2) - j(b_1 - b_2)$$

$$= G - jB$$

$$\text{or} \quad I_A = \frac{220\angle 0^\circ}{8.027\angle 29.29^\circ}$$

$$= 27.41\ \angle -29.29^\circ$$

$$= (\mathbf{23.9 - j13.4})\ \mathbf{A}$$

Branch B :

Resistance $R_B = 8\ \Omega$ and capacitance $C_B = 1000\ \mu F$

$$\text{Capacitive reactance } X_B = \frac{1}{2\pi f C_B} = \frac{1 \times 10^6}{2\pi \times 50 \times 1000} = 3.18\ \Omega$$

$$\therefore \quad \text{Impedance } Z_B = R_B - jX_B$$

$$= (8 - j3.18)\ \Omega$$

$$= 8.61\ \angle -21.7^\circ$$

$$\text{Branch current } I_B = \frac{V}{Z_B} = \frac{220\angle 0^\circ}{8.61\angle -21.7^\circ}$$

$$= 25.55\ \angle 21.7^\circ\ \text{A}$$

$$= (\mathbf{23.75 + j9.45})\ \mathbf{A}$$

$\therefore$ Total Current $I = I_A + I_B$ (phasor sum)

$$\therefore \quad I = (23.9 - j13.4) + (23.75 + j9.45)$$

$$= (47.65 - j\,3.95)\ \text{A}$$

$$= \mathbf{47.82\ \angle -4.75^\circ\,A}$$

Verification :

$$Y_A = \frac{1}{Z_A} = \frac{1}{7 + j\,3.929}$$

$$= \frac{7 - j3.929}{7^2 + 3.929^2}$$

$= (0.109 - j0.06)$ mho, after simplifying

$$Y_B = \frac{1}{Z_B}$$

$$= \frac{1}{8 - j3.18}$$

$$= \frac{8 + j3.18}{8^2 + 3.18^2}$$

$= (0.108 - j0.043)$ mho, on simplifying

$$Y_{eq} = Y_A + Y_B$$
$$= (0.109 - j0.06) + (0.108 - j0.043)$$
$$= (0.217 - j0.017) \text{ mho}$$
$$= 0.2175 \angle -4.78° \text{ mho}$$

Current $I = V\, Y_{eq}$

$$= (220\angle 0°)(0.2175 \angle -4.78°)$$

$= 47.85 \angle -4.749°$, which is practically the same as obtained earlier

Phasor Daigram

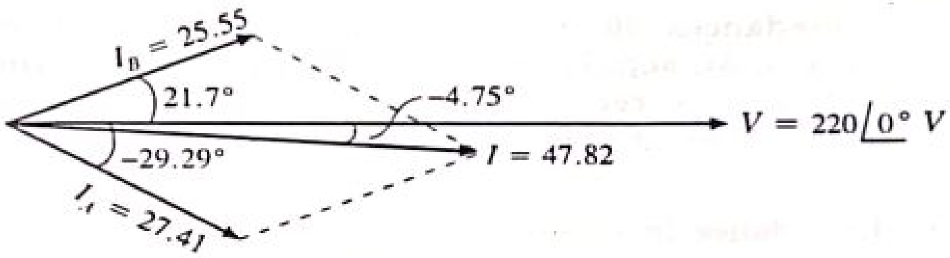


Fig. 3.71(b)

Problem 3.34

Find the reading of the ammeter when the voltmeter across the 3 ohm resistor in the circuit shown below reads 45 V. (July 88, B.U.)

Solution :

$$I_1 = \frac{45}{3} = 15 \text{ A}$$

We shall take this as reference quantity.

So $I_1 = 15 \angle 0°$

Obviously, the applied voltage is

$$V = 15 \angle 0° \times (3 - j3)$$
$$= 15 \angle 0° \times 4.24 \angle -45°$$
$$= 63.6 \angle -45°$$

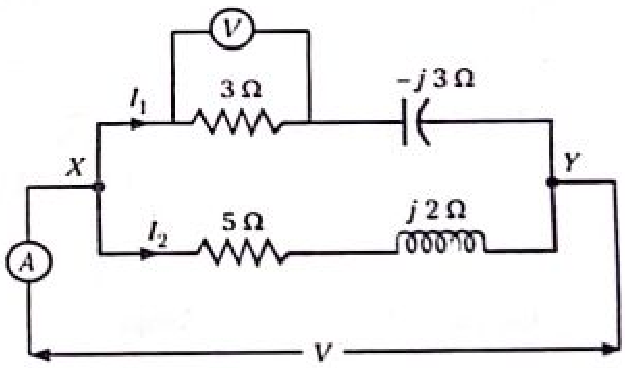


Fig. 3.72

$$I_2 = \frac{V}{Z_2} = \frac{63.6 \angle -45°}{(5 + j2)}$$

$$= \frac{63.6 \angle -45°}{5.4 \angle 21.8°} = 11.77 \angle -66.8°$$

$$= 11.77 (\cos 66.8° - j \sin 66.8°)$$

$$= 11.77 (0.3939 - j0.9191)$$

$$= 4.64 - j10.8$$

$$I = I_1 + I_2 = (15 + j0) + (4.64 - j10.8)$$

or $$I = 19.64 - j10.8$$

$$= \sqrt{(19.64)^2 + (-10.8)^2} \angle -\tan^{-1} 0.55$$

$$= \sqrt{385.7 + 116.6} \angle -28.8°$$

$$= 22.4 \angle -28.8°$$

Hence, ammeter reads **22.4 Amps.**

Problem 3.35

Two impedances 20 ∠−45° Ω and 30 ∠30° Ω are connected in seri across a certain AC supply and the resulting current is found to be 10 Am If the supply voltage remains unchanged, calculate the supply current wh the two impedances are connected in parallel. (Feb 96, B.

Solution :

Case I : Impedance in series

Impedance $Z_1 = 20\angle -45° = 20\,(\cos 45° - j \sin 45°)$

$$= 20\,(0.707 - j0.707)$$

$$= 14.14 - j\,14.14$$

Impedance $Z_2 = 30\angle 30° = 30\,(\cos 30° + j \sin 30°)$

$$= 30\,(0.866 + j0.5)$$

$$= 26 + j\,15$$

Total impedance $Z = Z_1 + Z_2$

$$= (14.14 - j14.14) + (26 + j15)$$

$$= 40.14 + j\,0.86$$

Current $I = \dfrac{V}{Z} = 10$ Amps

$\therefore$ Voltage applied, $V = 10\,Z$

$$= 10\,(40.14 + j\,0.86)$$

$$= 401.4 + 8.6$$

$$= 401.5 \angle \tan^{-1} 0.021$$

$$= 401.5 \angle 0.1°$$

Case II : Impedances in parallel

The same voltage *i.e.*, 401.5 $\angle 0.1°$ is applied to the parallel combination of the both the impedances.

$$\text{Total impedance } Z = \frac{Z_1 Z_2}{Z_1 + Z_2}$$

$$= \frac{(14.14 - j14.14)(26 + j15)}{40.14 + j0.86}$$

$$= \frac{579.74 - j\,155.54}{40.14 + j0.86}$$

$$= \frac{600 \angle -\tan^{-1} 0.268}{40.15 \angle \tan^{-1} 0.02}$$

$$= \frac{600 \angle -15°}{40.15 \angle 0.2°} = 14.95 \angle -15.2°$$

$$\text{Here, Current } I = \frac{V}{Z} = \frac{401.5 \angle 0.1°}{14.95 \angle -15.2°} = \mathbf{26.86 \angle 15.3°}$$

Problem 3.36

Two parallel circuits comprising of (i) a coil of resistance of 20 Ω and inductance of 0.07 H and (ii) a resistance of 50 Ω in series with a condenser of capacitance 60 μF are connected across 230 V, 50 Hz. Calculate the main current and power factor of the arrangement. (June 81, B.U.)

Solution :

With reference to Fig. 3.73.

Circuit A

Inductive Reactance $X_L = 2\pi f L$

$$= 2\pi \times 50 \times 0.07$$

$$= 22\ \Omega$$

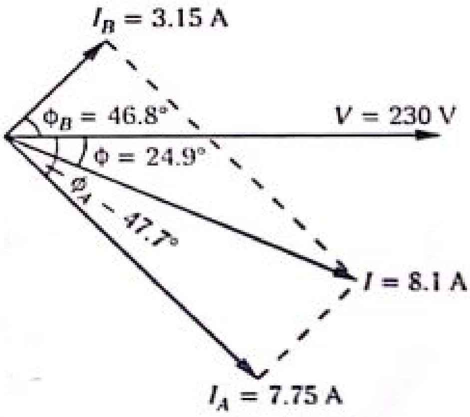


Fig. 3.73

Impedance of circuit A,

$$Z_A = \sqrt{R_A^2 + X_L^2}$$

$$= \sqrt{20^2 + 22^2} = 29.7\ \Omega$$

Current drawn by circuit A,

$$I_A = \frac{V}{Z_A} = \frac{230}{29.7} = 7.75\ \text{A}$$

Phase angle of circuit A,

$$\phi_A = \cos^{-1}\frac{R_A}{Z_A}$$

$$= \cos^{-1}\frac{20}{29.7}$$

$$= \cos^{-1} 0.673$$

$$= 47.7° \text{ (lag)} \quad \text{(for inductive circuit)}$$

Circuit B

Capacitive reactance $X_C = \frac{1}{2\pi f C}$

$$= \frac{1}{2\pi \times 50 \times 60 \times 10^{-6}}$$

$$= 53\ \Omega$$

Impedance of circuit B,

$$Z_B = \sqrt{{R_B}^2 + {X_C}^2}$$

$$= \sqrt{50^2 + 53^2} = 72.9\ \Omega$$

Current drawn by circuit B,

$$I_B = \frac{V}{Z_B} = \frac{230}{72.9}$$

$$= 3.15 \text{ A}$$

Phase angle of circuit B,

$$\phi_B = \cos^{-1}\frac{R_B}{Z_B}$$

$$= \cos^{-1} 0.685$$

$$= 46.8° \text{ lead, as circuit is capacitive}$$

Active component of resultant current,

$$I\cos\phi = I_A \cos\phi_A + I_B \cos\phi_B$$

$$= (7.75 \times 0.673) + (3.15 \times 0.685)$$

$$= 5.21 + 2.15 = 7.36 \text{ A}$$

Reactive component of resultant current,

$$I\sin\phi = I_A \sin\phi_A + I_B \sin\phi_B$$

$$= 7.75 \times (-0.739) + 3.15 \times 0.729$$

$$= -5.72 + 2.30 = -3.42 \text{ A}$$

Squaring both sides and adding them

$$I^2 = (I \cos \phi)^2 + (I \sin \phi)^2$$

$$I = \sqrt{(I \cos\phi)^2 + (I \sin \phi)^2}$$

$$= \sqrt{(7.36)^2 + (-3.42)^2}$$

$$= \sqrt{54.17 + 11.70} = \mathbf{8.1\ A}$$

Power Factor of the arrangement

$$= \frac{\text{Active component of resultant current}}{\text{Resultant current}}$$

$$= \frac{I \cos \phi}{I} = \frac{7.36}{8.1} = \mathbf{0.908\ (lag)}$$

Problem 3.37

Two impedances $Z_1 = (6 - j8)$ ohms and $Z_2 = (16 + j\,12)$ ohms are connected in parallel. If the total current of the combination is $(20 + j\,10)$ amperes, find (i) the voltage across the combination, (ii) the currents in the two branches. Draw the complete phasor diagram.

Solution :

$$Y = Y_1 + Y_2$$

$$= \frac{1}{6 - j8} + \frac{1}{16 + j12}$$

$$= \frac{(6 + j8)}{(6 - j8)(6 + j8)} + \frac{16 - j12}{(16 + j12)(16 - j12)}$$

$$= \frac{6 + j8}{100} + \frac{16 - j12}{400}$$

$$= (0.06 + j0.08) + (0.04 - j\ 0.03)$$

$$= 0.1 + j\ 0.05 = 0.1118 \angle 26°34'$$

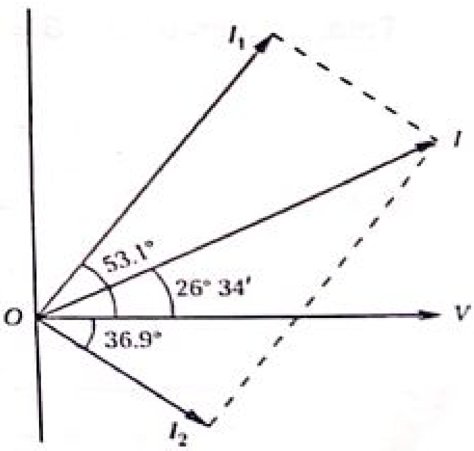


Fig. 3.74

$$I = 20 + j10 = 22.36 \angle 26°\ 34'$$

Now, $\quad I = VY$

$$\therefore \quad V = \frac{I}{Y} = \frac{22.36 \angle 26°34'}{0.1118 \angle 26°34'} = 200 \angle 0°$$

$$I_1 = VY_1 = (200 + j0)\,(0.06 + j0.08)$$

$$= 12 + j16 = 20 \angle \tan^{-1} 1.33 = \mathbf{20 \angle 53.1°}$$

$$I_2 = (200 + j0)\,(0.04 - j0.03)$$

$$= 8 - j6 = 10 \angle -\tan^{-1} 0.75 = \mathbf{10 \angle -36.9°}$$

Phasor diagram is given is Fig. 3.74.

Problem 3.38

A resistance of 10 ohms, an inductive reactance of 8 ohms and a capaci reactance of 15 ohms are connected in parallel across 120 volts, 50 c mains. Determine (i) total current, (ii) power factor of the circuit, (iii) pov (Aug/Sep 89, M

Solution :

We use the Admittance Method to solve this problem.

Reference is made to Fig. 3.75(a). We determine the total susceptance algebraically adding the susceptances in two of the branches. Inductive suscepta is negative and capacitive susceptance is positive. Thus,

$$\text{Inductive susceptance,} \quad -b_1 = -\frac{1}{8} = -0.125 \text{ Siemen}$$

$$\text{Capacitive susceptance,} \quad +b_2 = +\frac{1}{15} = +0.066 \text{ S}$$

$$\text{Total susceptance,} \quad B = (-b_1) + b_2$$

$$= -0.125 + 0.066$$

$$= -0.059 \text{ S}$$

$$\text{Conductance} \quad G = \frac{1}{10} = 0.1 \text{ Siemen}$$

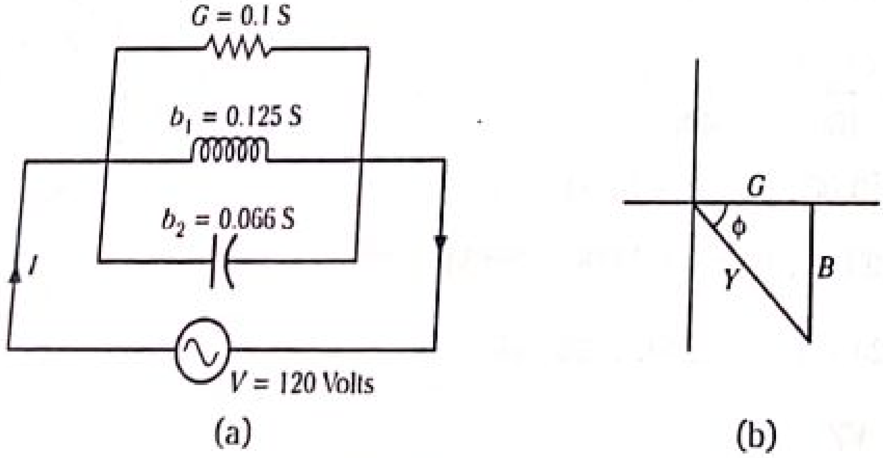


Fig. 3.75

Total Admittance of the circuit

$$Y = \sqrt{G^2 + B^2}$$

$$= \sqrt{(0.1)^2 + (-0.059)^2} = 0.116 \text{ Siemen}$$

The Admittance Triangle is given in Fig. 3.75(b).

Total current, $I = VY = 120 \times 0.116$

$= \mathbf{13.9\ A}$

Power Factor, $\cos\phi = \frac{G}{Y} = \frac{0.1}{0.116} = \mathbf{0.86}$ **(lagging)**

[**Note :** *As B has a negative sign, the p.f. is lagging*]

Power $= V I \cos\phi$

$= 120 \times 13.9 \times 0.86$

$= 1434$ watts or **1.434 KW**

roblem 3.39

Two impedances $Z_1 = 150 - j\,157\ \Omega$ and $Z_2 = 100 + j\,110\ \Omega$ are connected parallel across 200 V, 50 Hz supply. Find,

i) Branch Currents **(ii) Total current**

iii) Total power **(iv) Draw Vector Diagram.** **(Mar 94, B.U.)**

lution :

Impedance $Z_1 = 150 - j157 = \sqrt{150^2 + 157^2}\ \angle -\tan^{-1} 1.046$

$= 217\ \angle -46.3°$

Impedance, $Z_2 = 100 + j110$

$= \sqrt{100^2 + 110^2}\ \angle \tan^{-1} 1.1$

$= 148.66\ \angle 47.8°$

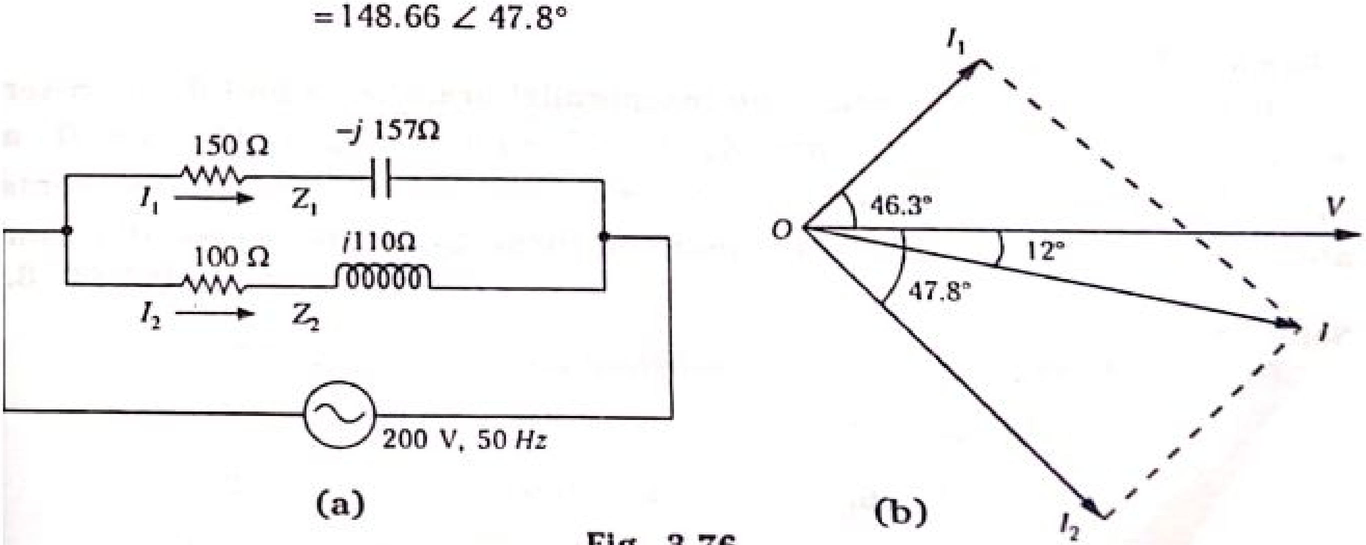


Fig. 3.76

$$V = 200 \angle 0°$$

$$\therefore \quad I_1 = \frac{V}{Z_1} = \frac{200 \angle 0°}{217 \angle -46.3°} = \mathbf{0.92 \angle 46.3°}$$

$$I_2 = \frac{V}{Z_2} = \frac{200 \angle 0°}{148.66 \angle -47.8°} = \mathbf{1.34 \angle -47.8°}$$

The total Impedance is given by :

$$Z = \frac{Z_1 Z_2}{Z_1 + Z_2} = \frac{(150 - j157)(100 + j110)}{250 - j47}$$

Simplifying the numerator and then multiplying both numerator and denominator by (250 + 47), we get

$$Z = 124 + j26.5 = \sqrt{124^2 + 26.5^2} \angle \tan^{-1} 0.212$$

$$= 126 \angle 12°$$

$$\text{Total Current } I = \frac{V}{Z} = \frac{200 \angle 0°}{126 \angle 12°} = \mathbf{1.58 \angle -12°}$$

$$\text{Total Power } P = VI \cos\phi = 200 \times 1.58 \times \cos 12°$$

$$= \mathbf{309 \text{ watts}}$$

Vector Diagram (drawn to Scale).

3.19 Series, Parallel Circuits

In such circuits, the parallel circuit is first reduced to an equivalent series circuit and then, as usual, combined with the rest of the circuit.

Problem 3.40

In a series parallel circuit, the two parallel branches A and B are in series with C. The impedances are $Z_A = (10 - j8)\,\Omega$, $Z_B = (9 - j6)\,\Omega$ and $Z_C = (3 + j2)\,\Omega$, and the voltage across C is $(100 + j0)$. Find the currents I_A and I_B and the phase difference between them. Draw the phasor diagram.

(83-84, B.U.)

Solution :

The circuit diagram and phasor diagram are given in Fig. 3.77.

$$Z_A = (10 - j8) = 12.8 \angle -\tan^{-1} 0.8 = 12.8 \angle -38° \, 40'$$

$$Z_B = (9 - j6) = 10.8 \angle -\tan^{-1} 0.666 = 10.8 \angle -33° \, 42'$$

$$Z_C = (3 + j2) = 3.6 \angle \tan^{-1} 0.666 = 3.6 \angle 33° \, 42'$$

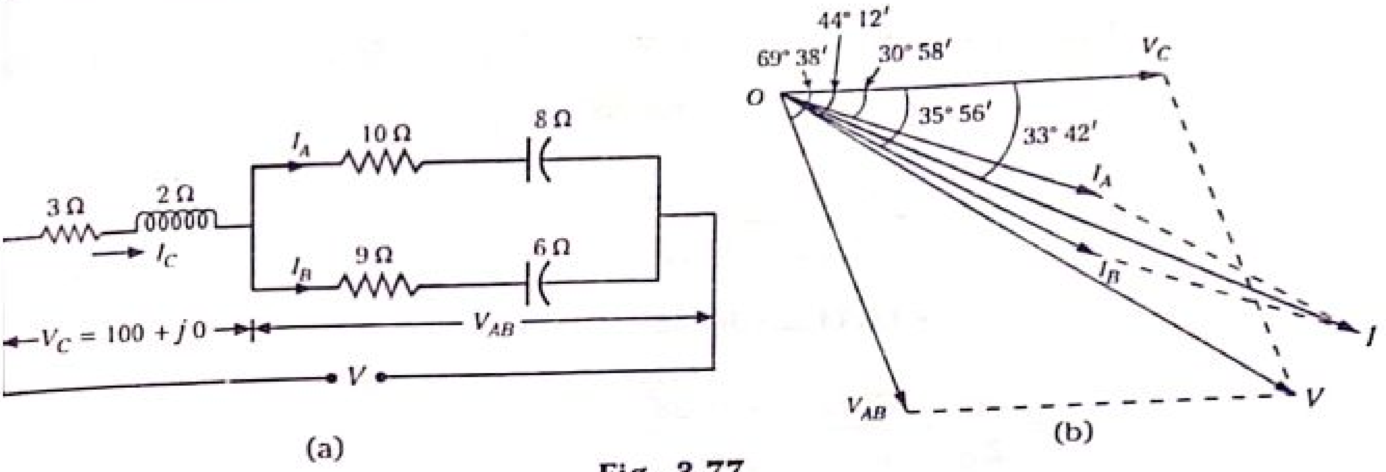


Fig. 3.77

$$V_C = I_C Z_C$$

or $$100\angle 0 = I_C \times 3.6 \angle 33°42'$$

$\therefore$ $$I_C = \frac{100\angle 0}{3.6\angle 33°\,42'} = 27.77\angle -33°\,42'$$

$$Z_{AB} = \frac{(10-j8)(9-j6)}{10-j8+9-j6}$$

$$= \frac{(10-j8)(9-j6)}{19-j14}$$

$$= \frac{(42-j132)(19+j14)}{(19-j14)(19+j14)}$$

$$= 4.75 - j3.44 = 5.86\angle -35°56'$$

$$Z = Z_C + Z_{AB}$$

$$= (3+j2) + (4.75 - j3.44)$$

$$= 7.75 - j1.44 = 7.88\angle -\tan^{-1} 0.185$$

$$= 7.88\angle -10°30'$$

$$I_C = \frac{V}{Z}$$

or $$V = I_C \times Z = 27.77\angle -33°42' \times 7.88\angle -10°30'$$

$$= 218.82\angle -44°12'$$

$$V_{AB} = I_C Z_{AB} = 27.77 \angle -33°42' \times 5.86 \angle -35°56'$$
$$= 162.73 \angle -69°38'$$

$$I_A = \frac{V_{AB}}{Z_A} = \frac{162.73 \angle -69°38'}{12.8 \angle -38°40'}$$
$$= \mathbf{12.71 \angle -30°58'}$$

$$I_B = \frac{V_{AB}}{Z_B} = \frac{162.73 \angle -69°38'}{10.8 \angle -33°42'}$$
$$= \mathbf{15 \angle -35°56'}$$

The phase difference between I_A and I_B is
$$= -30°58' - (-35°56')$$
$$= \mathbf{4° 58'}$$

Problem 3.41

In the arrangement shown in the figure below calculate the impedance AB and the phase angle between voltage and current. Also calculate the tc power consumed, if the applied voltage between A and B is 200 $\angle$(30°) vo

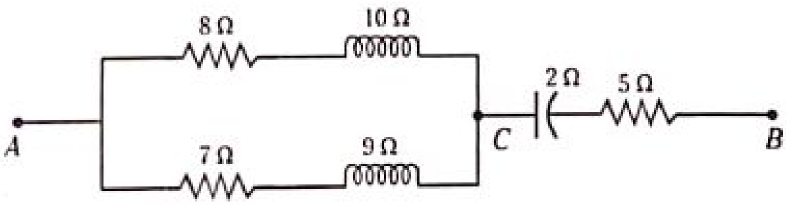


Fig. 3.78

Solution :

Impedance in the parallel arms :
$$Z_1 = 8 + j10 \quad \text{and} \quad Z_2 = 7 + j9$$

Impedance in the series arm $Z_3 = 5 - j2$

$$Z_{AC} = \frac{(8+j10)(7+j9)}{8+j10+7+j9}$$
$$= \frac{-34+j142}{15+j19} \cdot \frac{(15-j19)}{(15-j19)}$$
$$= \frac{2188+j2776}{586} = 3.73 + j4.73$$

Total Impedance $Z_{AB} = 3.73 + j4.73 + Z_3$

$$= 3.73 + j4.73 + 5 - j2$$

$$= 8.73 + j2.73$$

$$= \sqrt{8.73^2 + 2.73^2} \tan^{-1}\left(\frac{2.73}{8.73}\right)$$

$$= 9.15 \angle 17.3°$$

$$= \mathbf{9.15\ \Omega,\ 17.3°\ lag}$$

$$V = 200 \angle 30°$$

$$I = \frac{V}{Z} = \frac{200 \angle 30°}{9.15 \angle 17.3°} = 21.9 \angle 12.7°$$

The resistive component contributing to power is 8.73 Ω

$\therefore$ Power $= I^2 R = (21.9)^2 \times 8.73$

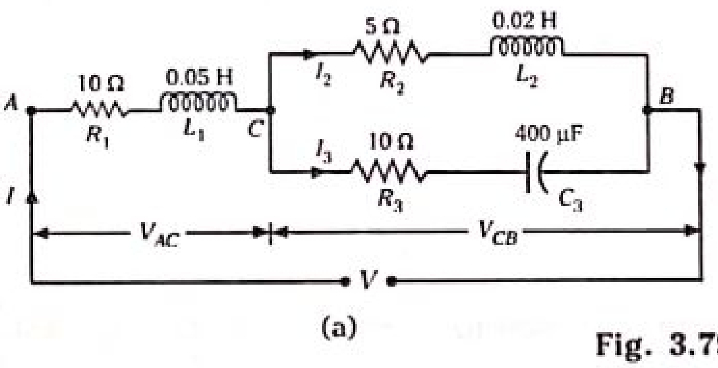
$= 4173$ watts

$= \mathbf{4.17\ kW}$

Problem 3.42

In the circuit shown below, determine the voltage at a frequency of 50 Hz to be applied across *AB* in order that the current in the circuit is 10 A. Draw the phasor diagram. (Nov/Dec 84, B.U., MQP-1, B.U.)

Solution :



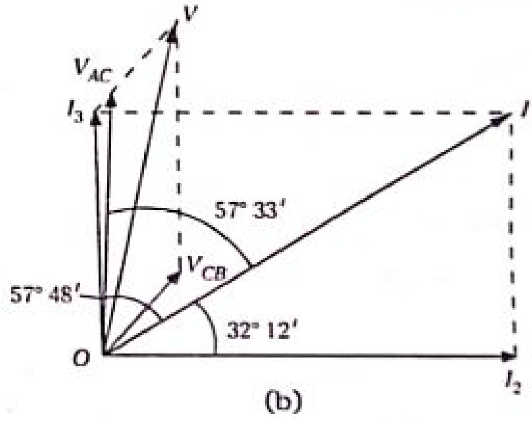


Fig. 3.79

Referring to Fig. 3.79 (a),

$$X_{L_1} = 2\pi \times f \times L_1 = 2\pi \times 50 \times 0.05 = 15.71\ \Omega$$

$$X_{L_2} = 2\pi \times 50 \times 0.02 = 6.284\ \Omega$$

$$X_{C_3} = \frac{1}{2\pi f C} = \frac{1}{2 \times \pi \times 50 \times 400 \times 10^{-6}} = 7.95\ \Omega$$

$$Z_1 = R_1 + jX_{L_1} = 10 + j15.71 = 18.6 \angle 57° 33'$$

$$Z_2 = R_2 + jX_{L_2} = 5 + j6.284 = 8 \angle 51°30'$$

$$Z_3 = R_3 - jX_{C_3} = 10 - j7.95 = 12.77 \angle -38°30'$$

$$Z_{CB} = \frac{(5+j6.284)(10-j7.95)}{5+j6.284+10-j7.95}$$

$$= \frac{(5+j6.284)(10-j7.95)}{15-j1.66}$$

$$= 6.42 + j2.25 = 6.8 \angle 19°18'$$

$$Z = Z_1 + Z_{CB}$$

$$= (10+j15.71) + (6.42 + j2.25)$$

$$= 16.42 + j17.96$$

$$= 24.36 \angle 47°36'$$

Now, $\frac{V}{I} = Z$

or $\frac{V}{10 \angle 0} = 24.36 \angle 47°36'$

$\therefore$ $\mathbf{V = 243.6 \angle 47°36'}$

$$V_{CB} = I \times Z_{CB} = 10 \angle 0 \times 6.8 \angle 19°18' = 68 \angle 19°18'$$

$$I_2 = \frac{V_{CB}}{Z_2} = \frac{68 \angle 19°18'}{8 \angle 51°30'} = 8.5 \angle -32°12'$$

$$I_3 = \frac{V_{CB}}{Z_3} = \frac{68 \angle 19°18'}{12.77 \angle -38°30'} = 5.32 \angle 57°48'$$

$$V_{AC} = IZ_1 = 10 \angle 0 \times 18.6 \angle 57°33' = \mathbf{186 \angle 57°33'}$$

Phasor Diagram is given in Fig. 3.79 (b).
(Please note that all angles are with reference to I, which is at 'O' angle).

Problem 3.43

The impedances of two parallel circuits can be represented by (20 + j 15) a n (-j 60) respectively. If the supply frequency is 50 Hz, find the resistance, inductance or capacitance of each circuit. (Mar 95, B.U.)

Solution :

1st Circuit

The circuit represented by (20 + j 15) is inductive because of the sign in the 'j' term. Resistance (seen directly in this term) = **20 Ω**.

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Also inductive reactance $X_L = 15 = 2\pi f L$

$$\therefore \quad L = \frac{15}{2\pi f} = \frac{15}{2\pi \times 50} = \mathbf{0.0478\ H}$$

2nd Circuit

This circuit, $(10 - j\,60)$, is capacitive as there is a minus sign before 'j' term. Resistance is **10 Ω** (as directly seen).

Capacitive Reactance $X_C = \dfrac{1}{2\pi f C} = 60$

$$\therefore \quad C = \frac{1}{2\pi f \times 60} = 0.000053 \text{ farad} = \mathbf{53\ \mu F}$$

3.20 Review Questions

Q1. (a) Explain what you understand by the following terms in respect of a sinusoidal waveform :

(i) Average Value (ii) Effective or R.M.S. Value

(iii) Form Factor (iv) Peak Factor

(Mar/Apr 88, M.U)

Q2. Derive an expression for Effective Value of a sinusoidal varying alternating quantity.
(June 81, Sep/Oct 87, Jan 93, Feb 96, B.U.)

Q3. Derive an expression for R.M.S. value of a sinusoidal wave.
(Feb/Mar 83, Aug/Sep 90, M.U., July 90, B.U.)

Q4. Define Average value and Effective value of an alternating current and find their relation with the maximum value if the alternating quantity is sinusoidal.
(Apr/May 87, M.U.)

Q5. Show that an alternating quantity can be represented by a rotating vector.
(Dec 86, B.U.)

Q6. What is meant by phase angle between two alternating quantities ? Show that the average power in an a.c. circuit is given by $P = VI \cos \phi$.
(Dec 84, Oct 85, July 88, B.U; Mar 89, M.U; Jan 93 B.U)

Q7. Explain the terms 'Phase' and 'Phase Difference' as applied to alternating quantities. Show the phasor diagram and waveform of voltage and current of a series R-L circuit energised by a sinusoidal voltage.

Q8. Show that current lags behind the voltage in a series R-L circuit. (Oct 85, B.U.)

Q9. Show that the current in a R-C circuit leads voltage by an angle ϕ. (Mar 95 B.U.)

Q10. Distinguish between lagging p.f. and leading p.f. in a.c. circuits. (Feb/Mar 90, M.U.)

Q11. What is meant by 'Power Factor' in a.c. circuits ? What is its significance in a.c. circuits ? (Apr/May 87, M.U; Mar 89, M.U; Aug/Sep 89 M.U.)

Q12. What is the necessity of power factor improvement in a power system ? Mention the methods available to improve power factor. (Aug/Sep 90, M.U.)

Q13. Establish the relationship between voltage and current in a *R-L-C* series circuit. Draw the phasor diagram. (Dec 81, B.U.)

Q14. Obtain an expression for the average power consumed by an a.c. circuit in terms of effective values of voltage, current and power factor. (Aug 82, B.U.)

Q15. Show that the voltage across a pure inductor and the current through it are displaced by 90°. Therefrom prove that the average power consumed by a pure inductor is zero. (Apr 95, Dec 86, B.U)

Q16. Show that the average power consumed in a pure capacitance is zero. (May/June 86, June/July 90, B.U; July 93, B.U.; Aug 94, Feb 96, B.U.)

Q17. Derive an expression for current, power and power factor in an *R-L* circuit.

Q18. Derive an expression for the power in a.c. circuit with inductance, resistance and capacitance in series. Draw the vector diagram of current and voltage and explain it. (Apr/May 87, M.U; Aug/Sep 89, M.U.)

Q19. Starting from fundamentals show that average power in a *R-L* circuit excited by sinusoidal current $I_m \sin \omega t$ is $VI \cos \phi$, where V is the voltage, I is current, both in RMS values, and ϕ is the phase angle between voltage and current. (Apr 97, B.U.)

3.21 Exercises - Problems

1. An e.m.f. of 400 sin (628 t) is applied to a series circuit, and the resulting current is 2.5 sin (628 t – 1.37°). Find (i) Frequency (ii) Phase angle between voltage and current (iii) Parameters of the circuit. (Nov 97, KUD)

   Answer : 100 Hz ; 1.37° ; R = 159.95 Ohms ; L = 6.37 mH

2. A voltage 100 sin (314 t) is applied to a circuit consisting of a 25 Ohm resistor and an 80 microfarads capacitor in series. Determine (i) an expression for the current, (ii) power consumed, (iii) voltage across the capacitor when the current is one-half of its maximum value. (1987, KUD)

   Answer : i = 2.12 sin (314 t + 57.87°) ; 56.25 W ; 42.2 V

3. An inductive coil takes 10 A and dissipates 1000 Watts, when connected to a supply of 250 V, 50 Hz. Calculate the inductance of the choke coil. (1983, KUD)

   Answer : 0.073 Henry

4. An *R-L* circuit has $R = 2\ \Omega$, L = 0.05 H. If a P.D. of 100 Volts at 50 Hz frequency is applied, calculate current, power and power factor. (1983, KUD)

   Answer : 6.314 A ; 79.746 W ; 0.1263

5. A coil having an inductance of 200 mH and a resistance of 4 Ω is connected in series with a capacitor across a 50 Hz supply. Calculate the capacitance required to give the circuit a power factor of 0.5 (i) lagging, (ii) leading. (May 91, KUD)

   Answer : (i) 57.943 mF, (ii) 45.63 mF

6. The power factor of a series R-L circuit is 0.5 and the power absorbed is 500 W when the input voltage is $e = 50 \sin (314\,t + 10°)$. Find (i) the sinusoidal expression for the current i, (ii) value of R and L.

(Dec 92, KUD)

Answer : $i = 40 \sin (314\,t - 50°)$; $R = 0.625\ \Omega$; $L = 3.447$ mH

7. A voltage of $(80 + j60)$ V is applied to a circuit and the current drawn is $(3 - j4)$ A. Determine (i) Resistance, (ii) Inductive Reactance, (iii) Power and (iv) Power Factor.

(Apr 89, Gulbaraga)

Answer : (i) $R = 0$, (ii) $X_L = 20\ \Omega$, (iii) Power = 0, (iv) P.F = 0

8. A resistor of 10 Ω, an inductor of 0.1 H and a capacitor of 50 μF are in series across a supply of 230 V, 50 Hz. Find (i) the impedance of the circuit, (ii) the current (iii) Power and P.F. and (iv) Voltage across the capacitor.

(May 90, Gulbraga)

Answer : (i) 33.76 Ω, (ii) 6.813 A leading,
(iii) 464.14 Watts, 0.2962 leading, (iv) 433.73 V

9. A coil of resistance 8 Ω and inductance 15 mH is connected in series with a capacitor of capacitance 150 μF, across a supply of 200 V, 50 Hz. Calculate (i) the impedance of the circuit, (ii) the current and (iii) the power consumed.

(Dec 89, Gulbarga)

Answer : (i) 18.346 Ω, (ii) 10.9 A, (iii) 950.7 Watts

10. The instantaneous values of current and voltage in a coil are given by $i = 28.28 \sin (314\,t - \pi/3)$ A and $v = 282.8 \sin 314\,t$ volts. Obtain i) the resistance, inductance and impedance of the coil, ii) the power consumed in the coil.

(Mar 91, M.U.)

Answer : (i) $R = 5\ \Omega$; L = 27.57 mH, Z = 100 Ω, (ii) 2000 Watts

11. A series circuit has a resistance of 10 Ohms, inductance of 40 mH and capacitance of 100 μF. It is supplied from 220 V, 50 Hz supply. Find the current and the power factor.

(1988, KUD)

Answer : 10.13 A ; 0.4607

12. The alternating current flowing through an inductive coil consists of an active component of 7.2 A and a reactive component of 5.4 A. The supply voltage is 200 volts. Find (i) the value of the supply current, (ii) the power factor, (iii) the power dissipated.

(1984, KUD)

Answer : 9 A ; 0.8 ; 1440 W

13. A capacitor of negligible resistance, connected to a 220 V variable frequency sinusoidal supply takes a current of 10 A, when the frequency is 50 Hz. A non-inductive resistor when connected to the same supply takes a current of 12 A. If the two are connected in series and placed across the supply, calculate the current taken and its phase angle, when the supply frequency is 50 Hz.

(1984, KUD)

Answer : 7.683 ; 50.2° lead

14. When a resistor and an inductor in series are connected to a 240 V supply, a current of 3 A flows lagging 37° behind the supply voltage, while the voltage across the inductor is 171 V. Find the resistance of the resistor and the resistance and reactance of the inductor. (Nov 89, KUD)

Answer : 33.26 Ohms ; 30.74 Ohms ; 48 Ohms

15. A resistance of 100 Ω and a capacitor of 50 μF capacitance are connected in series across a 200 V, 50 Hz supply. Find (i) Impedance, (ii) Current, (iii) Power Factor, (iv) Power. (Nov 87, Gulbarga)

Answer : (i) 118.6 Ω, (ii) 1.96 V, (iii) 0.845, (iv) 285.61 Watts

16. Two impedances $(4 + j10)\ \Omega$ and $(6 + j4)\ \Omega$ are connected in parallel across an a.c. supply, and dissipate 600 W. Calculate the power taken when the impedances are connected in series across the same supply.

(Aug 82, B.U.)

Answer : 135.424 W

17. Two circuits *A* and *B* are connected in parallel across a 220 V, 50 Hz supply. Circuit *A* consists of 100 Ω resistance in series with an inductance of 0.1 H. Circuit *B* consists of 100 μF capacitor in series with a resistor of 200 Ω. Find the current in each branch and the total current.

(Apr 85, B.U.)

Answer : $I_A = 2.099 \angle -17.438°$ **A,** $I_B = 1.0863 \angle 9.0428°$ **A,**
and $I = 3.109 \angle -8.4752°$ **A**

MODULE

3

(b) Domestic Wiring

3.22 Domestic Wiring

Wiring done in domestic premises (houses), for providing electrical power for lighting, fans and domestic appliances is called domestic wiring.

The primary objective of a wiring system is to distribute electrical energy to the various points at which it is required, duly considering the following :

a) **Electrical Safety :** This is the most important aspect – there must be no danger of leakage or of electric shock to persons using the supply.

b) **Mechanical Immunity :** A wiring system which is suitable for one type of building may not be suitable for another. The wiring selected for a particular type of building should be able to withstand weather changes for a long period and should be protected from physical damage during its usage.

c) **Permanence :** There should not be any undue deterioration in wiring due to the action of dampness, fumes, weather *etc*.

d) **Appearance :** In certain cases appearance or invisibility (concealed wiring) is important. However, in the case of factory wiring, appearance apart from neatness is usually not important.

e) **Cost :** The cost of wiring installation is an important consideration. The system chosen should depend upon the type of building and the purpose for which it is used, keeping economy in view.

3.23 Three - phase, 4 - wire system

The electric supply authority supplies power to the consumers through a low-voltage three-phase four wire distribution system. A 3-phase, 4- wire, Star connection is shown in Fig. 3.80.

If V is the voltage of each winding, then line voltage is $\sqrt{3}$ V. Usually, the phase voltage, i.e., voltage between any outer and the neutral for a symmetrical system is 230 V, so that the voltage between any two lines or outers is $\sqrt{3} \times 230 = 400$ V.

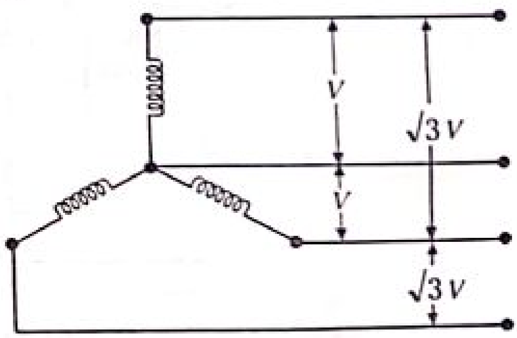


Fig. 3.80 Three-Phase, 4-Wire System

Single phase residential lighting loads are connected between the neutral and any of the line wires. These loads are connected symmetrically so that line wires are loaded equally. Hence the resultant current in the neutral wire is zero or at least minimum.

3.24 Service Connections/Service Mains

The supplier's distribution system brings power to the consumer through overhead lines or by means of underground cables to a spot outside the consumer's premises.

The line bringing electric power from supplier's low voltage distributor upto the energy meter installed at the consumer's premises is called *service connection* or *service mains*.

Service connection may be achieved by means of underground cables or by means of overhead conductors or cables. We shall take up overhead service connections with PVC or weather proof cables, described as follows :

Bare conductors are run from the supplier's pole to shackle insulators fitted to brackets fixed on a cross arm embedded into the wall of a two-storied building at an appropriate height (see Fig. 3.81)

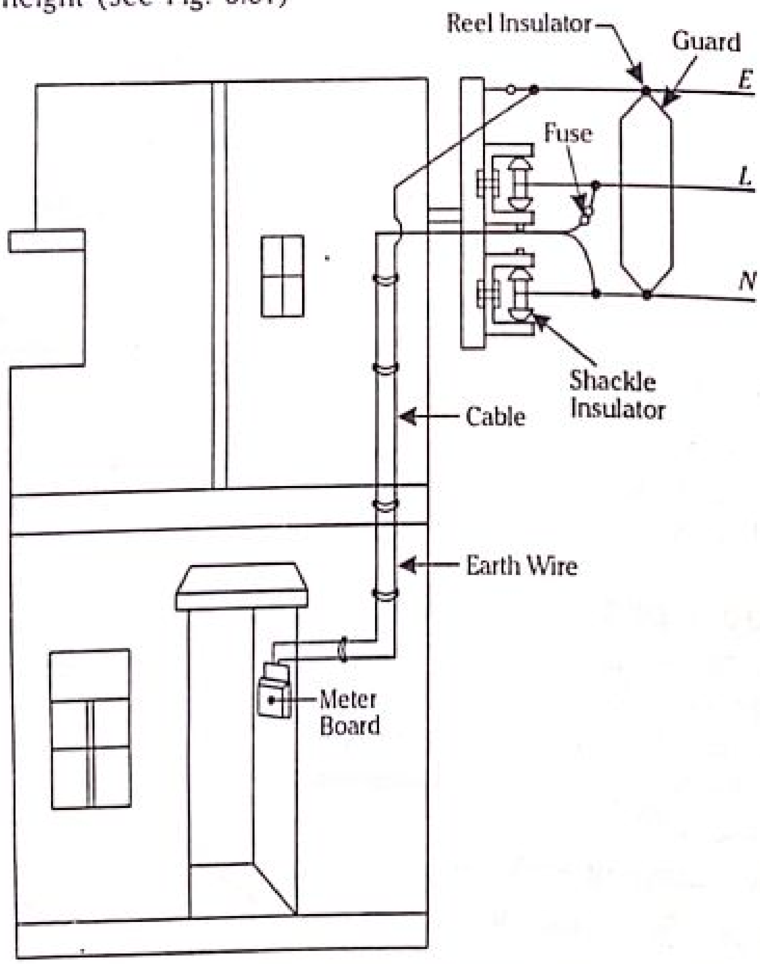


Fig. 3.81 : Service connections with PVC or weather proof cables

Thereafter service connections are taken from the bare conductors by means of PVC or weather-proof cables run on wooden battens or through a GI pipe.

3.25 Meter Board and Distribution Board

Now that the supplier's service line has been brought to the consumer's premises, it is now to be connected with the consumer's internal wiring. The supply authority has to charge the consumer for the electrical energy consumed. For this purpose the supplier's service lines will be connected to the input terminal of the energy meter, which has to be provided by the supply authority. After the energy meter the service line is connected to a cut-out (Fig. 3.82)

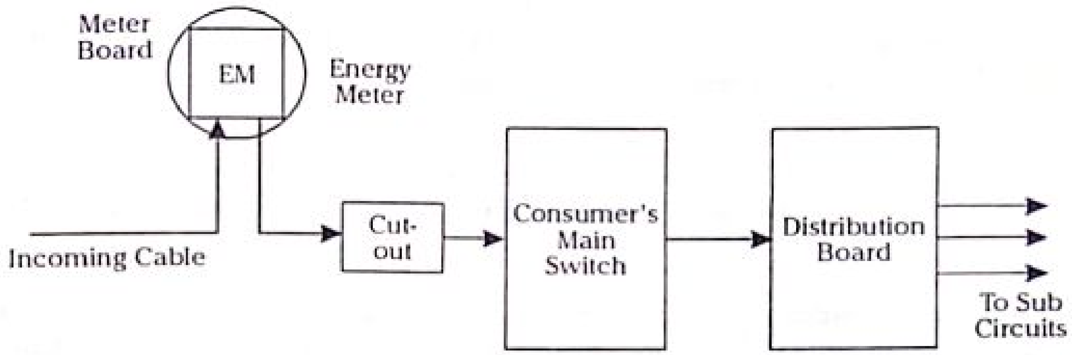


Fig. 3.82 : Block diagram of the meter board and the distribution board

The cut-out contains a fuse wire so that if the consumer draws more current than the rated current of the meter, the fuse will blow off, thus preventing damage to the meter. The cut-out also serves the purpose of enabling the supply authority to discontinue the supply should the consumer fail to pay his bill. The cut-out and the meter are the supply authority's property and are sealed. The consumer's distribution starts after the energy meter and the supply authority's cut-out. The leads from the output terminals of the energy meter via the cut-out are connected to the consumer's main switch. The energy meter is placed before the main switch because the main switch must be accessible to the consumer in case he wishes to switch off supply when required. Therefore the main switch should not be sealed.

The energy meter should be installed at such a place where it is readily accessible to both the consumer and the supply authority. It should be installed at such a height as to enable the meter to be read conveniently. The energy meter should either be provided with a protective covering enclosing it completely except for a glass window through which the readings are noted, or else it should be mounted inside a completely enclosed panel provided with hinged or sliding doors with a suitable locking arrangement.

Fuses should be provided to interrupt any short circuit current that may occur.

3.26 Conduit (Concealed) Wiring

The conduit could be made up seamless or welded tubes of steel or galvanized iron, in which case it is important to ensure that there is continuous bonding and efficient earthing. The VIR or PVC wires are carried inside these tubes. These tubes are routed along walls and ceiling and are rigidly secured by fixing them on wooden

guttas and fastening them with bolts and screws as shown.

There are two types of conduits - the open conduit and the concealed conduit. In the open conduit system, the tubes are routed along the surfaces of the walls and the ceiling, whereas in the concealed conduit system, the pipes are embedded within the plastering of the walls and ceiling and thus are hidden from view. First, the pipes are fixed at the required places and then the VIR/PVC wires are drawn through them.

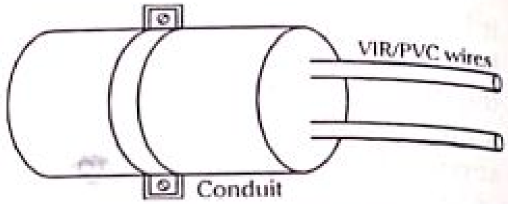


Fig. 3.83 Conduit Wiring

In case PVC pipes are employed, they can be easily bent, as they are quite flexible. Such a system, using PVC pipes, is often called as "Flexible Conduit System".

Advantages

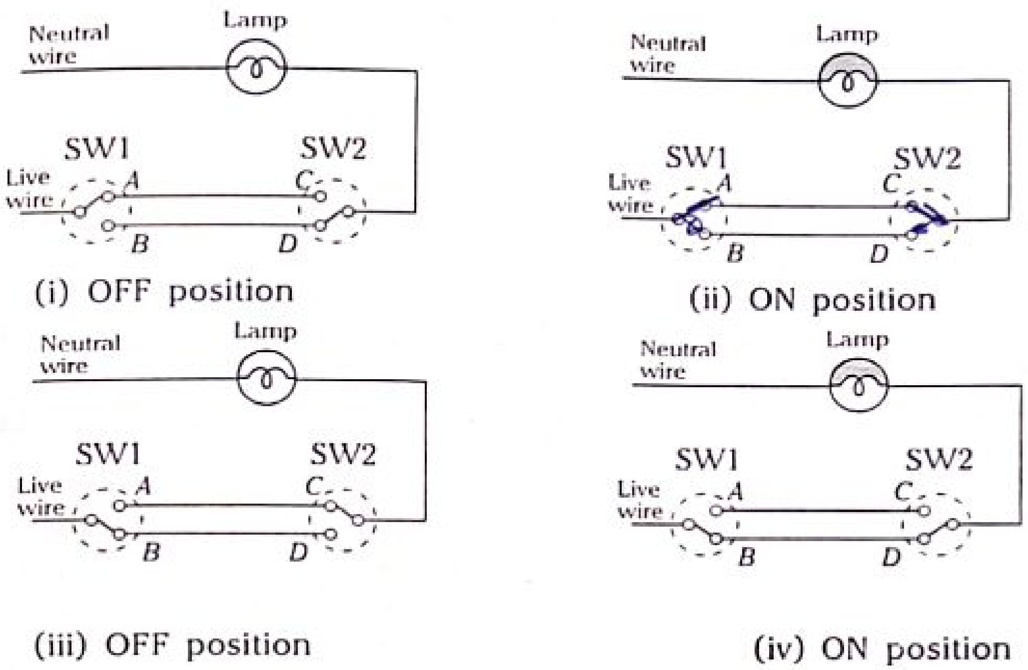
- The greatest advantage of the conduit system, when properly installed, is that fire risk is completely eliminated, as compared to any other system.
- It is permissible to "bunch" the lead and return wires of a circuit in a single tube, and also "looping back" wires carrying on the current to the next circuit.
- It provides complete protection to V.I.R. or P.V.C. wires against mechanical injury.
- This system is preferable in damp situations, and in places where gas is likely to be present.
- Readily inspectable for alteration and repairs.
- Earthing and electrical continuity easily assured.

Disadvantages

- In damp situations, water condenses and gathers in the tubes, lowering the insulation resistance and perhaps causing short circuits between the cables.
- In the case of A.C. supply, bunching of wires must be done, otherwise in long lengths there may be a heavy drop of pressure due to induction.
- Skilled workers are required to do the wiring.
- Conduit should not come in contact with gas pipes or water pipes and should be watertight in damp situations.

3.27 2-Way Control of Lamp

Staircase lighting is usually done by two-way control of lamp. Fig. 3.84 gives the wiring diagram to control one lamp from two places. Two 2-way switches are used in different positions. The operation of these switches is given as follows :



(i) OFF position

(ii) ON position

(iii) OFF position

(iv) ON position

Fig. 3.84 Two-Way Control of Lamp

Let the 2-way switch *SW*1 be at the bottom of the staircase and the 2-way switch *SW*2 be at the top of the staircase. Fig. 3.84(i) gives the OFF POSITION, where the lamp will be OFF, as the circuit from the live wire to the neutral is broken.

Now, a person, intending to go up the staircase, will operate *SW*1, so that the circuit is complete and the lamp will be ON, as given in Fig. 3.84 (ii).

After he reaches the top of the staircase, he will operate *SW*2, so that the lamp is switched OFF, as given in Fig. 3.84 (iii).

Later on, if he desires to come down the staircase, he will operate *SW*2, and the lamp will be switched ON, as given in Fig. 3.84(iv). The switching table is given alongside.

Switching Table

Sl. No.	Position of Switch *SW*1	Position of Switch *SW*2	Lamp ON or OFF
1.	*A*	*D*	OFF
2.	*B*	*D*	ON
3.	*B*	*C*	OFF
4.	A	C	ON

Similarly, in big halls, corridors or bedrooms, it may be necessary to control the lamp from two points. In such cases too, a two-way control of lamp is done.

3.28 3-Way Control of Lamp

Sometimes in very big corridors, godowns or workshops, it may be necessary to control a lamp from these three points. In such cases, the circuit connection requires two, two-way switches & an intermediate switch as shown in Fig. 3.85.

Fig. 3.85 shows a circuit diagram to control a lamp from three different positions. An Intermediate Switch couples together the two two-way switches *SW1* and *SW2*. It has four terminals *CDEF*. For one position it connects points *CD* and *EF*, which is called a straight connection. For another position, it connects points *CF* and *ED*, which is called a cross-connection.

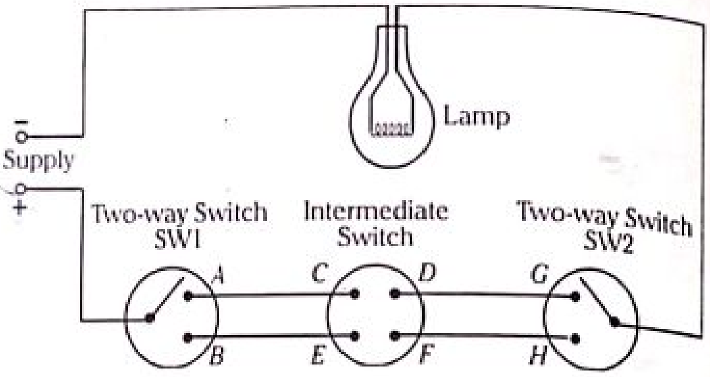


Fig. 3.85 Three-Way Control of Lamp

When the switches *SW1* and *SW2* are in positions *A* and *G* respectively and the Intermediate Switch is in the position of straight connection *i.e.*, when *CD* and *EF* are connected, the lamp circuit is completed and the lamp glows. Now, if the Intermediate Switch is changed over to the cross-connection mode, *i.e.*, when points *CF* and *ED* are connected, the lamp circuit becomes open and so the lamp is switched off. Now, if the position of *SW2* is changed from *G* to *H*, the lamp circuit is again closed and the lamp is switched ON. Thus, it is possible to control the lamp from three points. The table below shows the positions of the switches and the states of the lamp.

Switching Table

Sl. No.	Position of Switch *SW1*	Position of Intermediate *SW*	Position of Switch *SW2*	Lamp ON or OFF
1.	*A*	*CD, EF*	*G*	ON
2.	*A*	*CD, EF*	*H*	OFF
3.	*B*	*CD, EF*	*G*	OFF
4.	*B*	*CD, EF*	*H*	ON
5.	*A*	*CF, ED*	*G*	OFF
6.	*A*	*CF, ED*	*H*	ON
7.	*B*	*CF, ED*	*G*	ON
8.	*B*	*CF, ED*	*H*	OFF

3.29 Fuses

3.29.1 Definition

A fuse is a safety device, a weak link connected in series with the circuit, which melts whenever the current in the circuit exceeds the value of the fuse provided, either due to overload or short circuit, thus opening the circuit and protecting other materials in the circuit.

3.29.2 Need

In any power system, conductors are employed to convey power to the different installations, devices or accessories. During normal operation, these conductors carry current. The magnitude of the current depends on the load. The diameter of the wire is fixed, based upon the maximum current which the conductor is likely to carry continuously. As long as there is no overload or fault, there is no overheating of the conductor. However, if there is an excessive overload, or if a fault (or short circuit) occurs, the conductor will carry a very large current – much more than the normal current – which will cause overheating of the conductor and possible damage to appliances and devices. The insulation could get damaged or destroyed, causing considerable havoc. Hence the need is felt for a built-in mechanism by means of which the entire installation is saved from possible damage, in the event of a sudden short circuit or fault. *Fuses function as such a safety mechanism.*

3.29.3 Rating

Fuses are rated as follows :

(a) **Rated Carrying Current :** It is the maximum current which a fuse can carry without any undue heating and melting. It depends upon the permissible temperature rise of the contacts of the fuse holder, fuse and upon the deterioration of the fuse due to oxidation.

(b) **Fusing Current :** This is the minimum current at which a fuse element shall melt. The various factors on which the fusing current depends are as follows:

- (i) Material of the fuse element
- (ii) Its length
- (iii) Its diameter
- (iv) The cross-section of the fusing element
- (v) The type of enclosure

(c) **Fusing Factor :** This is defined as the ratio between the minimum fusing current and the rated carrying current. Its value is always more than one.

3.29.4 Requirements of a Fuse

It should be designed, keeping in view the following :

- (i) There should be no risk of flash-over to other conductors.
- (ii) There should be no risk of splashing of molten metal.
- (iii) The persons handling the fuses should not run the risk of electric shock.

It should be impossible for an arc to be maintained between terminals after the fuse has blown ; moreover, its base should be *incombustible* and *moisture-proof* and should be a good *insulator* too.

A fuse should not get *overheated* when the full-load current flows continuously through it.

The *current rating* of the fuse should not exceed the rating of the *smallest* cable protected, provided that a fuse having a rating less than *3 amperes* need not be used except in radio and accoustic circuits.

A fuse should not be fitted in an outlet socket. Branch fuses should be grouped in *accessible* places and *labelled* to indicate the circuits controlled.

3.29.5 Fusing Materials used

S.No	Fuse Material	Melting Point in °C	Resistivity in $\mu\Omega$-m
1	Tin	231.9	11.3×10^{-2}
2	Lead	327.4	21.0×10^{-2}
3	Zinc	419.5	6.1×10^{-2}
4	Aluminium	660.1	2.86×10^{-2}
5	Silver	960.8	1.64×10^{-2}
6	Copper	1084.8	1.72×10^{-2}

Tin and *lead* have a low melting point, *i.e.*, the *fusing factor* of a tin and lead fuse is reasonably low, as shown in the above Table.

The *standard* tin-lead alloy for fusible elements upto about 10 amps capacity is composed of 63% tin and 37% lead.

Copper wire has the disadvantage compared to tin or lead, that it operates at a rather high temperature when used with a reasonably low fusing factor.

Silver is best suited as fuse material for the following reasons :

(i) The coefficient of expansion for silver is quite small and hence the fuse does not blow when a current lesser than or equal to the rated value flows for a long length of time.

(ii) The conductivity of silver is not affected by continuous operation and by surges of current.

(iii) Unlike other metals that are usually used for fusible elements, silver wire cannot suffer from oxidation. Hence, the fuse blows quickly and isolates the circuit from damage and is especially suitable for circuits upto 5 amps.

3.29.6 Classification of Fuses

Fuses may be classified as follows :

(a) Low Voltage Fuses

(b) High Voltage Fuses

(a) **Low Voltage Fuses :** In order to limit the damage caused by fault currents of high value, the arc should be prevented from re-striking after passing through the zero value of the A.C. cycle. This is done by means of a semi-enclosed or enclosed fuse where the element is shrouded, so that the gases produced on melting of the fuse are cooled ; moreover, shrouding also prevents the scattering of molten metal.

(i) **Semi-enclosed rewirable fuses :** These fuses have one or more strands of fuse wire stretched between terminal blocks and are mounted on a moulded or porcelain handle. The use of these type of fuses is limited to power points where low values of fault currents are to be handled. Fuses of this type are made upto about 500 amperes rated current, but their low breaking capacity makes them unsuitable for power system applications.

(ii) **Enclosed Fuses :** Enclosed fuses have the element surrounded by an *asbestos* tube or by *incombustible* powder. The latter are called **cartridge** fuses ; the fusible elements are usually made of silver, which has a low ohmic loss and is immune to oxidation. These are surrounded by filling powder having good arc extinguishing properties, and are all encased in a ceramic body of high strength.

H.R.C Fuses : Modern cartridge fuses have a *high rupturing capacity*, are *non-deteriorating*, have a *consistent* fusing current, so that they can be graded to operate in correct sequence. The *fusing factor* for such type of fuses may lie between 1.4 and 2.

Fig. 3.86 shows an H.R.C Cartridge Fuse which consists of a ceramic body having metal end-caps to which fusible silver current carrying fuse elements are welded.

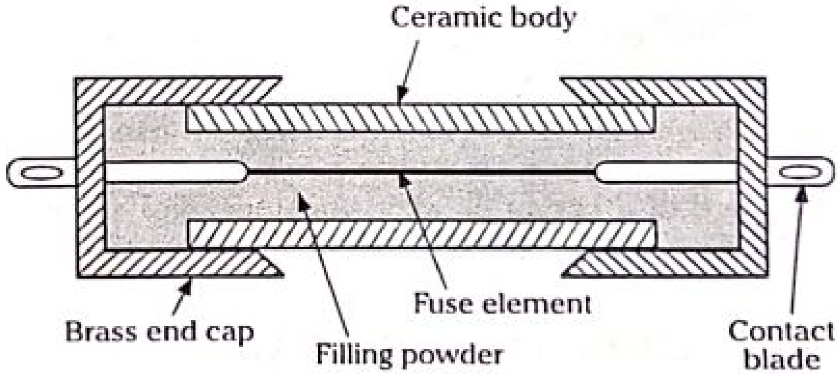


Fig. 3.86 H.R.C Cartridge Fuse

The space within the body surrounding the elements is completely filled with incombustible powder, usually quartz, which has excellent arc extinguishing properties.

The operation of the fuse is performed with the following steps :

(i) Melting of the fuse element.

(ii) Vaporization of the element.

(iii) Fusion of silver vapour and filling powder.

(iv) Extinction of arc due to fusion.

Advantages

(i) The capability of clearing high values of short circuit.

(ii) High speed operation.

(iii) Consistent performance.

(iv) Low cost as compared to other types of circuit interruptors.

Disadvantages

(i) Replacement is necessary after every operation.

(ii) The fuse assemblies are by nature prone to high temperature rise. Though this is not a disadvantage in the fuse, it is likely to affect associated switches etc. unless proper steps are taken to prevent it.

Applications of H.R.C Cartridge Fuse

Due to its high rupturing capacity, the Cartridge Fuse is effectively used in series with circuit breakers, the rupturing capacity of which is inadequate under modern supply conditions. In such a case, the fuse is set to operate more rapidly than the circuit breaker on fault currents in excess of the rupturing capacity of the breaker.

(b) High Voltage Fuses

High voltage fuse elements should be enclosed in a strong casing as the fuse may melt with explosive violence.

The element may be immersed in oil or carbon tetrachloride so as to extinguish the arc.

One type of high voltage fuse is specially designed to fuse at a current which is a little higher than its current rating, i.e., it has a *low fusing factor*, which enables it to be employed in place of a circuit breaker, together with an over-load trip for protection against *overload* as well as *short-circuit*.

3.30 Miniature Circuit Breakers (MCB's)

MCB's are equivalent to having a switch and a fuse in a single unit. They are normally used to make and break the supply in sub-circuits. In addition to the above function, it also protects the circuit against overloading or short circuiting. They are available in single, two, three or four-pole configurations with current ratings of 5A, 16A, 32A and so on. If the circuit current exceeds the capacity of the MCB, it trips off.

Miniature Circuit Breakers are used widely as protective devices in consumer premises and for group switching and protection of fluorescent lights in commercial and industrial buildings. Moulded case circuit breakers with ratings up to 3000 A and capable of interrupting currents up to 200 kA (for the larger ratings) are becoming popular for control of low-voltage networks.

3.31 Electric Shock

3.31.1 Meaning

The effects of electric current passing through the human body are as follows :

1. 1 mA causes only a faint tingle.
2. 5 mA causes a slight shock, but may not be painful at all.
3. 10 to 30 mA causes painful shock, leading to loss of muscular control.
4. 50 to 150 mA causes extremely painful shock. Higher current may cause tissue damage or heart fibrillation, leading to death (fibrillation is the rapid, irregular and synchronized contraction of the muscle fibres of the heart).

3.31.2 Causes

Electric shock may be caused due to the following reasons :

(a) By accidentally touching faulty appliances or frayed cords or extension leads.

(b) Due to lightning strike

(c) By coming into contact with a high enough voltage source, resulting in the passage of a high enough value of current through the muscles or nerves.

3.31.3 Precautions to be taken to prevent electric shocks

The following precautions should be taken to prevent persons from getting electric shocks.

(i) People should not work on exposed live conductors if at all possible. If this is not possible then insulated gloves and tools should be used.

(ii) If both hands make contact with surfaces or objects at different voltages, current can flow through the body from one hand to the other. This can lead the current to pass through the heart. Similarly, if the current passes from one hand (especially the left hand) to the feet, significant current will probably pass through the heart. Wearing insulated footwear and standing on wooden platforms or mats can reduce the risk of shock.

(iii) All metal parts of appliances which must be at zero potential must be properly earthed, as otherwise these metal parts could come in contact with live wires, thus acquiring high potentials.

If inadvertently a person were to touch these parts, he will experience

a severe electric shock. Hence such metal parts are connected to earth through a low resistance path. Usually, an earth pit is dug to a sufficient depth and a copper plate or copper rod of appropriate dimensions, to which the metal parts of the equipments are connected by a thick copper wire or galvanized iron wire, is buried inside the pit. In order to ensure that there is good electrical contact between the copper rod or plate and earth, materials such as salt, coal etc., are filled into the pit and then the pit is covered with earth. This ensures that the body of the equipment is always at earth potential, and in case of an earth fault, the leakage or fault current will immediately flow into the earth.

(iv) When the human skin becomes wet, it allows more current to flow than the dry human body would. Thus being in the bath or shower will not only ground oneself to the return path of the power mains but will lower the body's resistance as well. Under these circumstances, touching any metal switch or appliance that is connected to the power mains due to some fault in the appliance could result in electrocution. So, touching a live wire, or a switch, with a wet skin should be avoided.

(v) Install ground fault circuit interrupts (GFCIs) in wall outlets located in the bathroom, Kitchens, basements & garages.

(vi) Cover all electrical sockets with plastic safety caps.

(vii) Replace all worn cords and wiring.

(viii) Never use an electrical appliance like a radio or an iron near water.

3.31.4 Remedy

When we come to know that a person has received an electric shock, the following remedial action should be taken :

(i) Do not touch the victim with bare hands until the power supply is switched off. Be especially careful in wet areas, such as bathrooms, since the salts in water conduct electricity.

(ii) Shut off the supply immediately.

(iii) If the supply cannot be switched off immediately, use a non-conducting object such as a dry wooden rod, broom, chair, rug or rubber door-mat to push the victim away from the source of current. Never use a wet or metal object. If possible, stand on something dry, non-conducting surfaces such as a mat, folded newspapers or dry wooden bases to push the victim.

(iv) Once the victim is free, check him/her for a response, *i.e.*, for breathing and pulse. If they have stopped or dangerously slow, initiate artificial breathing. Treat third degree burns and call emergency services for an ambulance.

(v) If the breathing and pulse are steady, treat the victim for shock by elevating the feet and by covering the body with a blanket.

(vi) Talk calmly and reassuringly to the conscious victim.

3.32 Earthing

The earth's potential is taken as zero for all practical purposes, and therefore, any electrical machine, appliance or component, when connected to earth, attains zero potential, and is said to be *earthed*, and the voltage of this appliance will fall to zero if its voltage is higher or will increase to zero if its voltage is lower than the earth potential.

The neutral wire of an AC Supply System and the middle wire of the three-wire DC Distribution System are always earthed to maintain line voltage always constant. The bodies of electrical machinery or appliance are earthed so that, in the event of any leakage, the leakage current immediately flows to earth, the circuit fuses blow off and the machinery or appliance is disconnected from the supply. Thus, by providing such earthing, risk to human life and the risk of outbreak of fire are eliminated.

3.32.1 Necessity of Earthing

Earthing is necessary for the following reasons :

a) To protect the human being from disability or death from shock in case the human body comes into contact with the frame of any electrical machinery, appliance or component which is electrically charged due to leakage current or fault (See Fig. 3.87).

b) To maintain the line voltage constant.

c) To protect tall buildings and structures from atmospheric lightning strikes.

d) To protect all machines, fed from overhead lines, from atmospheric lightning.

e) To serve as the return conductor for telephone, telegraph and traction work.

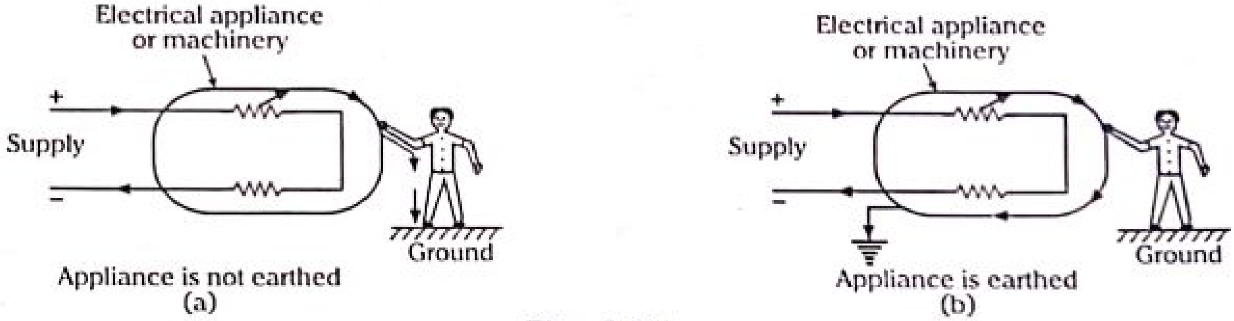


Fig. 3.87

Earthing is accomplished by connecting the electrical machinery appliance or component to earth by employing a good conductor called "*Earth Wire*" or "*Earth Electrode*".

3.32.2 Types of Earthing

Any pipe, plate or wire embedded in the earth is called *earth electrode*, and any current passing through it gets directly earthed. For effective and efficient earthing, the resistance offered by the earth electrode and by the soil in which the electrode is embedded, should be very low, offering minimum opposition to the passage of current through it, in order that the fuses in a faulty circuit blow off immediately. The various methods of earthing are

(i) Plate Earthing, (ii) Pipe Earthing, (iii) Earthing through water main, (iv) Horizontal Strip Earthing, (v) Rod Earthing.

We shall discuss only Plate Earthing and Pipe Earthing.

(i) Plate Earthing

Fig. 3.88 suggests the method of plate earthing. The earth connection is provided with the help of a copper plate (of size 60 cm × 60 cm × 3.18 mm) or Galvanized Iron plate (of size 60 cm × 60 cm × 6.3 mm) or Cast-iron plate (of the same size), embedded 3 metres into the ground. The plate is kept with its face vertical. Copper plates are found to be the most effective earth electrodes and are least affected by moisture (they do not get rusted).

However, they are very costly, and hence G.I. plates are preferred for normal work.

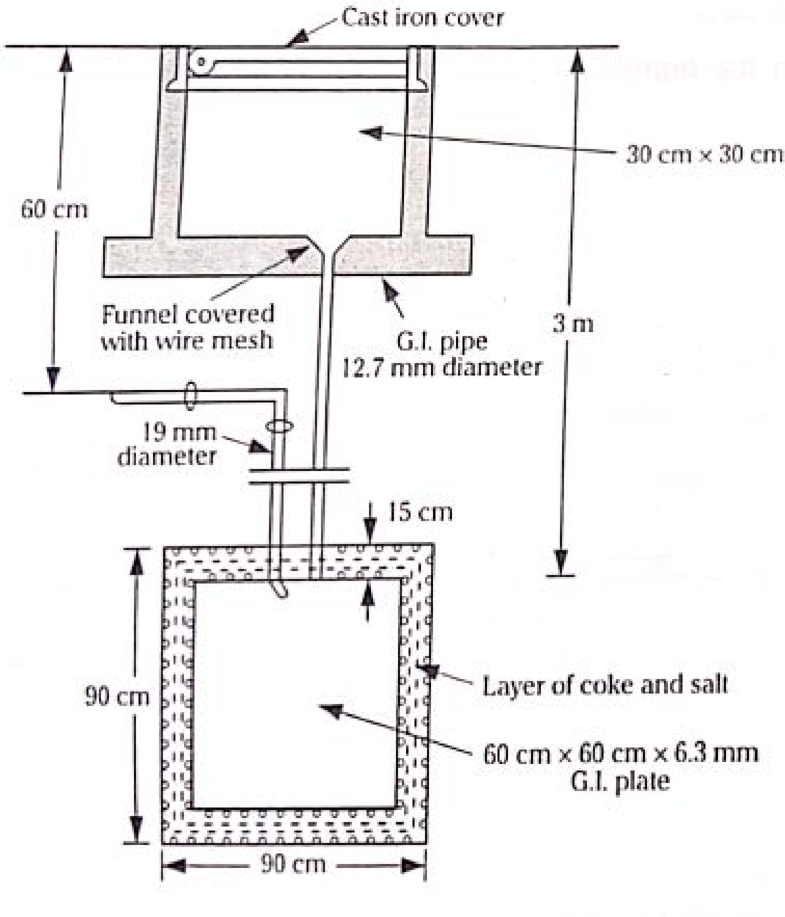


Fig. 3.88 Plate Earthing

The plate is so arranged that it is embedded in alternate layers of coke (or coal) and salt for minimum thickness of about 15 cm. The earth wire is drawn through a G.I. pipe and is perfectly bolted to the earth plate. The nuts and bolts must be made of copper for copper plate and of galvanized iron for G.I. plate.

The earth lead used must be G.I. wire or G.I. strip of sufficient cross-sectional area to carry the fault current safely. The earth wire is drawn through G.I. pipe of 19 mm diameter, at about 60 cm below the ground.

The G.I. pipe is fitted with a funnel on the top. To achieve affective earthing, salt water is poured periodically through the funnel.

The conductivity, *i.e.*, earthing efficiency, increases with the increase of the plate area and depth of embedding. The only disadvantage is that discontinuity of earth wire with the plate below the earth cannot be observed physically and hence is misleading and is likely to result in heavy loss in the event of a fault. If the resistivity of the soil is high, then it will be necessary to embed the plate vertically at a greater depth into the ground.

(ii) Pipe Earthing

Here, as shown in Fig. 3.89, a G.I. pipe of 38 mm diameter and 2 metres length is vertically embedded into the ground to serve as earth electrode, the depth depending on the soil condition. According to the Indian Standard, the pipe should go down to a depth of 4.75 m.

The pipe must be placed upright in wet ground. The pit area around the G.I. pipe is filled with salt and coal mixture for improving the soil condition and efficiency of the earthing system.

In summer, the soil becomes dry, in which case salt water is poured through the funnel connected to the main G.I. pipe through a 19 mm dia. pipe, to keep the soil wet.

The leading wire from the body of the apparatus to the earthing pit should be made of G.I. wire or G.I. strip of sufficient cross-sectional area to carry the fault current safely. The earth wire, from the 19 mm dia. G.I. pipe, should be carried in a conduit of G.I. pipe of dia. 12.7 mm, at a depth of about 60 cm below the ground.

The earth wires a re connected to the G.I. pipe above the ground level and can be physically inspected from time to time and continuity checks can be easily performed. This is an important advantage of Pipe Earthing over Plate Earthing. Further, the contact surface of the G.I. pipe with soil is greater as compared to the plate because of its circular section, and hence can handle heavier leakage current for the same electrode size.

The sole disadvantage of pipe-earthing it that the embedded pipe length has to be increased sufficiently in case the soil specific resistivity is of a high order, resulting in increased cost of material and excavation work. In ordinary soil condition, the range of earth resistance should be 2 to 5 ohms.

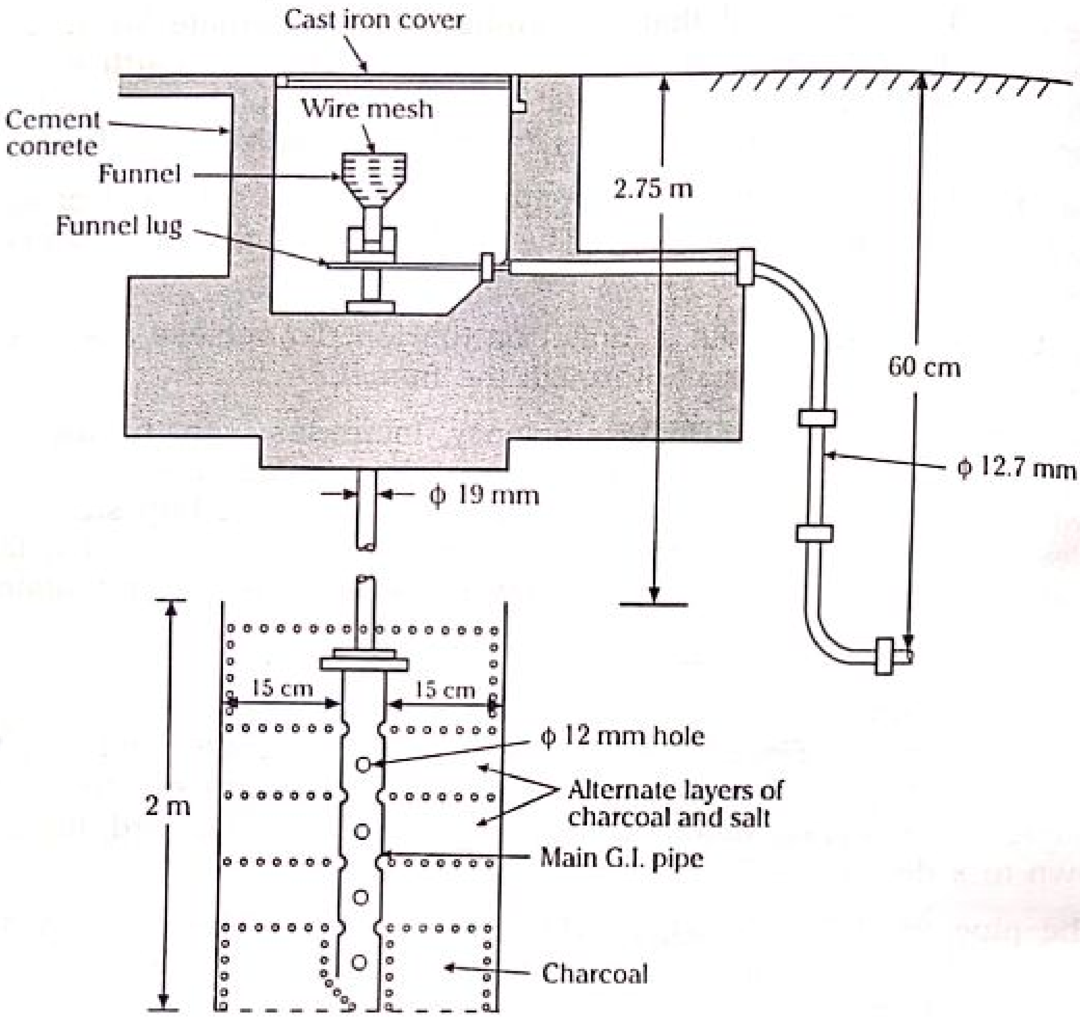


Fig. 3.89 Pipe Earthing

3.33 ELCB (Earth Leakage Circuit Breaker)

An *Earth Leakage Circuit Breaker (ELCB)* is a safety device used in electrical installations with high earth impedance to prevent shock. It detects small stray voltages on the metal enclosures of electrical equipment, and interrupts the circuit if a dangerous voltage is detected.

The *main purpose* of earth leakage protectors is to prevent injury to humans & animals due to electric shock.

ELCB's are of two types - *Voltage Operated and Current operated*. We shall however, discuss the former as it is more widely used.

A voltage operated ELCB detects a rise in potential between the protected interconnected metalwork (equipment frames, conduits, enclosures) and a distant isolated earth reference electrode. It operates at a detected potential of around 50 volts to open a main breaker and isolate the supply from the protected premises.

A voltage operated ELCB has a second terminal for connecting to the remote reference earth connection.

The earth circuit is modified when an ELCB is used; the connection to the earth rod is passed through the ELCB by connecting to its two earth terminals. One terminal goes to the installation earth CPC (circuit protective conductor, aka earth wire) and the other to the earth rod (or sometimes other type of earth connection).

The disadvantages of the voltage-operated ELCB are the requirement for a second connection, and the possibility that any additional connection to earth on the protected system can disable the detector.

Operation : An ELCB is a specialised type of latching relay that has a building's incoming mains power connected through its switching contacts so that the ELCB disconnects the power in an earth leakage (unsafe) condition.

The ELCB detects the fault currents from live to the earth (ground) wire within the installation it protects. If sufficient voltage appears across the ELCB's sense coil, it will switch off the power and will remain off until manually reset. A voltage sensing ELCB does not sense fault currents from live to any other earthed body.

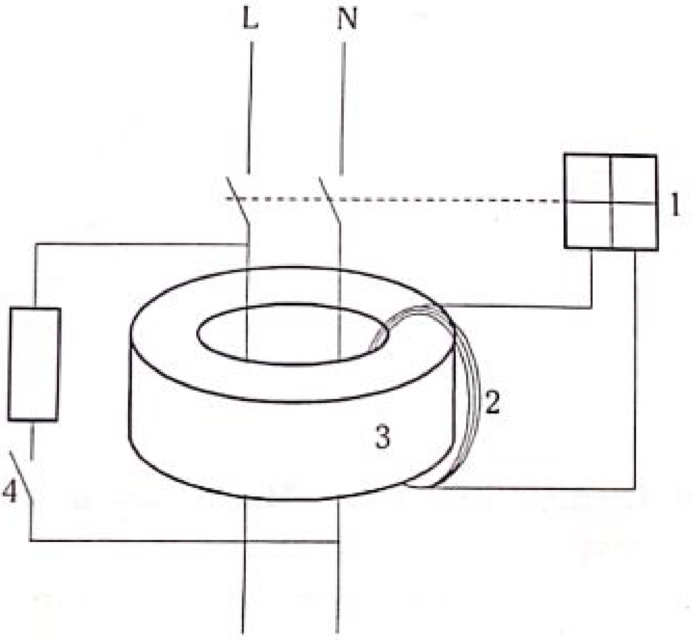
3.34 Residual-Current Circuit Breaker (RCCB) or a Residual Current Device (RCD)

A residual current circuit breaker (RCCB) or a Residual Current Device (RCD) is an electrical wiring device that disconnects a circuit whenever it detects that the electric current is not balanced between the energized conductor and the return neutral conductor. Such an imbalance may indicate current leakage through the body of a person who is grounded & accidentally touching the energized part of the circuit. A lethal shock can result from these conditions. RCCBs are designed to disconnect quickly enough to prevent injury caused by such shocks. They are not intended to provide protection against overcurrent (overload) or all short circuit conditions.

Purpose and Operation : RCCBs are designed to disconnect the circuit if there is a leakage current. By detecting small *leakage currents* (typically 5 to 30 milliamperes) and disconnecting quickly enough (< 300 ms), they may prevent electrocution. They are an essential part of the automatic disconnection of supply (ADS), i.e. to switch off when a fault develops, rather than rely on human intervention, this is one of the essential tenets of modern electrical practice. There are RCDs with intentionally slower responses & lower sensitivities, designed to protect equipment or avoid starting electrical fires, but not disconnect unnecessarily for equipment which has greater leakage currents in normal operation. To prevent electrocution, RCDs should operate within 5-40 milliseconds with any leakage currents (through a person) of greater than 30 milliamperes, before electric shock can drive the heart into ventricular fibrillation, the most common cause of death through electric shock. By contrast, conventional circuit breakers or fuses only break the circuit when the total current is excessive (which may be thousands of times the leakage current an

RCD responds to). A small leakage current, such as through a person, can be a very serious fault, but would probably not increase the total current enough for a fuse or circuit breaker to break the circuit, and certainly not to do so fast enough to save a life.

RCDs operate by measuring the current balance between two conductors using a differential current transformer (Fig. 3.90)



1. Elctromagnet with help electronics
2. Current transformer secondary winding
3. Transformer core
4. Test switch

L - Live conductor

N - neutral conductor

Fig. 3.90 Principle of operation

This measures the difference between current flowing through the live conductor and that returning through the neutral conductor. If these do not sum to zero, there is a leakage of current to somewhere else (to earth/ground, or to another circuit), and the device will open its contacts. It is important to realise that operation does not require a fault current to return via the earth wire in the installation, the trip will operate just as well if the return path is via plumbing, contact with terra firma or any other current path. Automatic disconnection and a measure of shock protection is therefore still provided even if the earth wiring of the installation is damaged or incomplete.

Residual current detection is complementary to over-current detection. Residual current detection cannot provide protection for overload or short-circuit currents, except for the special case of a short circuit from live to ground (not live to neutral).

For a RCD used with three-phase power, all three live conductors and the neutral (if fitted) must pass through the current transformer.

3.35 Review Questions

Q 1. Write short notes on :

(a) Service mains

(b) Miniature Circuit Breakers

Q 2. Draw the block diagram of the Meter Board and the Distribution Board and explain in brief.

Q 3. Write explanatory notes on ELCB, & RCCB.

Q 4. What is Domestic Wiring ? How do you do it with respect to the lamp (i) from two points (ii) from three points ?

Q 5. Give the wiring diagram for the two-way control of a lamp and explain.

Q 6. What is the purpose of a fuse? What are the requirements of a good fuse?

Q 7. Write a brief explanatory note on different types of fuses.

Q 8. (a) What do you understand by 'Electric Shock' ?

(b) What are the causes of electric shock ?

(c) What are the precuations to be taken to prevent electric shocks ?

Q 9. Write short notes on :

(a) Fusing materials used

(b) Remedial measures to be taken in the event of electric shock suffered by a person.

Q 10. What do you mean be Earthing ? Explain why installations must be earthed. (Jan 93, B.U.)

Q 11. What is the necessity for earthing electrical equipment ? Give a cross-sectional view of an earthing arrangement. (July 93, B.U.)

Q 12. What is the purpose of earthing electrical appliances ? Explain the different methods of earthing.

Q 13. What is Earthing ? Why is it necessary ? Explain its performance. (April 97, B.U.)

MODULE

4

(a) Three Phase Circuits

4.1 Necessity and Advantages of Three-Phase Systems

1. In a single phase a.c. generation, a number of armature coils are connected to form one winding, which when rotated in a magnetic field generates a voltage called *single-phase voltage*. But it is found that in many applications, the single-phase system is not very satisfactory. For instance, a single-phase induction motor is not self-starting unless it is fitted with an auxiliary winding. Hence it is found that a three-phase induction motor is more suitable as it is self-starting and has better efficiency and power factor than its single-phase counterpart.
2. The output of a three-phase machine is always greater than that of a single-phase machine of the same size of frame. So, for a given size and voltage, a three-phase machine (e.g a 3-phase alternator) occupies less space and costs less than the single-phase machine having the same rating.
3. To transmit and distribute a given amount of power over a given distance, a three-phase transmission line requires less copper than a single-phase line.
4. Three-phase motors have an absolutely uniform torque, whereas single-phase motors (except commutator motors) have a pulsating torque.
5. Single-phase motors (except commutator motors) are not self-starting. Three-phase motors are self-starting.
6. The pulsating nature of the armature reaction in single-phase alternators causes difficulty with parallel running unless the poles are fitted with exceedingly heavy dampers. Three-phase generators work in parallel without difficulty.
7. The connection of single-phase generators in parallel gives rise to harmonics, whereas three-phase generators can be conveniently connected with causing generation of harmonics.
8. In the case of a three-phase star system, two different voltages can be obtained, one between lines and the other between line and phase, whereas in the case of a single phase system only one voltage can be obtained.

9. In a single-phase system, the instantaneous power is a function of time and hence fluctuates with respect to time. This fluctuating power causes considerable vibrations in single-phase motors. Hence performance of single-phase motors in poor; whereas instantaneous power in a symmetrical three-phase system is constant.

4.2 Generation of Three Phase Voltages

In the 3-phase system, there are three equal voltages of the same frequency but displaced from one another by 120° electrical. These voltages are produced by a three-phase generator which has three identical windings or phases displaced 120° electrical apart. When these windings are rotated in a magnetic field, e.m.f is induced in each winding or phase. These e.m.f.s are of the same magnitude and frequency but are displaced from one another by 120° electrical.

Consider three electrical coils $a_1\,a_2$, $b_1\,b_2$ and $c_1\,c_2$ mounted on the same axis but displaced from each other by 120° electrical. Let the three coils be rotated in an anticlockwise direction in a bipolar magnetic field with an angular velocity of ω radians/sec, as shown in Fig. 4.1. Here, a_1, b_1 and c_1 are the start terminals and a_2, b_2 and c_2 are the end terminals of the coils.

When the coil $a_1\,a_2$ is in the position AB shown in Fig. 4.1, the magnitude and direction of the e.m.f.s induced in the various coils is as under :

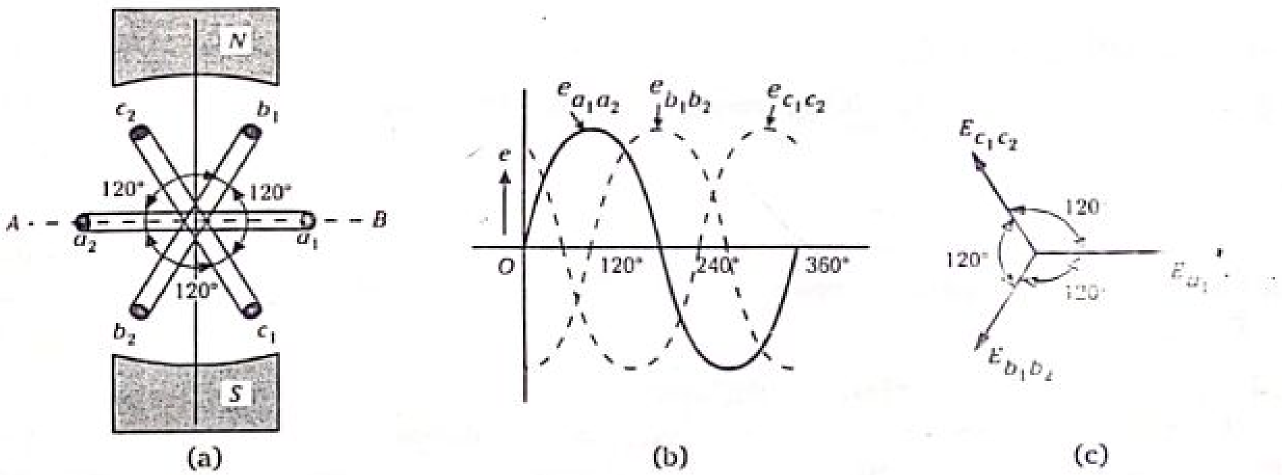


Fig. 4.1

a) E.m.f. induced in coil $a_1\,a_2$ is zero and is increasing in the positive direction. This is indicated by $e_{a_1a_2}$ wave in Fig. 4.1 (b).

b) The coil $b_1\,b_2$ is 120° electrically behind coil $a_1\,a_2$. The e.m.f. induced in this coil is negative and is approaching maximum negative value. This is shown by the $e_{b_1b_2}$ wave.

c) The coil $c_1\, c_2$ is 240° electrically behind $a_1\, a_2$ or 120° electrically behind coil $b_1 b_2$. The e.m.f. induced in this coil is positive and is decreasing. This is indicated by wave $e_{c_1c_2}$.

Thus, it is apparent that the *e.m.f.'s induced in the three coils are of the same magnitude and frequency but displaced 120° electrical from each other.*

Vector Diagram : The r.m.s. values of the three phase voltages are shown vectorially in Fig. 4.1 (c).

Equations : The equations for the three voltages are :

$$e_{a_1a_2} = E_m \sin \omega t$$

$$e_{b_1b_2} = E_m \sin\left(\omega t - \frac{2\pi}{3}\right); \qquad e_{c_1c_2} = E_m \sin\left(\omega t - \frac{4\pi}{3}\right)$$

4.3 Definition of Phase Sequence

The order in which the voltages in the phases reach their maximum positive values is called the phase sequence. For example, in Fig. 4.1 (a), the three coils $a_1\, a_2$, $b_1\, b_2$ and $c_1\, c_2$ are rotating in anticlockwise direction in the magnetic field. The coil $a_1\, a_2$ is 120° electrical ahead of coil $b_1\, b_2$ and 240° electrical ahead of coil $c_1\, c_2$. Therefore, e.m.f. in coil $a_1\, a_2$ leads the e.m.f. in coil $b_2\, b_2$ by 120° and that in coil $c_1\, c_2$ by 240°. It is evident from Fig. 4.1 (b) that $e_{a_1a_2}$ attains maximum positive first, then $e_{b_1b_2}$ and $e_{c_1c_2}$. In other words, the order in which the e.m.f.s in the three phases $a_1\, a_2$, $b_1\, b_2$ and $c_1\, c_2$ attain their maximum positive values is a, b, c. Hence, the phase sequence is a, b, c.

4.3.1 Naming the Phases

The 3 phases may be numbered (1, 2, 3) or lettered (a, b, c) or specified colours ($R\,Y\,B$). By normal convention, sequence $R\,Y\,B$ is considered positive and $R\,B\,Y$ negative.

4.4 Double-Subscript Notation

It is necessary to employ some systematic notation for the solution of a.c. circuits and systems containing a number of e.m.f.s. acting and currents flowing so that the process of solution is simplified and less prone to errors.

It is normally preferred to employ double-subscript notation while dealing with a.c. electrical circuits. In this system, the order in which the subscripts are written indicates the direction in which e.m.f. acts or current flows.

For example, if e.m.f. is expressed as E_{ab}, it indicates that e.m.f. acts from a to b ; if it is expressed as E_{ba}, then the e.m.f. acts in a direction opposite to that in which E_{ab} acts. (Fig. 4.2) *i.e.*, $E_{ba} = -E_{ab}$.

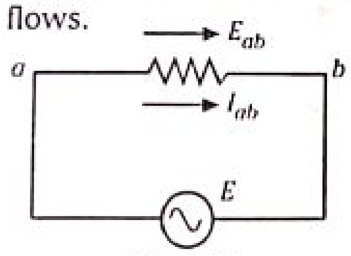


Fig. 4.2

Similarly, I_{ab} indicates that current flows in the direction from a to b but I_{ba} indicates that current flows in the direction from b to a ; *i.e.*, $I_{ba} = -I_{ab}$.

4.5 Three-Phase Balanced Supply and Load (Star and Delta)

When a balanced generating supply, where the three phase voltages are equal, and the phase difference is 120° between one another, supplies balanced equipment load, where the impedances of the three phases or three circuit loads are equal, then the current flowing through these three phases will also be equal in magnitude, and will also have a phase difference of 120° with one another. Such an arrangement is called a *balanced load*.

4.6 Relationship between Line & Phase Quantities & Expression for Power for Balanced Three phase Star Connection

This system is obtained by joining together similar ends, either the start or the finish; the other ends are joined to the line wires, as shown in Fig. 4.3 (a). The common point N at which similar (start or finish) ends are connected is called the *neutral* or *star point*. Normally, only three wires are carried to the external circuit, giving a 3-phase, 3-wire, star-connected system; however, sometimes a fourth wire known as neutral wire, is carried to the neutral point of the external load circuit, giving a 3-phase, 4-wire connected system.

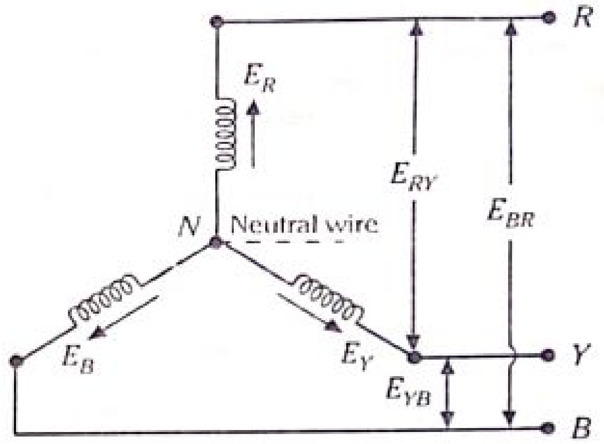


Fig. 4.3 (a) Connection Diagram

The voltage between any line and the neutral point, *i.e.*, voltage across the phase winding, is called the *phase voltage*; while the voltage between any two outers is called *line voltage*. Usually, the neutral point is connected to earth.

In Fig. 4.3 (a), positive directions of e.m.f.s are taken from star point outwards. The arrow heads on e.m.f.s and currents indicate the positive direction. Here, the 3-phases are numbered as usual : R, Y and B indicate the three natural colours red, yellow and blue respectively.

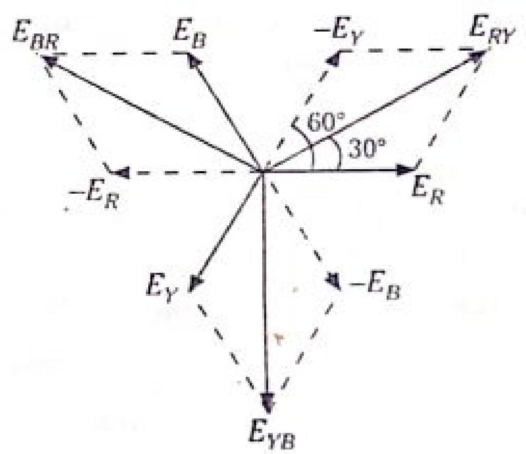


Fig. 4.3 (b) Vector Diagram of Line and Phase voltages.
3-Phase Star-Connected System

By convention, sequence RYB is taken as positive and RBY as negative.

In Fig. 4.3 (b), the e.m.f.s induced in the three phases are shown vectorially. In a star-connection there are two windings between each pair of outers and due to joining of similar ends together, the e.m.f.s induced in them are in opposition.

Hence the potential difference between the two outers, known as *line voltage*, is the vector difference of phase e.m.f.s of the two phases concerned.

For example, the potential difference between outers R and Y or line voltage E_{RY}, is the vector difference of phase e.m.f.s E_R and E_Y or vector sum of phase e.m.f.s E_R and $(-E_Y)$.

i.e., $E_{RY} = E_R - E_Y$ (Vector difference)

or $E_{RY} = E_R + (-E_Y)$ (Vector sum)

As phase angle between vectors E_R and $(-E_Y)$ is 60°,

∴ from vector diagram shown in Fig. 4.3 (b),

$$E_{RY} = \sqrt{E_R^2 + E_Y^2 + 2\,E_R\,E_Y \cos 60°}$$

Let $E_R = E_Y = E_B = E_P$ (phase voltage)

Then line voltage $E_{RY} = \sqrt{E_P^2 + E_P^2 + (2\,E_P\,E_P \times 0.5)} = \sqrt{3}\,E_P$

Similarly, potential difference between outers Y and B or line voltage $E_{YB} = E_Y - E_B = \sqrt{3}\,E_P$ and potential difference between outers B and R, or line voltage $E_{BR} = E_B - E_R = \sqrt{3}\,E_P$.

In a balanced star system, E_{RY}, E_{YB} and E_{BR} are equal in magnitude and are called *line voltages*.

$$\therefore \quad \boldsymbol{E_L = \sqrt{3}\,E_P}$$

Since, in a star-connected system, each line conductor is connected to a separate phase, so the current flowing through the lines and phases are the same.

i.e., Line Current I_L = Phase current I_P

If the phase current has a phase difference of ϕ with the phase voltage,

Power output per phase $= E_P\,I_P \cos\phi$

Total power output, $P = 3\,E_P\,I_P \cos\phi$

$$= 3\,\frac{E_L}{\sqrt{3}}\,I_P \cos\phi$$

$$= \sqrt{3}\;E_L\;I_L \cos\phi$$

i.e., Power = $\sqrt{3}$ × line voltage × line current × power factor

Apparent power of 3-phase star-connected system

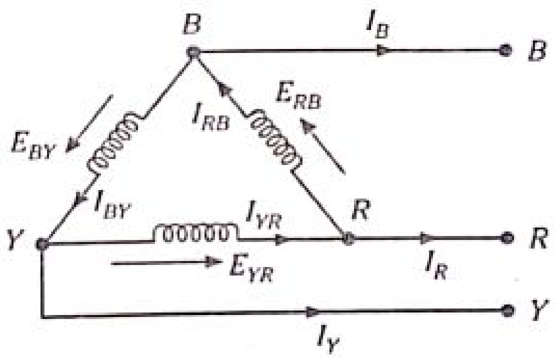
$$= 3 \times \text{apparent power per phase}$$

$$= 3 E_P I_P = 3 \times \frac{E_L}{\sqrt{3}} \times I_L = \sqrt{3} E_L I_L$$

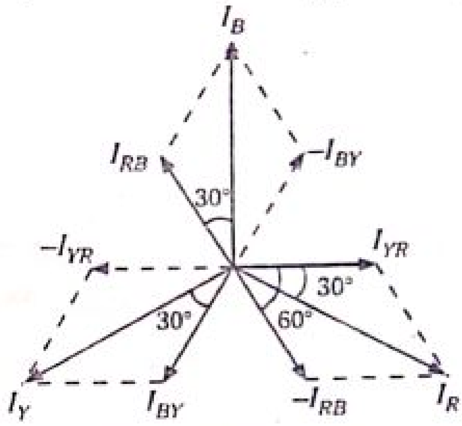
4.7 Relationship between Line and Phase Quantities and Expression for Power for Balanced Three Phase Delta Connection

When the starting end of one coil is connected to the finishing end of another coil, as shown in Fig. 4.4 (a), delta or mesh connection is obtained. The direction of the e.m.f.s is as shown in the diagram.

From Fig. 4.4 it is clear that line current is the vector difference of phase currents of the two phases concerned. For example, the line current in red outer I_R will be equal to the vector difference of phase currents I_{YR} and I_{RB}. The current vectors are shown in Fig. 4.4 (b).



(a) Connection Diagram



(b) Vector Diagram

Fig. 4.4

Referring to Fig. 4.4 (a) and (b),

Line current, $I_R = I_{YR} - I_{RB}$ (Vector difference)

$= I_{YR} + (-I_{RB})$ (Vector sum)

As the phase angle between currents I_{YR} and $-I_{RB}$ is 60°

$$\therefore \quad I_R = \sqrt{I_{YR}^2 + I_{RB}^2 + 2 I_{YR} I_{RB} \cos 60^\circ}$$

For a balanced load, the phase current in each winding is equal and let it be = I_P.

$$\therefore \quad \text{Line current, } I_R = \sqrt{I_P^2 + I_P^2 + 2 I_P I_P \times 0.5} = \sqrt{3} I_P$$

Similarly, line current, $I_Y = I_{BY} - I_{YR} = \sqrt{3} I_P$

and line current, $I_B = I_{RB} - I_{BY} = \sqrt{3}\, I_P$

In a delta network, there is only one phase between any pair of line outers, so the potential difference between the line outers, called the *line voltage*, is equal to *phase voltage*.

i.e., Line voltage, E_L = Phase voltage, E_P

Power output per phase = $E_P I_P \cos\phi$,

where $\cos\phi$ is the power factor of the load.

$$\text{Total power output, } P = 3\, E_P I_P \cos\phi$$

$$= 3\, E_L \frac{I_L}{\sqrt{3}} \cos\phi$$

$$= \sqrt{3}\, E_L I_L \cos\phi$$

i.e., Total power output = $\sqrt{3}$ × Line voltage × Line current × p.f.

Apparent power of 3-phase delta-connected system

= 3 × apparent power per phase

$$= 3 E_P I_P = 3 E_L \frac{I_L}{\sqrt{3}} = \sqrt{3}\, E_L I_L$$

4.8 Measurement of Power in 3-Phase Circuits : Two Wattmeter Method - Balanced Load

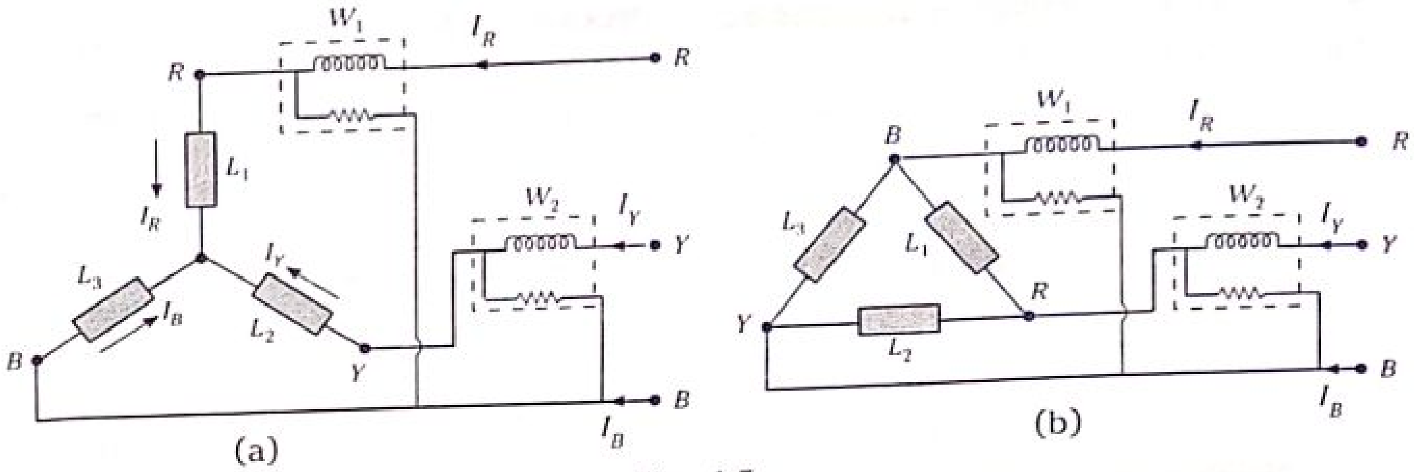
This method is normally used for measuring power, in 3-phase, 3-wire balanced load circuits. As shown in Fig. 4.5, the current coils in the two wattmeters are inserted in *any two* lines and the potential coil of each wattmeter is joined to the third line.

We shall consider a star-connected load for our discussion: however, it will be equally applicable to a delta-connected load because a delta connected load can be replaced by an equivalent star-connected load.

In the case of balanced load, (where impedances of all the 3 phases are equal) we can find the power factor of the load from the two wattmeter readings. We shall take the star-connected load of Fig. 4.5 (a) to be inductive, the vector diagram for which is given in Fig. 4.6. We shall consider r.m.s. values in our discussion.

Let the three phase voltages have r.m.s. values of E_R, E_Y and E_B and let the r.m.s. values of currents be I_R, I_Y and I_B. Here, we show these currents lagging behind their respective phase voltages by ϕ.

Current through wattmeter W_1 [Fig. 4.5 (a)] is = I_R



(a) (b)

Fig. 4.5

The potential difference across the voltage coil of W_1 is

$$E_{RB} = E_R - E_B \qquad \text{– vectorially}$$

This E_{RB} is the vectorial resultant of E_R and $-E_B$, as shown in Fig. 4.6. We see that the phase difference between E_{RB} and I_R = $(30° - \phi)$.

$\therefore$ Reading of $W_1 = I_R \, E_{RB} \cos(30° - \phi)$

Similarly, we see frcm Fig. 4.5 (a) that current through $W_2 = I_Y$ and p.d. across $W_2 = E_{YB} = E_Y - E_B$ – vectorially

Again E_{YB} is the vectorial resultant of E_Y and $-E_B$, as shown in Fig. 4.6. The angle between I_Y and E_{YB} is $(30° + \phi)$.

Reading of $W_2 = I_Y \, E_{YB} \cos(30° + \phi)$

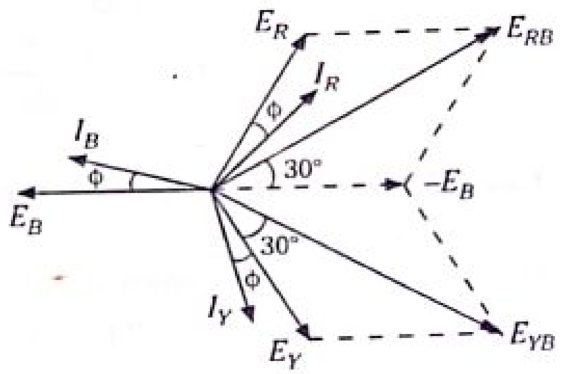


Fig. 4.6

As the load is balanced

$E_{RB} = E_{YB}$ = Line voltage E_L and

$I_Y = I_R$ = Line current I_L

$$\therefore \quad W_1 = E_L I_L \cos(30° - \phi) \quad \text{and} \quad W_2 = E_L I_L \cos(30° + \phi)$$

$$\therefore \quad W_1 + W_2 = E_L I_L \cos(30° - \phi) + E_L I_L \cos(30° + \phi)$$

$$= E_L I_L (\cos 30° \cos\phi + \sin 30° \sin\phi + \cos 30° \cos\phi - \sin 30° \sin\phi)$$

$$= E_L I_L (2 \cos 30° \cos\phi) = \sqrt{3}\, E_L I_L \cos\phi$$

= Total power in the 3-phase load.

Thus, the total power absorbed in the 3-phase load is given by the sum of the two wattmeter readings. **Thus, it has been shown that two wattmeters are sufficient to measure power in a three-phase balanced circuit.**

4.9 Power Factor - Balanced 3-Phase Load

Case 1 : Lagging Power Factor

As we have discussed in Sec 4.8 for a *lagging power factor* :

$$W_1 + W_2 = E_L I_L \cos(30° - \phi) + E_L I_L \cos(30° + \phi)$$

$$= \sqrt{3}\, E_L I_L \cos\phi \qquad \text{---(i)}$$

Similarly, $\quad W_1 - W_2 = E_L I_L \cos(30° - \phi) - E_L I_L \cos(30° + \phi)$

$$= E_L I_L \left(2 \times \sin\phi \times \frac{1}{2}\right)$$

$$= E_L I_L \sin\phi \qquad \text{---(ii)}$$

Dividing eqn.(ii) by (i), we get

$$\tan\phi = \frac{\sqrt{3}\,(W_1 - W_2)}{(W_1 + W_2)} \qquad \text{---(iii)}$$

If $\tan\phi$, and hence ϕ, is known, the power factor, $\cos\phi$, can be found from mathematical tables. We should bear in mind that if W_2 is negative (*i.e.*, its reading is taken after reversing the pressure coil), then the expression (iii) above will become

$$\tan\phi = \sqrt{3}\,\frac{W_1 - (-W_2)}{W_1 + (-W_2)} = \sqrt{3}\,\frac{W_1 + W_2}{W_1 - W_2} \qquad \text{---(iv)}$$

Case 2 : Leading Power Factor

Referring to Fig. 4.7 and following the reasoning similar to what is done earlier,

$$W_1 = E_L I_L \cos(30° + \phi)$$

and $\quad W_2 = E_L I_L \cos(30° - \phi)$

$\therefore \quad W_1 + W_2 = \sqrt{3}\, E_L I_L \cos\phi$

$$W_1 - W_2 = -E_L I_L \sin\phi$$

$\therefore \quad \tan\phi = -\sqrt{3}\,\dfrac{W_1 - W_2}{W_1 + W_2}$

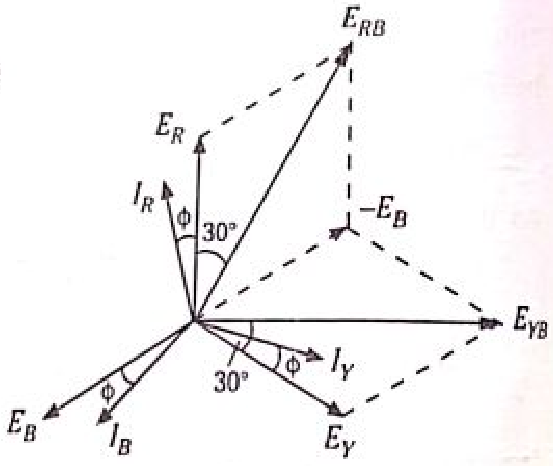


Fig. 4.7

It is apparent that if ϕ is greater than 60°, then the phase angle between E_{RB} and I_R exceeds 90°, and hence W_1 will give a negative reading. But W_2 will indicate a positive reading even if ϕ = 90°

4.10 Solved Problems on Three-Phase Circuits

Problem 4.1

A balanced star-connected load is supplied from a balanced 3-phase, 400 V, 50 Hz system. The current in each phase is 30 A and lags 30° behind the phase voltage. Find (i) the total power, (ii) phase voltage. Draw the phasor diagram. (Dec. 81, B.U.)

Solution :

Line voltage, $E_L = 400$ V

a) Phase voltage, $E_P = \frac{E_L}{\sqrt{3}} = \frac{400}{\sqrt{3}} = \mathbf{231\ V}$

Phase current, $I_p = 30$ A

Power factor, $\cos\phi = \cos 30° = 0.866$ (lag)

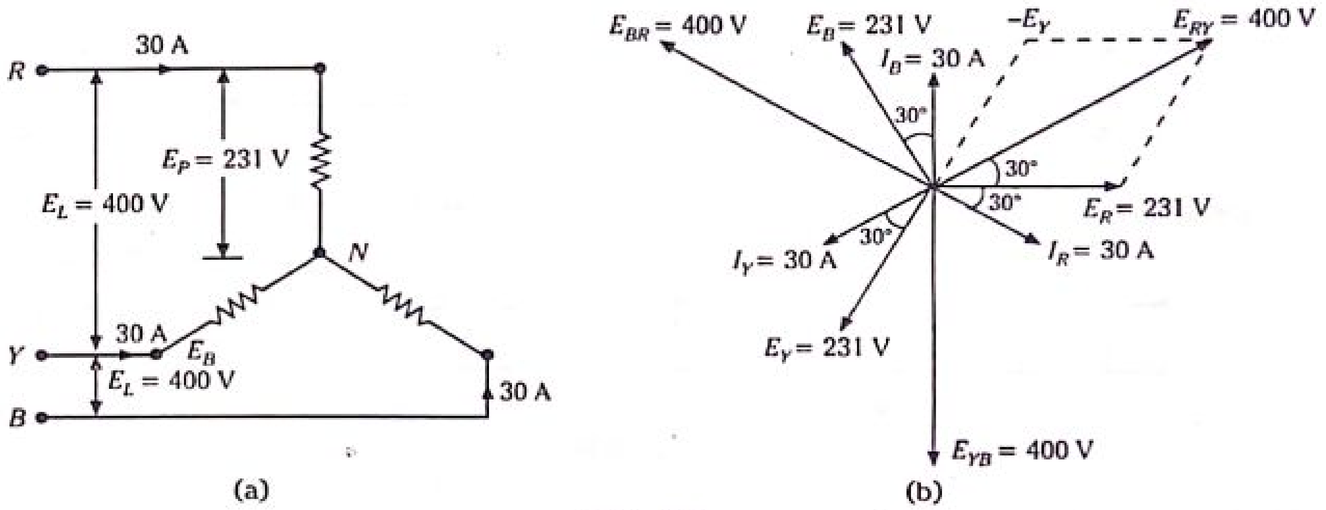


Fig. 4.8

b) Total Power $3\, E_P\, I_P \cos\phi = 3 \times 231 \times 30 \times 0.866$

$= 18000$ watts or **18 kW.**

Problem 4.2 ✶✶✶✶

Three equal impedances, each having a resistance of 8 ohms and inductive reactance of 6 ohms are connected in i) star, (ii) delta, across a 3-phase, 440 V system. Find (i) Phase current, (ii) Line current, (iii) Total power consumed. (Jan 93, B.U.)

Solution :

Impedance of each coil $Z_P = \sqrt{8^2 + 6^2} = 10\ \Omega$

i) Star connection [Fig. 4.9 (a)]

Line voltage, $E_L = 400$ V

Phase Voltage, $E_P = \dfrac{440}{\sqrt{3}} = 254$ V

$\therefore$ Phase current, $I_P = \dfrac{E_P}{Z_P} = \dfrac{254}{10} = 25.4$ A

Line current, I_L = Phase current I_P = **25.4 A**

Total power consumed, $P = \sqrt{3}\ E_L\ I_L\ \cos\phi$

Now, p.f., $\cos\phi = \dfrac{R}{Z_P} = \dfrac{8}{10} = 0.8$

$\therefore$ $P = \sqrt{3} \times 400 \times 25.4 \times 0.8 = 15485$ W = **15.485 kW**

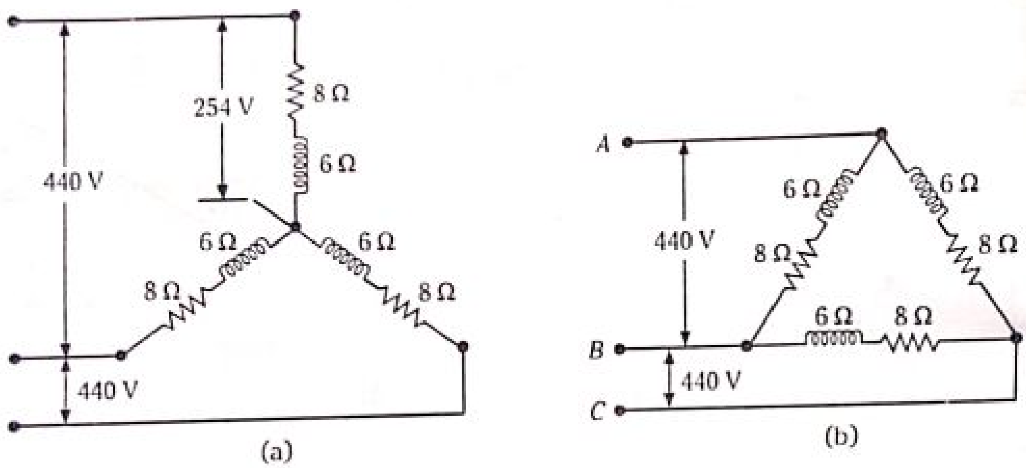


Fig. 4.9

ii) Delta connection [Fig. 4.9 (b)]

Here $E_P = E_L = 440$ V

$$I_P = \frac{E_P}{Z_P} = \frac{440}{10} = \mathbf{44\ A}$$

Line current $I_L = \sqrt{3} \times$ Phase Current I_P

$= \sqrt{3} \times 44 = \mathbf{76.2\ A}$

Total power consumed, $P = \sqrt{3} \times E_L\ I_L \cos\phi$

$= \sqrt{3} \times 440\ \ 76.2 \times 0.8$

$= 46457$ W = **46.457 kW**

Problem 4.3

A balanced 3-phase, star connected load of 150 kW takes a leading current of 100 A with line voltage of 1100 V, 50 Hz. Find the circuit constants of the load per phase. (Aug 94, B.U.)

Solution :

As the load is star-connected, phase voltage

$$E_p = \frac{\text{Line Voltage}}{\sqrt{3}}$$

$$= \frac{1100}{\sqrt{3}} = 635 \text{ volts}$$

Power $P = \sqrt{3}\ E_L\ I_L\ \cos\phi$

Given

Line current $I_L = 100$ A

$\therefore$ $150{,}000 = \sqrt{3} \times 1100 \times 100 \times \cos\phi$

$\therefore$ $\cos\phi = \dfrac{150{,}000}{\sqrt{3} \times 1100 \times 100} = 0.787$

Now, given that current is leading and hence we know that the reactance in the load is capacitive.

Impedance/Phase, $Z_p = \dfrac{E_p}{I_p}$

In a star-connected circuit, $I_p = I_L = 100$ A

$\therefore$ $Z_p = \dfrac{635}{100} = 6.35$ ohms

$\therefore$ Resistance in each phase, $R_p = Z_p \cos\phi$

$$= 6.35 \times 0.787$$

$$= \mathbf{5\ \Omega}$$

Reactance $X_p = \sqrt{Z_p^2 - R_p^2}$

$$= \sqrt{6.35^2 - 5^2} = 3.9\ \Omega$$

As this reactance is capacitive, $\dfrac{1}{\omega C}$ or $\dfrac{1}{2\pi f C} = 3.9$

or $C = \dfrac{1}{2\pi \times 50 \times 3.9}$

$= 0.000810$ F

$= \mathbf{810\ \mu F}$

Problem 4.4

Three similar resistors are connected in star across a 400 V, 3 phase supply. The line current is 10 Amps. Calculate

i) the value of each resistor

ii) the line voltage required to give the same line current if the resistors are connected in delta. (Feb 96, B.U.)

Solution :

Star Connection

$$I_L = I_P = 10\,A$$

$$E_p = \frac{400}{\sqrt{3}} = 231\text{ V}$$

$$\therefore \quad R_p = \frac{231}{10} = \mathbf{23.1\ \Omega}$$

Delta Connection

$$I_L = 10\text{ A (given)}$$

$$I_p = \frac{10}{\sqrt{3}} = 5.77\text{ A}$$

$$R_p = 23.1\,\Omega$$

$$\therefore \quad E_p = I_p \times R_p$$

$$= 5.77 \times 23.1$$

$$= \mathbf{133.3\ V}$$

Problem 4.5

A balanced Delta-connected load of $(8 + j6)\,\Omega$ per phase is supplied from a 3-phase 440 V source. Find the line current, power factor, power per phase and total power.

Solution :

Given $E_L = 440$ V,

Z_p (Impedance per phase) $= (8 + j6)\,\Omega$

i) For a Delta-connected system,

Phase Voltage = Line voltage

or $E_P = E_L = 440$ V

$$Z_P = (8 + j6)\,\Omega$$

$$\therefore \quad \text{Magnitude of } Z_P = \sqrt{(8^2 + 6^2)} = 10\,\Omega$$

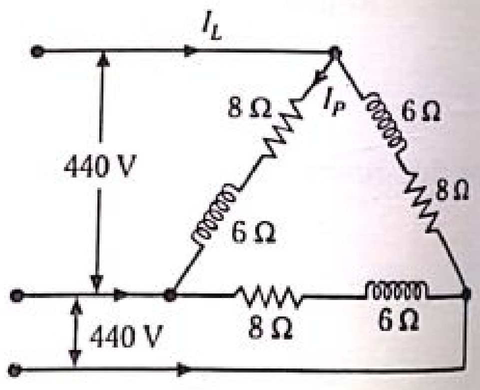


Fig. 4.10

$\therefore$ Phase Current $I_P = \dfrac{E_P}{Z_P}$

$$= \frac{440}{10} = 44 \text{ A}$$

Line Current $= \sqrt{3} \times$ Phase Current

i.e., $I_L = \sqrt{3} \times 44 =$ **76.21 A**

ii) Power Factor $= \cos\phi = \dfrac{R}{Z_P} = \dfrac{8}{10} =$ **0.8**

iii) Power / Phase $= E_p I_p \cos\phi$

$$= 440 \times 44 \times 0.8 = \textbf{15.488 kW}$$

iv) Total Power $= 3 \times 15.488$ kW

$= \textbf{46.464 kW}$

(Alternatively, Total Power, $P = \sqrt{3}\, E_L I_L \cos\phi$

$= \sqrt{3} \times 440 \times 76.21 \times 0.8$

$=$ **46.464 kW** - as before)

Problem 4.6

A balanced Star-connected load of (8 + *j*6) Ohms per phase is connected to a 3-phase, 230 V supply. Find the line current, power factor, power, reactive volt-amperes and total volt-amperes.

Solution :

Given Impedance/Phase, $Z_p = (8 + j6)$ Ohms

Line Voltage, $E_L = 230$ V

I) For the Star-connected system,

Phase Voltage, $E_P = \dfrac{E_L}{\sqrt{3}}$

$$= \frac{230}{\sqrt{3}} = 132.79 \text{ V}$$

Magnitude of $Z_P = \sqrt{(8^2 + 6^2)} = 10\ \Omega$

Phase Current, $I_P = \dfrac{E_P}{Z_P}$

$$= \frac{132.79}{10} = 13.279 \text{ A}$$

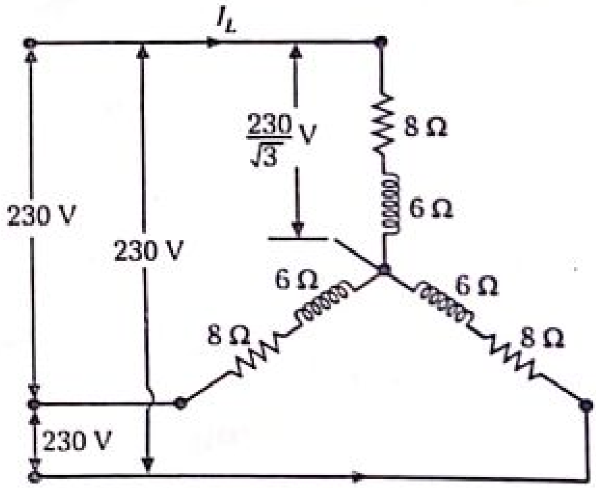


Fig. 4.11

Since $I_L = I_P$, Line Current I_L = **13.279 A**

ii) Power Factor, $\cos\phi = \frac{R}{Z} = \frac{8}{10} = \mathbf{0.8}$

iii) Total Power, $P = \sqrt{3}\, E_L\, I_L \cos\phi$

$$= \sqrt{3} \times 230 \times 13.279 \times 0.8$$

$$= \mathbf{4232\ Watts}$$

iv) Reactive Volt Amperes $= 3\, E_P\, I_P \sin\phi$

$$= 3 \times 132.79 \times 13.279 \times 0.6$$

$$= \mathbf{317\ VAR}$$ **(Reactive Volt-Amperes)**

v) Total Volt-Amps $= \frac{\text{Total Power in Watts}}{\cos\phi}$

$$= \frac{4232}{0.8}$$

$$= \mathbf{5290\ Amps}$$

Problem 4.7

A balanced, 3-phase, Star-connected load of 150 kW takes a leading current of 100 A, with a line voltage of 1100 V, 50 Hz. Find the circuit constants of the load per phase. (Apr 85, B.U., March Mysore)

Solution :

Given E_L = 1100 volts, ϕ = 50 Hz,

Total 3-phase power, P = 150 KW and I_L = 100 A leading

Since the current is leading, each phase of the load must be a series combination of R and C.

We have $I_P = I_L$ (Star Connection)

$= 100$ A

3-phase power $P = 3\, I_P^2\, R$

$\therefore \quad 150 \times 1000 = 3 \times 100^2 \times R$

$\therefore \quad R = \frac{150 \times 1000}{3 \times 100^2} = \mathbf{5\ \Omega}$ **resistance/phase**

Now $Z_P = \frac{E_P}{I_P} = \frac{\frac{1100}{\sqrt{3}}}{100} = 6.35\ \Omega$

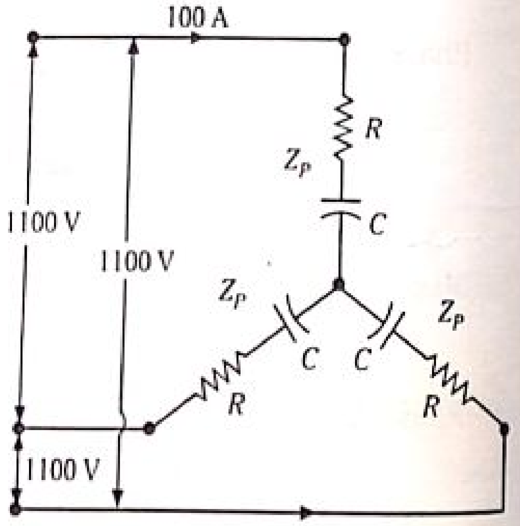


Fig. 4.12

Also $Z^2 = R^2 + X_C{}^2$

where X_C = Capacitive reactance

$$\therefore \quad X_C = \sqrt{(Z^2 - R^2)} = \sqrt{6.35^2 - 5^2}$$

$$= 3.9144$$

$$X_C = \frac{1}{2\pi f C}$$

or $C = \dfrac{1}{2\pi f X_C}$

or Capacitance $C = \dfrac{1}{2\pi \times 50 \times 3.9144}$

$$= 813.175 \times 10^{-6} \text{ F}$$

$$= \mathbf{831.175\ \mu F}$$

So, the circuit constants are R = 5 Ω and C = 831.175 μF.

Problem 4.8

Calculate the current flowing into each terminal and in each phase of the winding of a 3-phase Delta-connected Induction Motor developing an output of 250 HP, at 2300 V between the terminals at a power factor of 0.75 and efficiency of 85%. **(Feb 88, B.U.)**

Solution :

Given output of the Induction Motor = 250 HP

E_L = 2300 V, p.f. = 0.75 and Motor Efficiency = 85 %

Input to the motor $= \dfrac{\text{Output in KW}}{\text{Efficiency}}$

$$= \frac{250 \times 0.7355}{0.85} \qquad [1 \text{ HP} = 0.7355 \text{ kW}]$$

$$= 216.324 \text{ kW}$$

Power input to the 3-phase Induction Motor is given by the expression

$$P = \sqrt{3}\, E_L\, I_L \cos\phi$$

∴ Line Current, *i.e.*, current flowing into each terminal,

$$I_L = \frac{P}{\sqrt{3}\, E_L \cos\phi} = \frac{216.324 \times 1000}{\sqrt{3} \times 2300 \times 0.75}$$

$$= \mathbf{72.4\ A}$$

For Delta-connection, $I_P = \dfrac{I_L}{\sqrt{3}}$

Current in each phase, $I_P = \dfrac{72.4}{\sqrt{3}} = \mathbf{41.8\ A}$

Problem 4.9

With relevant phasor diagrams show that the two wattmeters read equal values when used to measure total power in a 3-phase balanced resistive circuit. **(Feb 96, B.U.)**

Solution :

The phasor diagram is shown in Fig. 4.13, where E_R, E_Y and E_B are the r.m.s. values of phase voltages.

As the circuit is resistive, the respective r.m.s. values of currents I_R, I_Y and I_B are in phase with the above voltages, or in other words, the power factor ϕ is zero.

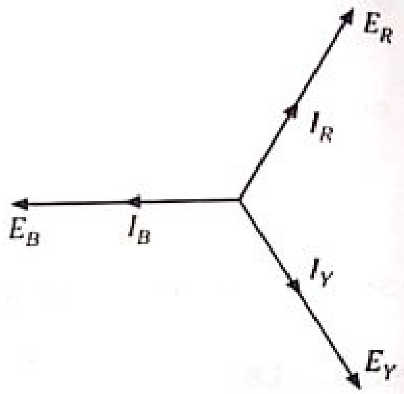


Fig. 4.13

In the case of lagging Power Factor,

$$\tan\phi = \frac{\sqrt{3}\,(W_1 - W_2)}{(W_1 + W_2)}$$

or $$0 = \frac{\sqrt{3}\,(W_1 - W_2)}{(W_1 + W_2)}$$

or $W_1 - W_2 = 0$, *i.e.*, the two wattmeters read equal values.

Similarly, in the case of leading p.f.,

$$\tan\phi = \frac{-\sqrt{3}\,(W_1 - W_2)}{(W_1 + W_2)}$$

Again, $\tan\phi = \tan 0° = 0$

Hence $W_1 = W_2$, as before.

Problem 4.10

The power to a 3-phase induction motor was measured by two wattmeter method and the readings were 3400 and –1200 watts respectively. Calculate the total power and power factor.

(Mar 95, B.U.)

Solution :

$$\tan\phi = \frac{\sqrt{3}\,(W_1 - W_2)}{(W_1 + W_2)}$$

$W_1 = 3400$ watts, $\quad W_2 = -1200$ watts

$$\therefore \quad \tan\phi = \frac{\sqrt{3}\,(3400+1200)}{3400-1200}$$

$$= \frac{\sqrt{3}\times 4600}{2200} = 3.62$$

$\therefore \quad \phi = 74.5°$

$\therefore \quad$ Power factor , $\cos\phi =$ **0.267**

Total power $= W_1 + W_2 = 3400 - 1200$

$=$ **2200 Watts**

Problem 4.11

A balanced three phase star-connected load draws power from a 440 V supply. The two wattmeters connected indicate W_1 = 4.2 KW and W_2 = 0.8 KW. Calculate the power, power factor and current in the circuit.

(Apr/May 87, M.U.)

Solution :

$W_1 = 4.2$ KW $= 4200$ W

$W_2 = 0.8$ KW $= 800$ W

$\therefore \quad$ Total Power $W_1 + W_2 = 4200 + 800 =$ **5000 W**

As per Sec. 4.9, Eqn.(iii),

$$\tan\phi = \frac{\sqrt{3}\,(W_1 - W_2)}{(W_1 + W_2)}$$

$$= \frac{\sqrt{3}\,(4200 - 800)}{4200 + 800}$$

$$= 1.177$$

Referring to the Trigonometrical tables,

$\phi = 49.6°$

$\therefore \quad$ Power Factor, $\phi = \cos 49.6° =$ **0.65**

$\therefore \quad$ Total Power, $P = \sqrt{3}\, E_L\, I_L \cos\phi$

or $\quad 5000 = \sqrt{3} \times 440 \times I_L \times 0.65$

$$\therefore \quad I_L = \frac{5000}{\sqrt{3} \times 440 \times 0.65}$$

$$= \mathbf{10.1\ A}$$

Problem 4.12

A 440 V, 3-phase A.C. motor has an output of 80 H.P and operates at a power factor of 0.866 with an efficiency of 90 %. Calculate,

i) The current in each phase of the motor if the motor is delta connected

ii) The readings of the two wattmeters connected in the lines to measure the input power.

Solution :

Output = 80 h.p.= (80 × 0.746) = 59.68 kW

$$\therefore \quad \text{Motor input} = \frac{\text{Output}}{\text{efficiency}} = \frac{59.68}{0.90} = 66.3\ \text{kW}$$

$\cos \phi = 0.866$, so $\phi = 30°$

Total power, $P = 66300\ \text{W} = \sqrt{3}\ E_L\ I_L \cos \phi$

$$= \sqrt{3} \times 440 \times 0.866 \times I_L$$

$$\therefore \quad I_L = \frac{66300}{\sqrt{3} \times 440 \times 0.866}$$

$$= 100.5\ \text{A}$$

i) Current in each phase, $I_P = \frac{I_L}{\sqrt{3}} = \frac{100.5}{\sqrt{3}} = \mathbf{58\ A}$

Using the formula (assuming a lagging power factor),

$$\tan \phi = \frac{\sqrt{3}\,(W_1 - W_2)}{(W_1 + W_2)}$$

Now the total power = $(W_1 + W_2)$ = 66.3 kW

$$\therefore \quad \tan 30° = \frac{\sqrt{3}\,(W_1 - W_2)}{66.3}$$

$$\text{or} \quad 0.577 = \frac{\sqrt{3}\,(W_1 - W_2)}{66.3}$$

or $(W_1 - W_2) = 22.1$ kW

Now $W_1 + W_2 = 66.3$ kW

$2W_1 = 88.4$

$W_1 = \mathbf{44.2\ kW}$ and $W_2 = \mathbf{22.1\ kW}$

Problem 4.13

Three coils each of impedance of 20 ∠60° are connected in star to 400 V, 3-Phase, 50 Hz supply. Find the reading on each of the two wattmeters connected to measure the power input. **(Mar 94, B.U.)**

Solution :

Given $Z_p = 20$, $\phi = 60°$ and $E_L = 400$ V

$$\therefore \quad R_p = Z_p \cos\phi = 20 \times \cos 60° = 10$$

$$E_p = \frac{E_L}{\sqrt{3}} = \frac{400}{\sqrt{3}} = 231 \text{ V}$$

$$I_p = \frac{E_p}{Z_p} = \frac{231}{20} = 11.55 \text{ A} = I_L$$

$$\text{Total power taken} = \sqrt{3}\, E_L\, I_L \cos\phi$$

$$= \sqrt{3} \times 400 \times 11.55 \times 0.5$$

$$= 4000 \text{ W}$$

i.e., $\quad W_1 + W_2 = 4000$

Now $\quad \tan\phi = \dfrac{\sqrt{3}\,(W_1 - W_2)}{(W_1 + W_2)}$

$$\tan 60° = \frac{\sqrt{3}\,(W_1 - W_2)}{4000}$$

$$\sqrt{3} = \frac{\sqrt{3}\,(W_1 - W_2)}{4000}$$

$\therefore \quad W_1 - W_2 = 4000$

Now $\quad W_1 + W_2 = 4000$

$\therefore \quad W_2 = \mathbf{0\ W}$ and $W_1 = \mathbf{4000\ W}$

Problem 4.14

Two wattmeters are used to measure the power in a 3-phase balanced system. What is the power factor when

i) both the meters read equal,
ii) both the meters read equal, but one is negative,
iii) one reads twice the other,
iv) one of the meters reads zero.

(Aug/Sep 90, M.U.)

Solution :

Case 1 : Both meters read equal.

Equation (iii) of Sec. 4.9 gives

$$\tan\phi = \frac{\sqrt{3}\,(W_1 - W_2)}{(W_1 + W_2)}$$

Here, $W_1 = W_2$

$\therefore \quad \tan\phi = 0$

$\therefore \quad \phi = 0°$

$\therefore \quad$ Power Factor, $\cos\phi = \cos 0° = 1$

Case 2 : Both the meters read equal, but one is negative.

Let W_2 read negative, in which case equation (iv) of Sec 4.9 gives

$$\tan\phi = \frac{\sqrt{3}\,(W_1 + W_2)}{(W_1 - W_2)}$$

As both the metres read equal.

$$\tan\phi = \sqrt{3}\,\frac{W_1 + W_1}{W_1 - W_1} = \frac{\sqrt{3} \times 2W_1}{0} = \infty$$

$\therefore \quad \phi = 90°$

Power Factor, $\cos\phi = \cos 90° = \mathbf{0}$

Case 3 : One reads twice the other

We take the equation

$$\tan\phi = \sqrt{3}\,\frac{(W_1 - W_2)}{(W_1 + W_2)}$$

In this case, $W_1 = 2W_2$

Substituting in the eqn.(i) above

$$\tan\phi = \frac{\sqrt{3}\,(2W_2 - W_2)}{2W_2 + W_2}$$

$$= \frac{\sqrt{3} \times W_2}{3W_2}$$

$$= 0.577$$

$\therefore \quad \phi = 30°$

$\therefore \quad$ Power Factor, $\cos\phi = \cos 30°$

$= \mathbf{0.866}$

Case 4 :

Let W_2 read zero

Now $$\tan\phi = \frac{\sqrt{3}\,(W_1 - W_2)}{(W_1 + W_2)}$$

$$\therefore \quad \tan\phi = \frac{\sqrt{3}\,W_1}{W_1} = \sqrt{3}$$

$\therefore \quad \phi = 60°$

$\therefore$ Power factor, $\cos\phi$ = **0.5**

Problem 4.15

Each of the two wattmeters connected to measure the input to a 3-phase circuit reads 10 kW on a balanced load when the power factor is unity. What does each instrument read when the power factor falls to

i) 0.866 lagging,

ii) 0.5 lagging, the total 3-phase power remaining unchanged.

(Aug. 95, B.U., Mar. 91, Mysore U.)

Solution :

i) When the power factor is unity.

$\cos\phi = 1$ or $\phi = 0$

$\therefore \quad \tan\phi = 0$

Thus, in the equation,

$$\tan\phi = \frac{\sqrt{3}\,(W_1 - W_2)}{W_1 + W_2} \qquad \text{---(i)}$$

or $$0 = \frac{\sqrt{3}\,(W_1 - W_2)}{W_1 + W_2}$$

or $W_1 - W_2 = 0$

or $W_1 = W_2 = 10$ kW (given)

$\therefore$ The total power $W_1 + W_2 = 20$ kW

ii) Power Factor is 0.866 lagging

$\cos\phi = 0.866$

$\therefore \quad \phi = 30°$ and so $\tan\phi = 0.5774$

Substituting the values in the equation (i) above,

$$0.5774 = \frac{\sqrt{3}\,(W_1 - W_2)}{(W_1 + W_2)}$$

As total power remains unchanged,

$$W_1 + W_2 = 20 \text{ kW}$$

$$\therefore \quad 0.5774 = \frac{\sqrt{3}\,(W_1 - W_2)}{20}$$

$$\therefore \quad (W_1 - W_2) = 6.6 \qquad \text{---(ii)}$$

$$W_1 + W_2 = 20 \qquad \text{---(iii)}$$

From the above equations (ii) and (iii),

$$W_1 = \mathbf{13.33\ kW}$$

and $W_2 = \mathbf{6.67\ kW}$

iii) Power Factor is 0.5 lagging

$$\cos\phi = 0.5$$

$$\therefore \quad \phi = 60°$$

$$\therefore \quad \tan\phi = \tan 60° = 1.732$$

Again substituting in equation (i) above,

$$1.732 = \frac{\sqrt{3}\,(W_1 - W_2)}{20}$$

$$\therefore \quad W_1 - W_2 = 20$$

$$W_1 + W_2 = 20$$

$$\therefore \quad 2W_1 = 40$$

$$\therefore \quad W_1 = \mathbf{20\ kW}$$

and $W_2 = \mathbf{0\ \ kW}$

Problem 4.16

A 500 V, 3-phase motor has an output of 50 H.P and operates at a power factor of 0.85 with an efficiency of 90 %. Calculate the reading on each of the two wattmeters connected to measure the input.

Solution :

Motor output = 50 H.P.

1 H.P. = 746 W

$\therefore$ 50 H.P = 50 × 746 = 37300 W

Given efficiency = 90 % = 0.90

$$\therefore \quad \text{Motor input} = \frac{37300}{0.90}$$

$$= 41444 \text{ W}$$

$$= \mathbf{41.444\ kW}$$

$\therefore \quad W_1 + W_2 = 41.444$ KW

$\cos\phi = 0.85$; $\phi = 31.8°$

$\tan\phi = 0.62$

Using the formula : $\tan\phi = \sqrt{3}\,\dfrac{W_1 - W_2}{W_1 + W_2}$

or $\quad 0.62 = \sqrt{3}\,\dfrac{W_1 - W_2}{41.444}$

$\therefore \quad W_1 - W_2 = 14.83$

But $\quad W_1 + W_2 = 41.444$

$2W_1 \quad = 56.274$

$\therefore \quad W_1 =$ **29.137 kW** and $W_2 =$ **13.307 kW**

Problem 4.17

Two wattmeters connected in a balanced system indicate 4.5 kW and -0.5 kW. What is the total power and power factor of the circuit ?

Solution :

Given $W_1 = 4.5$ kW and $W_2 = -0.5$ kW

Now, $\quad \tan\phi = \dfrac{\sqrt{3}\,(W_1 - W_2)}{(W_1 + W_2)}$

or $\quad \tan\phi = \dfrac{\sqrt{3}\,(4.5 + 0.5)}{(4.5 - 0.5)}$

$$= \sqrt{3} \times \left(\frac{5}{4}\right) = 2.1651$$

$\therefore \quad \phi = 65.2087°$

Thus p.f. $= \cos\phi =$ **0.4193**

Total Power $= W_1 + W_2 = 4.5 + (-0.5)$

$= $ **4 kW**

Problem 4.18

Two wattmeters connected to measure the input to a balanced 3-phase circuit indicate 2500 W and 500 W, respectively. Find the power and p.f. of the circuit (a) when both readings are positive and (b) when the latter reading is obtained after reversing the connections to the current coil.

Solution :

Given $W_1 = 2500$ W and $W_2 = 500$ W

a) **When both readings are positive**

Now, 3-phase Power $P = W_1 + W_2$

$= 2500 + 500$

$= 3000$ W $=$ **3 kW**

$$\text{Also, } \tan\phi = \frac{\sqrt{3}\,(W_1 - W_2)}{(W_1 + W_2)}$$

$$= \frac{\sqrt{3}\,(2500 - 500)}{3000} = \frac{\sqrt{3} \times 2000}{3000}$$

$$= 1.1547$$

$\therefore \quad \phi = 49.1066°$

$\therefore \quad$ p.f. $= \cos\phi = \cos 49.1066° =$ **0.6547**

b) $W_1 = 2500$ W, $W_2 = -500$ W, as the reading is taken after reversing the current coil connections

$\therefore \quad$ Power $P = W_1 + W_2 = 2500 - 500$

$= 2000$ W $=$ **2 kW**

$$\text{Also, } \tan\phi = \frac{\sqrt{3}\,(W_1 - W_2)}{W_1 + W_2} = \frac{\sqrt{3}\,(2500 + 500)}{2000}$$

$$= \frac{\sqrt{3} \times 3000}{2000} = 2.5981$$

$\therefore \quad \phi = 68.9483°$

$\therefore \quad$ Power Factor, $\cos\phi = \cos 68.9483 =$ **0.3592**

Problem 4.19

A 3-phase, 400 V motor takes an input of 40 kW at 0.45 p.f. lagging. Find the reading of the two single phase wattmeters connected to measure the input.

Solution :

Given $E_L = 400$ Volts and Power Input $= 40$ kW

p.f. $= \cos\phi = 0.45$ lagging

Let W_1 and W_2 be the two wattmeter readings.

$\therefore \quad$ Input power $= W_1 + W_2 = 40$ kW (given) ---(i)

Now, given $\cos\phi = 0.45$

$\therefore \quad \phi = 63.2563°$

$\therefore \quad \tan\phi = 1.9845$

Now, $\tan\phi = \dfrac{\sqrt{3}(W_1 - W_2)}{W_1 + W_2}$

or $1.9845 = \dfrac{\sqrt{3}(W_1 - W_2)}{40}$

or $W_1 - W_2 = \dfrac{40 \times 1.9845}{\sqrt{3}} = 45.83$ kW ---(ii)

Adding (i) and (ii), we have

$2W_1 = 85.83$ or $W_1 = \mathbf{42.915\ kW}$

$\therefore\ W_2 = 40 - W_1 = 40 - 42.915 = \mathbf{-2.915\ kW}$

Problem 4.20

Each branch of a 3-phase star connected load consists of a coil of resistance 4.2 ohms and reactance 5.6 ohms. The load is supplied at a line voltage of 415 V, 50 Hz. The total power supplied to the load is measured by the two wattmeter method. Find the wattmeter readings.

Solution :

Given $E_L = 415$ V, $\phi = 50$ Hz

Impedance per phase, $Z_P = R + jX_L$

$= (4.2 + j5.6)$ ohms

Let W_1 and W_2 be the two wattmeter readings.

For a star-connected system, we have

$$E_P = \frac{E_L}{\sqrt{3}} = \frac{415}{\sqrt{3}} = 239.6 \text{ volts}$$

Phase current, $I_P = \dfrac{E_P}{Z_P}$

Magnitude of $Z_P = \sqrt{4.2^2 + 5.6^2} = 7$ ohms

$$\therefore\quad I_P = \frac{239.6}{7} = 34.23 \text{ A}$$

Line current $I_L = I_P = 34.23$ A

Power Factor, $\cos\phi = \dfrac{R}{Z} = \dfrac{4.2}{7} = 0.6$

$$\therefore\quad \phi = 53.13°$$

$\therefore$ Power Input, $P = \sqrt{3}\, E_L\, I_L \cos\phi$

$$= \sqrt{3} \times 415 \times 34.23 \times 0.6$$

$$= 14.76 \text{ kW}$$

$\therefore$ $W_1 + W_2 = 14.76 \text{ kW}$ ---(i)

Now, $\tan\phi = \tan 53.13° = 1.33$

$$= \frac{\sqrt{3}(W_1 - W_2)}{W_1 + W_2}$$

or $$1.33 = \frac{\sqrt{3}(W_1 - W_2)}{14.76}$$

or $$W_1 - W_2 = \frac{1.33 \times 14.76}{\sqrt{3}} = 11.33 \text{ kW} \quad \text{---(ii)}$$

Solving eqns.(i) and (ii) we have

W_1 **= 13.33 kW** and W_2 **= 1.43 kW**

Problem 4.21

The power input to a 2000 V, 50 Hz, 3-phase motor running on full load at an efficiency of 90 % is measured by two wattmeters which indicate 300 kW and 100 kW respectively. Calculate (i) the input, (ii) the power factor, (iii) the line current and (iv) H.P. Output.

(Mysore, Feb. 84)

Solution : Given $W_1 = 300$ kW and $W_2 = 100$ kW, $E_L = 2000$ V, $f = 50$ Hz, Full-Load Efficiency = 90 %

i) Power Input to the motor = $W_1 + W_2$

i.e., $P = 300 + 100 =$ **400 kW**

ii) Now, $$\tan\phi = \frac{\sqrt{3}\,(W_1 - W_2)}{W_1 + W_2}$$

$$= \frac{\sqrt{3}\,(300 - 100)}{300 + 100} = 0.866$$

$\therefore$ $\phi = \tan^{-1} 0.866 = 40.9°$

$\therefore$ Power Factor, $\cos\phi =$ **0.7559**

iii) 3-Phase power $P = \sqrt{3}\, E_L\, I_L \cos\phi$

$$\text{or} \quad I_L = \frac{400 \times 10^3}{\sqrt{3} \times 2000 \times 0.7559} = \mathbf{152.76\ A}$$

iv) Output of the motor = Input × efficiency

$$= 400 \times 0.9 \text{ kW}$$

$$= \frac{400 \times 0.9}{0.7355} \text{ HP (metric)}$$

$$= \mathbf{489.46\ H.P}$$

Problem 4.22

A 440 V, 3-phase a.c. motor has an output of 80 H.P with an efficiency of 90% and power factor 0.866. Calculate (i) the current in each phase of the motor, if the motor is delta-connected, (ii) the readings of two wattmeters connected in the lines to measure the input power. (B.U., Jan 93)

Solution :

Given E_L = 440 V, Motor Output = 80 HP

Motor Efficiency = 90 %; p.f. = 0.866

i) Motor output = 80 HP = 80 × 735.5 W

$$\therefore \quad \text{Input to the motor} = \frac{\text{Motor Output}}{\text{Efficiency}} = \frac{80 \times 735.5}{0.9}$$

$$= 65{,}377.78 \text{ Watts}$$

Power input to 3-phase motor is given as

$$P = \sqrt{3}\, E_L\, I_L \cos\phi$$

$$\text{or} \quad \text{Line current } I_L = \frac{P}{\sqrt{3} \times E_L \times \cos\phi} = \frac{65{,}377.78}{\sqrt{3} \times 440 \times 0.866}$$

$$= 99 \text{ A}$$

As the motor is delta connected, phase current $I_P = \dfrac{I_L}{\sqrt{3}} = \dfrac{99}{\sqrt{3}} = 57.2$ A

ii) If W_1 and W_2 are the readings of the wattmeters connected in the lines to measure the input power, we have

Input power = $W_1 + W_2$ = 65,377.78 watts ---(i)

Power Factor $\cos\phi$ = 0.866

$\therefore \quad \phi = 30°$ and $\tan\phi = \tan 30° = 0.5774$

$$\text{Now,} \quad \tan\phi = \frac{\sqrt{3}(W_1 - W_2)}{W_1 + W_2}$$

or $$0.5774 = \frac{\sqrt{3}\,(W_1 - W_2)}{65,377.78}$$

or $$W_1 - W_2 = 21,794.5$$

Solving eqns.(i) and (ii) we have

$$W_1 = \mathbf{43586.14 \text{ watts}}$$

$$W_2 = \mathbf{21,791.64 \text{ watts}}$$

These are the readings of the two wattmeters.

Problem 4.23

The power input to a 3-phase Induction Motor running on 400 V, supply was measured by two wattmeter method, and the readings 3000 W and −1000 W. Calculate (i) Total Input Power (ii) p.f. (iii) current. (B.U. Jul

Solution :

Given $E_L = 400$ V, $f = 50$ Hz, $W_1 = 3000$ W and $W_2 = -1000$ W

i) Total 3-phase power $P = W_1 + W_2$

$$= 3000 - 1000$$

$$= \mathbf{2000 \text{ Watts}}$$

ii) $$\tan\phi = \frac{\sqrt{3}(W_1 - W_2)}{W_1 + W_2}$$

$$= \frac{\sqrt{3}\,[3000 - (-1000)]}{3000 + (-1000)}$$

$$= \frac{\sqrt{3} \times 4000}{2000} = 3.464$$

$\therefore$ $\phi = \tan^{-1} 3.464 = 73.9°$

$\therefore$ Power Factor, $\cos\phi = \cos 73.9° = \mathbf{0.2773}$

iii) Now, $P = \sqrt{3}\, E_L\, I_L \cos\phi$

$\therefore$ Line Current $$I_L = \frac{P}{\sqrt{3}\, E_L \cos\phi}$$

$$= \frac{2000}{\sqrt{3} \times 400 \times 0.2773}$$

$$= \mathbf{10.4 \text{ A}}$$

4.11 Review Questions

Q1. Show that in a 3-phase star-connected system, the line voltage is $\sqrt{3}$ times the phase voltage. (May/June 86, B.U.)

Q2. Establish the relationship between phase and line values of voltages and currents in 3-phase star and delta-connected circuits. Show the phasor diagram neatly.
(June 81, B.U; Apr/May 87, M.U; Mar/Apr 88, Feb/Mar 90, M.U; July 93, B.U;)

Q3. When do we say that the system of an a.c 3-phase voltage is a balanced 3-phase system ?

Q4. Derive an expression for power in a 3-phase balanced circuit.
(Apr/May 87, M.U.; Mar/Apr 88, M.U.)

Q5. Show that the power in a balanced 3-phase circuit can be measured by two wattmeters. Draw the circuit and vector diagrams.
(Aug 85, B.U; May/June 86, B.U; Apr/May 87, M.U; Mar 89 & Aug/Sep89, M.U.; June/July 90, B.U; Jan 93, B.U; Aug 94, Mar 95, Aug 95, B.U)

Q6. Write a short note on advantages of 3-phase systems. (Aug 95, B.U.)

Q7. Obtain an expression for power in a 3-phase system connected in star in terms of line voltage, line current and phase angle. (Apr 97, B.U.)

Q8. With relevant diagrams show that two wattmeters are enough to measure three-phase power. (Mar 99, V.T.U)

Q9. Draw the circuit diagram and vector diagram for the measurement of power in a 3-phase star-connected system using two wattmeters.
(Aug/Sep 99, V.T.U)

4.12 Exercises - Problems

1. A balanced Star-connected load is supplied from a balanced 3-phase, 400 V, 50 Hz system. The current in each phase is 30 A and lags 30° behind the phase voltage. Find (a) total power (b) phase voltage. Draw the phasor diagram.
[June 81, B.U.]
Answer : (a) 18 kW (b) 230.94 V

2. A balanced, 3-phase, star-connected load of 150 kW takes a leading current of 100 A at 1100 V, 50 Hz. Find the parameters of the load per phase. [1987, KUD]
Answer : R = 5 Ohms; C = 813 mF

3. Three coils, each of resistance 6 Ohms and Inductive Reactance 8 Ohms are connected in Delta across 400 volts, 3-phase lines. Calculate the line current and the power absorbed. Take phase sequence *R-Y-B*. [1986, KUD]
Answer : 69.282 A ; 28.8 kW

4. Each phase of a 3-phase Star-connected load is a coil of resistance 20 ohms and inductance 0.05 H. Calculate the power drawn from a 400 V, 3-phase, 50 Hz sinusoidal supply. [Nov 84, KUD]
Answer : 4947.64 Watts

5. Three similar resistors are connected in star across 400 V, 3-phase lines. The line current is 10 A. Calculate (i) the value of each resistor (ii) the line voltage required to give the same line current if the resistors were connected in delta. [Apr 84, KUD]

Answer : (i) R = 23.094 Ohms (ii) 132.81 Volts

6. Three identical impedances are connected in delta to a 3-phase supply of 400 volts. The line current is 34.65 amps and the total power taken from the supply is 14.4 KW. Calculate the resistance and reactance values of the impedance. [May 89, KUD]

Answer : R = 12 Ohms ; Reactance = 16 Ohms

7. Three 100 Ω resistors are connected, first in Star and then in Delta, across a 415 V, 3-phase supply. Calculate the line and phase currents in each case, and also the power taken from the source. [Aug 82, B.U.]

Answer : Star Connection : I_L = 2.396 A, I_P = 2.396 A and P = 1722.25 W

Delta Connection : I_L = 7.188 A; I_P = 4.15 A and P = 5166.74 W

8. A balanced delta connected load of (8 + *j*6) Ohms per phase is connected to a 3-phase, 230 V supply. Find the line current, power factor, power, reactive volt-amperes and total volt-amperes [May 91, KUD]

Answer : 39.83 A ; 0.8 ; 12.69 kW ; 7.61 KVAR ; 14.8 KVA

9. A 3-phase Delta-connected load consumes a power of 200 kW, taking a lagging current of 200 A at a voltage of 2000 V, 40 Hz, between lines. Find the parameters of each phase. What would be the power consumed, if the load were connected in star?

Answer : R = 5 Ω ; L = 52.785 mH ; 66.6752 kW

10. When three equal impedances are connected in Delta across a balanced 3-phase, 400 V, 50 Hz supply, the line current drawn is 20 A at a lagging power factor of 0.3. Calculate the circuit constants. [Nov 87, Gulbarga]

Answer : R = 10.392 Ω ; L = 0.1052 H

11. Three identical coils, having a resistance of 10 ohms and an inductance of 0.05 H each, are connected in star across a 3-phase, 400 V, 50 Hz balanced supply. Calculate the line current and the power consumed. What will be the readings of the two wattmeters connected to measure the total power ? [1985, KUD]

Answer : 12.4 A ; 4613.35 W ; W_1 = 214.48 W ; W_2 = 4398.87 W

12. The power input to a 3-phase, balanced, star-connected series *R-L* circuit is measured by the two-wattmeter method. When the supply voltage is 400 V, 50 Hz, the readings of the two wattmeters are 5200 W and 1800 W respectively. Determine the power, power factor, line current, *R* and *L*. [Nov 91, KUD]

Answer : 7 kW ; 0.7652 ; 13.2 A ; 13.39 Ohms, 35.86 mH.

13. Two wattmeters are used to measure the power input to a 3-phase Induction Motor and the readings are 10 kW and 2.5 kW, and the latter reading is obtained after the reversal of the current coil terminals. Calculate (i) Power and Power Factor (ii) the Line Current at 400 V supply (iii) If the two wattmeter readings are the same, what does it mean regarding power factor of the load ? [Nov 90, KUD]

Answer : (i) 7.5 kW, 0.3273 (ii) 33 A (iii) Unity P.F,

14. Three equal star-connected inductors take 8.kW at 0.8 p.f. lag when connected to 460 V, 3-phase, 3-wire supply. Calculate the readings of the two wattmeters used to measure the power. **Answer : W_1 = 2.268 kW ; W_2 = 5.732 kW**

15. The input power to a 3-phase motor was measured by the two wattmeter method. The readings are 5200 W and −1700 W and voltage was 400 V. Calculate the total power, power factor and line current. [Nov 89, KUD]

Answer : 3500 W ; 0.2811 ; 17.97 A

16. A 3-phase, 400 V motor has an output of 50 HP. It operates with an efficiency of 90 % and a power factor of 0.8. Find the readings of two wattmeters connected to measure the input power. **Answer : 11.584 kW and 29.277 kW**

17. A 3-phase, 440 V motor has a p.f. of 0.8. Two wattmeters are used to measure the power input of 35 kW. Find the readings of the wattmeters.

Answer : 25 kW and 10 kW

18. Three coils, each of impedance 25 $\angle -60°$ ohms are connected in Delta across a 400 V, 3-phase, 50 Hz supply. Find :
    i) The current in each phase
    ii) The line current
    iii) Total power consumed.
    iv) Draw the phasor diagram. [Apr 97, B.U.]

19. A balanced Star-connected load each having resistance of 10 ohms and inductive reactance of 30 ohms is connected to a 400 V, 50 Hz, 3-phase supply. Two wattmeters are connected to measure the power. Determine the reading of each wattmeter. Draw the circuit diagram and phasor diagram. [Apr 97, B.U.]

Answer : 2190 W, −583 W

MODULE

4

(b) Three Phase Synchronous Generator

4.13 Basic Principle of Operation

The most commonly used machine for generation of alternating current power for commercial as well as for domestic purposes is the **synchronous generator**, which is also called an a.c. generator or alternator. It operates on the same fundamental principle of electromagnetic induction as a d.c. generator i.e., *when a conductor moves across a magnetic field or vice-versa an e.m.f. is induced in the conductor.*

Just as in the case of a d.c. generator, the synchronous generator also has an armature winding and a field winding. However, the two major differences between the two are as follows :

(a) The armature winding of a synchronous generator is an a.c. winding, i.e., it is connected to produce a.c. supply, and the field is connected to d.c. supply. Therefore, no commutator is required in an alternator, which makes its construction simpler than that of a d.c. generator.

(b) In a d.c. generator, the armature winding rotates and the field system is stationary, whereas in the case of a synchronous generator, the armature winding is mounted on a stationary element called the **stator** and the field windings on a rotating element called the **rotor**. A simple sketch giving the basic construction of a synchronous generator is given in Fig 4.14.

The stator consists of a cast-iron frame which houses the armature core having slots on its inner periphery for accommodating the armature conductors. The rotor resembles a flywheel with alternate N & S poles fixed on its outer rim. The magnetic poles are excited (or magnetised) by the direct current supplied by a d.c. source of 125 or 250 V. Often, the exciting (or magnetising) current is obtained from a small d.c. shunt generator which is mounted on the shaft of the synchronous generator itself. As the field magnets are rotating, this current is supplied through two slip-rings.

When the rotor is rotated by the prime-mover, the stator windings or conductors are cut by the magnetic flux of the rotor poles, hence an e.m.f. is induced in the stator conductors. The rotor poles being alternately North and South, they induce an alternating e.m.f. The frequency of this induced e.m.f. is given by $f = \frac{NP}{120}$ Hz, where 'P' is the number of poles and 'N' is the speed in revolutions per minute

and its diection can be found by using Fleming's Right-hand rule. The e.m.f. generated in the stator conductors is taken out from 3 leads connected to the stator windings as shown in Fig 4.14.

4.14 Advantages of Stationary Armature (and rotating field)

The advantages of having a stationary armature and a rotating field system are:

1. The synchronous generator (or alternator) generates 11000 volts or 33000 volts in India, and therefore it is easier to insulate the stationary armature winding for such high a.c.voltages.
2. The output current can be passed directly from fixed terminals on the stator (or armature windings) without having to be taken out through brush contacts.
3. It is more easy to brace the armature windings against any deformation which could be produced by the mechanical stresses set up due to short circuit current and the large centrifugal forces produced during rotation, as the armature is stationary.
4. The field windings of the rotor are supplied with a D.C. voltage of 110 or 220 volts, generated by the pilot exciter, through two brushes which are set to slide on two slip rings fixed to the shaft of the synchronous generator. As this is a low-power, low-voltage d.c. circuit, it is easy to insulate these slip-rings.
5. The stator (armature) conductors are placed in slots and can be easily insulated for the high voltage generation.
6. Radical and axial ventilating ducts can be provided by enlarging the stator core of the armature. This ventilation and cooling arrangement can be easily made if stationary stator (armature) is used.

4.15 Constructional Features & Types of Rotors

The basic details of construction of an synchronous generator are shown in Fig. 4.14.

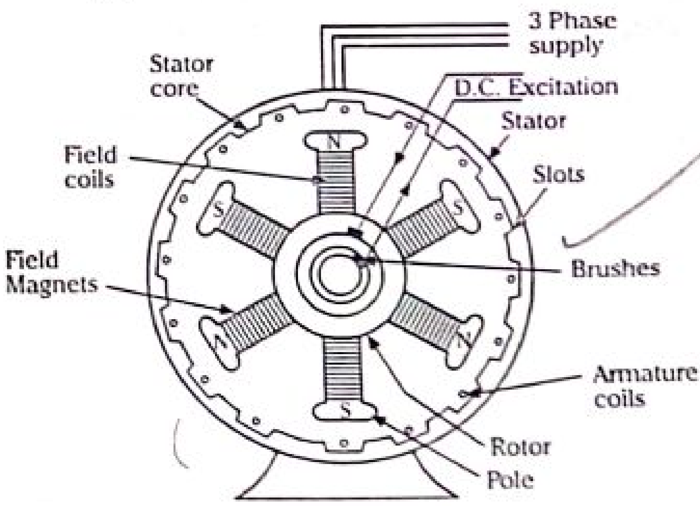


Fig. 4.14

1. Stator : The armature is an iron ring, formed of laminations of special magnetic iron or steel alloy (silicon steel), having slots on its inner periphery to accommodate armature conductors and is called the **stator**. The entire structure is held in a frame which is of cast-steel or welded steel plates. As the field rotates in-between the stator, the flux of the rotating field cuts

the core of the stator continuously and causes eddy current losses in the stator core. The stator core is laminated to minimize these losses due to eddy currents. The laminations are stamped out in complete rings (for smaller machines) or in segments (for larger machines), and are insulated from each other with paper or varnish. The stampings also have openings which make axial and radial ventilating ducts provide efficient cooling.

Slots provided on the stator are of three types : (i) Wide open type slot, (ii) semi-closed type slot and (iii) wholly closed type slot, as shown in Figs. 4.15(a), 4.15(b), and 4.15(c) respectively.

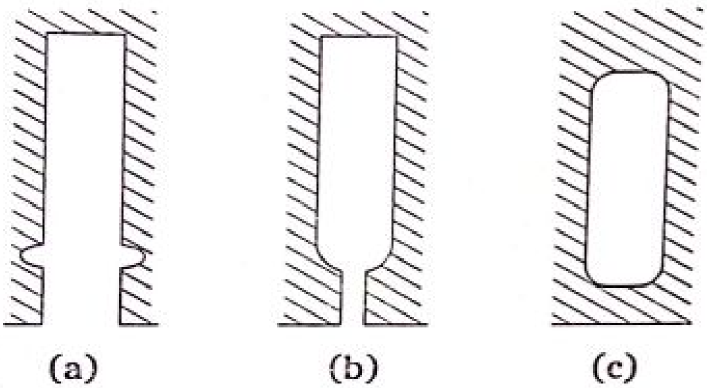


Fig. 4.15

The wide-open slots are more commonly used because the coils can be form-wound and insulated prior to being placed in the slots, ensuring a more satisfactory winding method. This type of slots also facilitate easy removal and replacement of defective coils. However, this type of slots suffer from the disadvantage of distributing the air-gap flux into tufts or branches that produce ripples in the wave of generated e.m.f. The semi-closed type of slots are better in this respect, but do not allow the use of form-wound coils. The wholly closed type of slots do not disturb the air-gap flux but (i) they tend to increase the inductance of the windings, (ii) it is necessary to thread through armature conductors, thus increasing initial labour and cost of winding and (iii) the problem of end connection crops up. For these reasons, these wholly-closed slots are seldom used.

2. Rotor : The field system is similar to that of a d.c. generator, which is excited from a separate source of 125 or 250 V d.c. supply. The excitation is usually provided from a small d.c. shunt or compound generator known as an exciter, mounted on the shaft of the synchronous generator itself. The field system of the alternator is rotated within the armature ring and is known as rotor. The exciting current is supplied to the rotor through two slip-rings and brushes. The polarities of the field produced is alternately North and South. The power rating of the exciter is ordinarily 0.3 to 1% of power rating of the synchronous generator.

Rotors are of two types, namely

i) Salient-pole type.

ii) Smooth cylindrical type or non-salient pole type.

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i) Salient Pole Type : The salient pole type rotors are used in low and medium-speed alternators (120 to 400 r.p.m.) because of the following reasons :

a) The salient field poles would cause an excessive windage loss if driven at high speed and would tend to produce noise.

b) Salient pole construction cannot be made strong enough to withstand the mechanical stresses to which they may be subjected at higher speeds.

In this, the rotor is like a flywheel and a number of alternate North and South poles are bolted to it as shown in Fig. 4.16. The salient or projecting poles are made of thick steel laminations, rivetted together, and are fixed to the rotor by a dove-tail joint. The pole faces are usually provided with slots for damper windings. These dampers are useful in preventing hunting. The pole faces are so shaped that the radial air-gap length increases from the pole centre to the pole tips so that the flux distribution over the armature is sinusoidal and the waveform of generated e.m.f. is sinusoidal. The field coils are placed on the pole pieces and connected in series. The ends of the field windings are connected to a d.c. source through slip-rings carrying brushes and mounted on the shaft of the field structure.

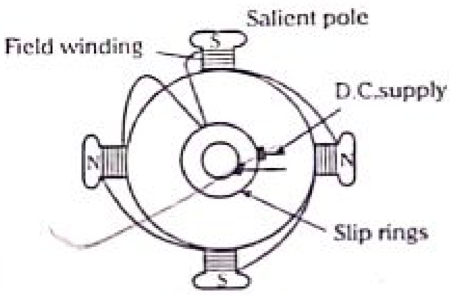


Fig. 4.16

The salient pole field structure has the following special features :

i) They have large diameter and short axial length.

ii) The poles and pole-shoes are laminated to minimize heating due to eddy currents.

iii) The pole-shoes cover about 2/3 of the pole pitch.

iv) These are used with hydraulic turbines or diesel engines.

ii) Smooth Cylindrical Type or Non-Salient Pole Type :

The rotors of this type are used in very high speed turbo alternators (synchronous generators driven by steam turbines).

The rotor consists of a smooth solid forged steel cylinder having a number of slots milled out at intervals along the outer periphery (and parallel to the shaft) for accommodating field coils. Such rotors could be designed for 2-pole (or 4-pole) turbo-generators running at 3600 r.p.m. (or 1800 r.p.m.) The rotor of a 4-pole turbo-generator is shown in Fig. 4.17. The regions forming the central polar areas are left unslotted, as shown in Fig. 4.17.

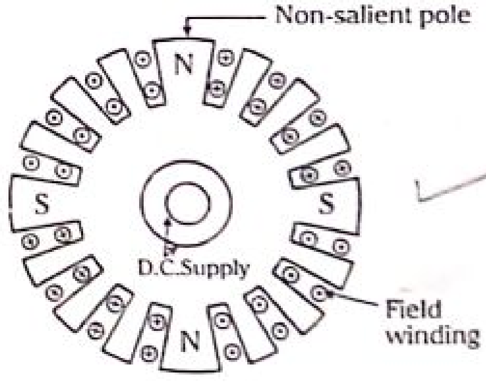


Fig. 4.17

Field windings occupy the slots as shown. They are so arranged around the central polar areas that flux density is maximum on the central polar area and

gradually reduces on either side. It is clear that the poles are non-salient, *i.e.*, they do not project out from the surface of the rotor.

In order to ensure that the peripheral velocity is not excessive, the diameters of such rotors have to be very small (about 1 metre); however, the axial length has to be very long, for better dynamic balance and stable operation. Windage losses are also reduced.

As in the earlier case of the salient pole type of rotor, d.c. excitation to the field windings is provided through slip-rings and brushes.

4.16 Concept of Winding Factors

There are two factors related to the armature winding of a synchronous generator which cause a slight reduction in the generation of e.m.f. These factors are the *Pitch Factor* and the *Distribution Factor*, which are covered below in some detail.

4.16.1 Pitch Factor

Referring to Fig. 4.18, the coil sides occupying slot nos. 1 and 7 comprise a full-pitched winding. However, if these coil sides occupy slots 1 and 6, then we have a short-pitched winding as the coil span is equal to 5/6 of a pole pitch. Put in another way, the winding falls short by 1/6 pole-pitch or by 180°/6 = 30°.

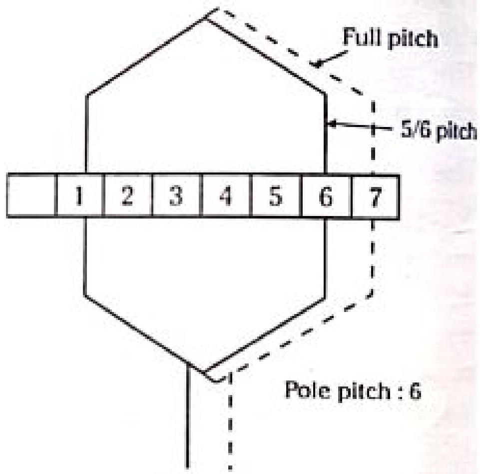
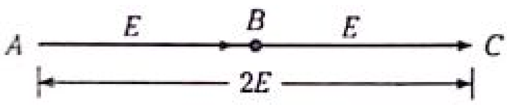


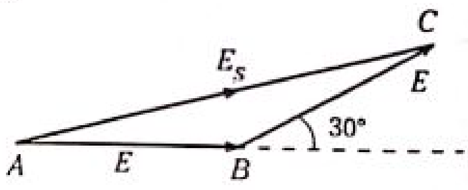
Fig. 4.18

We have seen that, in the case of a full-pitched winding, the total e.m.f. induced in a coil is the direct sum of the e.m.f. induced in its two sides.

Referring to Fig. 4.19(a), the total e.m.f. induced in the full-pitched coil = $2E$, where 'E' is the e.m.f. induced in each coil-side. However, in a coil which is short-pitched by 30° (electrical), then, as shown in Fig. 4.19(b), their resultant is E_s, which is the vector sum of the two voltages 30° (electrical) apart.



(a)



(b)

Fig. 4.19

$$\therefore \quad E_S = 2E\cos\frac{30°}{2} = 2E\cos 15°$$

$$\therefore \quad \text{Pitch Factor} \quad k_c = \frac{\text{vector sum}}{\text{arithmetic sum}}$$

$$= \frac{E_S}{2E} = \frac{2E\cos 15°}{2E}$$

$$= \cos 15° = 0.966$$

Thus, we may enunicate a general rule that, *if the coil span falls short of full-pitch by an angle α (electrical), then* **Pitch Factor $k_c = \cos\frac{\alpha}{2}$**.

Advantages of Short-pitched Windings.

1. We are able to obtain a better waveform of the generated e.m.f., as it can be made to have a shape very similar to that of a sinusoidal wave, thereby reducing, or entirely eliminating, distorting harmonics.
2. As high-frequency harmonics are either reduced or entirely removed, eddy current and hysteresis losses are reduced, enhancing efficiency.
3. Less copper is used.

4.16.2 Distribution Factor

In each phase of an armature winding, the coils are not placed together or concentrated in one slot, but are distributed in other slots; hence the coils occupying these slots are displaced from each other by a particular angle. As a result, the e.m.fs induced in the coils sides are not in phase with each other but differ by an angle equal to the angular displacement of the slots. The total e.m.f. is less than what it would have been if the coils had been concentrated in one slot.

It is, therefore, necessary to introduce a correction factor k_d, which is less than unity. Fig. 4.20 gives the vector diagram for a coil group of four coils with a consecutive displacement of β electrical degrees. The e.m.f of the coil group is the closing side AB of the polygon formed by the e.m.f.s of the individual coils. If E_c is the e.m.f. per coil, then the e.m.f. of this group is

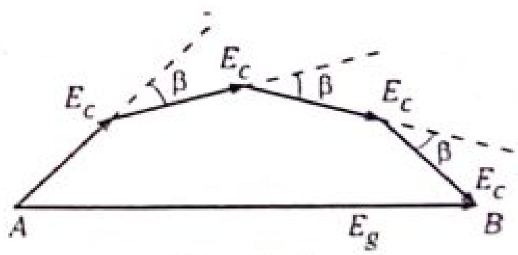


Fig. 4.20

$$E_g = E_c\frac{\sin\left(\frac{4\beta}{2}\right)}{\sin\left(\frac{\beta}{2}\right)}$$

If the coil e.m.f.s had all been in phase, the group e.m.f. would have been $4E_c$. Hence by distributing the winding, the e.m.f is reduced in the ratio

$$k_d = \frac{\sin\left(\frac{4\beta}{2}\right)}{4\sin\left(\frac{\beta}{2}\right)}$$

If we take a general case, where there are 'm' sections, then

$$k_d = \frac{\sin\left(\frac{m\beta}{2}\right)}{m\sin\left(\frac{\beta}{2}\right)}$$

k_d *is called the Distribution Factor.*

Advantages of Distributed winding (Disadvantages of Concentrated Winding)

A winding concentrated in one slot per pole per phase would have the following disadvantages :

(i) The size of such slots and the number of conductors per slot would be so great that it would be difficult to prevent the insulation on the conductors in the centre of the slot from becoming overheated, since most of the heat generated in the slots has to flow radially out to the steel core.

(ii) The waveform of the emf would be similar to that of the flux distribution around the inner periphery of the stator; and, in general *this would not be sinusoidal*.

4.17 Frequency of Generated e.m.f

Let us take an alternator whose rotor is being driven at a constant speed of 'N' rpm. Let the no. of poles = P, and the frequency of the generated e.m.f be = f.

In one complete revolution of the rotor, each of the 'N' and 'S' poles move past all the stator conductors. When one pair of 'N' and 'S' poles moves past on armature conductor, the e.m.f. induced in the conductor undergoes one full cycle. Therefore, in one full revolution of the field system, since ($P/2$) pairs of poles sweep past every armature conductor, the e.m.f induced undergoes $P/2$ cycles.

In one second, there are $N/60$ full revolutions of the rotor,

Therefore, the number of cycles of the induced e.m.f./second

= No. of cycles/revolution × No. of revolutions/sec.

$$= \frac{P}{2} \times \frac{N}{60} = \frac{NP}{120} \text{ cycles per sec}$$

i.e., Frequency of Generated e.m.f, $f = \dfrac{NP}{120}$ Hz

If n = Revolutions per second, then the above expression may be written as

$$f = \frac{nP}{2}$$

4.18 E.M.F. Equation

Let Z = No. of conductors or coil sides in series per phase

P = Number of poles

ϕ = Flux per pole (in webers)

f = Frequency of induced E.M.F. (in Hz)

N = Rotational speed of rotor (r.p.m.)

In one revolution of the rotor, each stator conductor is cut by $P\phi$ webers.

The time taken to complete one revolution is = 60/N second

$$\therefore \quad d\phi = P\phi \quad \text{and} \quad dt = \frac{60}{N} \text{ second}$$

So, average e.m.f. induced per conductor

$$= \frac{d\phi}{dt} = \frac{P\phi}{\frac{60}{N}} = \frac{\phi NP}{60} \text{ volt} \qquad \text{—(i)}$$

Frequency, $f = \dfrac{NP}{120}$ or $N = \dfrac{120f}{P}$

Substituting this value of N in (i), we get

Average e.m.f. induced per conductor $= \dfrac{\phi P}{60} \times \dfrac{120f}{P} = 2f\phi$ volts

For Z conductors in series/phase, we have

Average e.m.f induced/phase = $2f\phi Z$ volts ---(ii)

If T = No. of coils or turns per phase, then the no. of conductors, $Z = 2T$, which, when substituted in eqn (ii) gives

Average e.m.f. induced /phase = $4f\phi T$ volts

One factor that should be considered is the *Form Factor* (k_f) of the space distribution of flux, assumed to be sinusoidal, in which case its value is 1.11, *i.e.*, $\boldsymbol{k_f = 1.11}$.

$\therefore$ R.M.S. value of e.m.f induced/phase

= Form Factor (k_f) × Average e.m.f. induced/phase

= $1.11 \times 4f\phi T$ volts —(iii)

Had all the coils been full-pitched (instead of being short-pitched) and concentrated in one slot (instead of being distributed in several slots under poles) the expression (iii) above would have been the actual value of the induced voltage in a phase. However as this is not so, the induced e.m.f. is reduced in the ratio of the following two factors :

a) *Pitch Factor* $k_c = \cos \frac{\alpha}{2}$

b) *Distribution Factor,* $k_d = \dfrac{\sin\left(\frac{m\beta}{2}\right)}{m \sin\left(\frac{\beta}{2}\right)}$

∴ R.M.S. value of e.m.f induced/phase, actually available

$$= 4.44 \; k_c \; k_d \; f \, \phi \, T \text{ volts}$$

$$= 4 \; k_f \; k_c \; k_d \; f \, \phi \, T \text{ volts} \qquad (\because \; k_f = 1.11)$$

In the case of star-connected alternator, the line voltage is $\sqrt{3}$ times the phase voltage derived above.

4.19 Illustrative Examples on E.M.F. Equation

Problem 4.24

A 6-pole, 3-phase, 50 Hz alternator has 12 slots per pole and 4 conductors per slot. A flux of 25 mWb is sinusoidally distributed along the air-gap. Determine the line e.m.f if the alternator is star-connected.

Given : Winding Factor k_d = 0.96 ; Pitch Factor k_c = 1. (July 93, B.U.)

Solution :

Flux per pole = 25 mWb = 0.025 Wb.

Total No. of slots = Slots per pole × No. of poles = 12 × 6 = 72

∴ No. of slots per phase $= \frac{72}{3} = 24$

No. of conductors per slot = 4

∴ No. of conductors per phase = 4 × 24 = 96

∴ No. of turns per phase $T = \dfrac{\text{conductors per phase}}{2} = \dfrac{96}{2} = 48$

Pitch Factor $k_c = 1$

For sinusoidal distribution of flux,

Form Factor $k_f = 1.11$

∴ E.M.F. generated per phase,

$E_{ph} = 4 \, k_f \, k_c \, k_d \, f \, \phi \, T$ volts

$= 4 \times 1.11 \times 1 \times 0.96 \times 50 \times 0.025 \times 48$

$=$ **255.75 volts**

$\therefore$ Line E.M.F., $E_L = \sqrt{3}\ E_{ph} = \sqrt{3} \times 255.75$

$=$ **443 volts**

Problem 4.25

A 3-phase, 16-pole alternator has a star-connected winding with 144 slots and 10 conductors per slot. The flux per pole is 0.03 Wb and the speed is 375 r.p.m. Find the frequency and the phase and the line electromotive force. Given : Winding Factor k_d = 0.96 ; Pitch Factor k_c = 1.

(June 81, Aug. 94, Aug. 95, B.U.)

Solution :

Flux per pole, $\phi = 0.03$ Wb

Frequency $f = \dfrac{NP}{120} = \dfrac{375 \times 16}{120} = 50\,\text{Hz}$

No. of slots per phase $= \dfrac{144}{3} = 48$

No. of conductors per slot $= 10$

$\therefore$ No. of conductors/phase $= 48 \times 10 = 480$

$\therefore$ Turns per phase $T = \dfrac{\text{conductors per phase}}{2} = \dfrac{480}{2} = 240$

Assuming sinusoidal distribution of flux,

Form Factor, $k_f = 1.11$

$\therefore$ E.M.F. generated per phase

$E_{ph} = 4\ k_f\ k_c\ k_d\ f\ \phi\ T$ volts

$= 4 \times 1.11 \times 1 \times 0.96 \times 50 \times 0.03 \times 240$

$=$ **1534 volts**

$\therefore$ Line e.m.f. $= \sqrt{3} \times 1534 =$ **2657 volts**

Problem 4.26

A 12-pole, 500 RPM, star connected alternator has 60 slots with 20 conductors per slot. The flux per pole is 0.02 weber and is distributed sinusoidally. The winding factor is 0.93. Calculate (i) frequency, (ii) phase e.m.f., (iii) line e.m.f. Assume coil is full-pitched.

Given : Winding Factor k_d = 0.97. **(Jan 93, B.U.)**

Solution :

Flux per pole, ϕ = 0.02 weber

Frequency $f = \frac{NP}{120} = \frac{500 \times 12}{120} = \mathbf{50\ Hz}$

No. of slots per phase $= \frac{60}{3} = 20$

No. of conductors per slot = 20

$\therefore$ No. of conductors per phase = 20 × 20 = 400

$\therefore$ Turns per phase $T = \frac{\text{conductors per phase}}{2} = \frac{400}{2} = 200$

Assuming the coil is full-pitched, Pitch Factor $k_c = 1$
Assuming sinusoidal distribution of flux,

Form Factor, $k_f = 1.11$

$\therefore$ E.M.F. generated per phase,

$E_{ph} = 4\, k_f\, k_c\, k_d\, f\, \phi\, T$ volts

$= 4 \times 1.11 \times 1 \times 0.97 \times 50 \times 0.02 \times 200$

= 816 volts

$\therefore$ Line E.M.F. $= \sqrt{3} \times E_{ph} = \sqrt{3} \times 861 = \mathbf{1492\ V}$

Problem 4.27

A 6-pole, 3-phase, star-connected alternator has an armature with 90 slot and 8 conductors per slot, and revolves at 1000 r.p.m., the flux per pole bein 0.05 weber. Calculate the e.mf. generated if the winding factor is 0.97 and a the conductors in each phase are in series. (Aug, 83, B.U.

Solution :

Flux per pole , ϕ = 0.05 weber

Frequency, $f = \frac{NP}{120} = \frac{1000 \times 6}{120} = 50\,\text{Hz}$

Total number of slots = 90

Total number of conductors = 90 × 8 = 720

No. of conductors per phase $= \frac{720}{3} = 240$

No. of turns/phase, $T = \frac{240}{2} = 120$

Winding Factor, $k_d = 0.97$

Assuming full-pitched winding, Pitch Factor $k_c = 1$

For sinusoidal distribution of flux,

Form Factor, $k_f = 1.11$

$\therefore$ E.M.F. generated/phase,

$$E_{ph} = 4\, k_f\, k_c\, k_d\, f\, \phi\, T$$

$$= 4 \times 1.11 \times 1 \times 0.97 \times 50 \times 0.05 \times 120$$

$$= \mathbf{1292 \text{ volts}}$$

Line E.M.F., $E_L = \sqrt{3} \times E_{ph}$

$$= \sqrt{3} \times 1292 = \mathbf{2237.75 \text{ volts}}$$

Problem 4.28

A star-connected, 3-phase, 4-pole, 50 Hz alternator has a single layer winding in 24 slots. There are 50 turns in each coil and the flux per pole is 5×10^{-2} weber. Find the open circuit line voltage.

Given : Winding Factor $k_d = 0.966$. **(Mar 95, B.U.)**

Solution :

Assuming full pitched coil, Pitch Factor $k_c = 1$

Assuming sinusoidal e.m.f., Form Factor $k_f = 1.11$

$$\text{Total number of turns} = \frac{\text{No. of slots}}{2} \times \frac{\text{No. of turns}}{\text{coil}}$$

$$= \frac{24}{2} \times 50 = 600$$

$\therefore$ Total no. of turns/phase, $T = \dfrac{600}{3} = 200$

RMS value of voltage/phase

$$= 4\, k_f\, k_c\, k_d\, f \phi T \text{ volts}$$

$$= 4 \times 1.11 \times 1 \times 0.966 \times 50 \times \left(5 \times 10^{-2}\right) \times 200$$

$$= 2144.5 \text{ volts}$$

$\therefore$ Line voltage $= \sqrt{3} \times 2144.5 = \mathbf{3714 \text{ volts}}$

Problem 4.29

A 3-phase star-connected alternator with 12 poles generates 1100 volts on open circuit at a speed of 500 RPM. Assuming 180 turns/phase, a distribution

factor of 0.96 and full-pitched coils, find the useful flux per pole.

(Feb. 96, B.U.

Solution :

Given Line e.m.f. = 1100 volts

$\therefore$ E.M.F. per phase, $E_{ph} = \dfrac{1100}{\sqrt{3}} = 635$ volts

$$f = \frac{NP}{120} = \frac{500 \times 12}{120} = 50 \text{ Hz}$$

$$E_{ph} = 4\, k_f\, k_c\, k_d\, f \phi T \qquad \text{---(i)}$$

For sinusoidal distribution of flux.

Form Factor $k_f = 1.11$

For full-pitched winding,

Pitch Factor $k_c = 1$

Distribution Factor $k_d = 0.96$

T = No. of turns/phase = 180

ϕ = Flux per pole (in webers)

Substituting in expression (i) above,

$635 = 4 \times 1.11 \times 1 \times 0.96 \times 50 \times \phi \times 180$

$\therefore$ Flux/pole, $\phi = \dfrac{635}{4.44 \times 0.96 \times 50 \times 180}$

$= \mathbf{0.0165\ Wb}$

Problem 4.30

Calculate the phase e.m.f induced in a 4-pole, 3-phase, 50 Hz, star-connecte alternator with 36 slots and 30 conductors per slot. The flux per pole is 0.0 weber. Assume winding factor of 0.95.

(July 88, B.U.

Solution :

Total number of slots = 36

$\therefore$ No. of slots per phase $= \dfrac{36}{3} = 12$

No. of conductors per slot = 30

$\therefore$ No. of conductors/phase = 12 × 30 = 360

$\therefore$ No. of turns/phase, $T = \dfrac{360}{2} = 180$

Flux per pole, ϕ = 0.05 weber

Assuming sinusoidal distribution of flux.

Form Factor, k_f = 1.11

Winding Factor, k_d = 0.95

Assuming full-pitch winding, Pitch Factor k_c = 1

$\therefore$ Phase e.m.f., $E_{ph} = 4\, k_f\, k_c\, k_d\, f \phi T$ volts

$= 4 \times 1.11 \times 1 \times 0.95 \times 50 \times 0.05 \times 180$

= 1898 volts

Problem 4.31

Determine the phase and line values of the induced e.m.f. in a 4-pole, 3-phase, 50 Hz star-connected alternator with 36 slots and 30 conductors per slot. Flux per pole is 50 mWb and the winding factor is 0.95.

(March 94, B.U.)

Solution :

Flux per pole ϕ = 50 mWb = 0.05 Wb

Speed $N = \dfrac{120 f}{P} = \dfrac{120 \times 50}{4} = 1500$ r.p.m

No. of slots per phase $= \dfrac{36}{3} = 12$

No. of conductors per slot = 30

$\therefore$ No. of conductor per phase = 12 × 30 = 360

$\therefore$ No. of turns per phase, $T = \dfrac{\text{Conductors/phase}}{2}$

$= \dfrac{360}{2} = 180$

Assuming coil is full-pitched, Pitch factor k_c = 1

Given Winding factor, k_d = 0.95

Assuming sinusoidal distribution of flux,

Form Factor k_f = 1.11

$\therefore$ EMF generated per phase.

$E_{ph} = 4\, k_f\, k_c\, k_d\, f \phi T$ Volts

$= 4 \times 1.11 \times 1 \times 0.95 \times 50 \times 0.05 \times 180$

= 1898 volts

$$\text{Line EMF} = \sqrt{3}\, E_{ph}$$

$$= \mathbf{3287\ volts}$$

Problem 4.32

A 3-phase, 50 Hz, 16-pole generator with star-connected winding h 144 slots with 10 conductors/slot. The flux/pole 24.8 mWb is sinusoidal distributed. The coils are full-pitched. Find (i) speed, (ii) the line e.m.f. Given : Winding factor k_d = 0.96. **(Aug 96, B.U**

Solution :

Flux per pole, ϕ = 24.8 mWb (given)

= 0.0248 Wb

i) Speed $N = \dfrac{120f}{p} = \dfrac{120 \times 50}{16} = \mathbf{375\ RPM}$

ii) No. of slots per phase $= \dfrac{144}{3} = 48$

No. of conductors per slot = 10 (given)

∴ No. of conductors per phase = 48 × 10 = 480

∴ Turns per phase, $T = \dfrac{\text{conductors/phase}}{2}$

$$= \frac{480}{2} = 240$$

As the coils are full-pitched, Pitch Factor k_c = 1

As flux is sinusoidally distributed, Form Factor, k_f = 1.11

∴ E.M.F generated per phase,

$$E_{ph} = 4\, k_f\, k_c\, k_d\, f \phi T \text{ Volts}$$

$$= 4 \times 1.11 \times 1 \times 0.96 \times 50 \times 0.0248 \times 240$$

$$= 1268.5 \text{ volts}$$

∴ Line e.m.f. $= \sqrt{3} \times 1268.5$

$$= \mathbf{2197\ Volts}$$

Problem 4.33

Calculate the line e.m.f. induced in a 4-pole, 3-phase, 50 Hz star connecte alternator with 48 slots and 20 conductors per slot. The flux per pole i 0.05 Weber. Assume Winding Factor 0.96.

(Apr. 97, B.U.

Solution :

Total number of slots = 48

$\therefore$ No. of slots per phase = $48/3 = 16$

No. of conductors per slot = 20

$\therefore$ No. of conductors per phase = $16 \times 20 = 320$

$\therefore$ No. of turns/phase, $T = \dfrac{320}{2} = 160$

Flux per pole, ϕ = 0.05 Weber

Assuming sinusoidal distribution of flux.

Form Factor, k_f = 1.11

Winding Factor, k_d = 0.96

Assuming full-pitched winding, Pitch Factor k_c = 1

$\therefore$ Phase e.m.f $E_{ph} = 4\, k_f\, k_c\, k_d\, f\, \phi\, T$ Volts

$= 4 \times 1.11 \times 1 \times 0.96 \times 50 \times 0.05 \times 160 = 1705$ volts

$\therefore$ Line e.m.f $= \sqrt{3} \times E_{ph}$ (Star-connection)

$= \sqrt{3} \times 1705 = \mathbf{2953}$ **Volts**

4.20 Losses in Generator

The total power loss P_{tl} in the synchronous generator is the sum of the following losses :

1. Copper loss in the rotor
2. Copper loss in the stator
3. Iron loss in the rotor
4. Iron loss in the stator
5. Friction and Windage loss

4.21 Efficiency of a Synchronous Generator

The efficiency of a synchronous generator is given by

$$\eta = \frac{P_{out}}{P_{in}} = \frac{P_{out}}{P_{out} + P_{tl}}$$

where P_{out} = Power Output of the synchronous generator in W,

P_{in} = Power Input to the synchronous generator in W,

and P_{tl} = Total power loss in the synchronous generator

(which is the total of the five losses mentioned in the previous Section 7.8)

The total input power given to the synchronous generator is

$$P_{in} = P_{mech} + P_f$$

where P_{mech} = mechanical power input to the synchronous generator through the prime mover

& P_f = electrical power input to the field winding.

4.22 Voltage Regulation of a Synchronous Generator

A synchronous generator is always designed to provide a particular terminal voltage when supplying its rated current at a specified power factor - usually unity or 0.8 lagging (and at times leading). Fig. 7.8 shows the relationship between the terminal voltage and load current of an alternator at different loads. Let *OE* be the full-load current and *OD* the rated terminal voltage of a synchronous generator. If the field current is adjusted to give the terminal voltage *OD* when the generator is supplying current *OE* at unity power factor, then when the load is removed but with the field current and speed maintained unchanged, the terminal voltage rises to *OB*. If the field current is once again adjusted to give the terminal voltage *OD* when the generator is supplying current *OE* at 0.8 p.f. lagging, then when the load is removed, but with the field current and speed again kept unchanged, the terminal voltage rises to *OC*. Now if a similar operation is repeated for 0.6 p.f. leading, it is seen that on removal of load, the terminal voltage falls to *OA*.

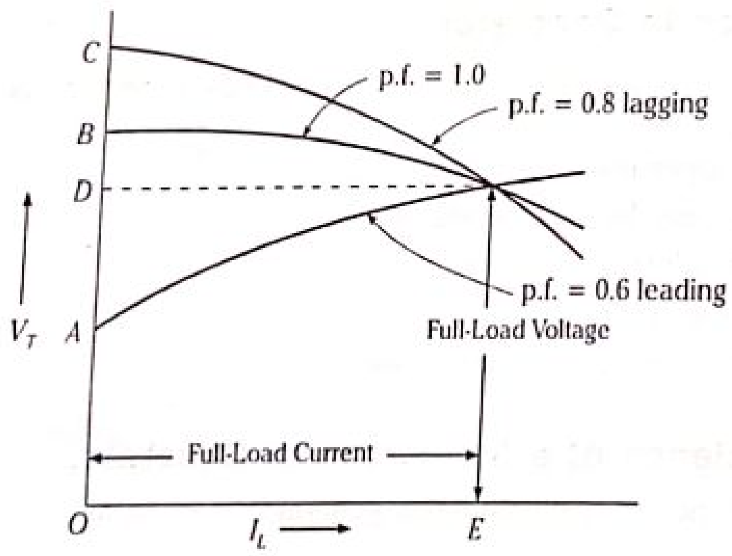


Fig. 4.21 Variation of terminal voltage of a synchronous generator at different p.f. loads

It is seen that the *change of terminal voltage* from full-load to no-load is more in the case of lagging and leading power factor as compared to unity p.f. load. This is because of the demagnetising or magnetising effect of armature reaction on the main field flux.

The variation of the terminal voltage between full-load and no-load, expressed as a per-unit value or as a percentage of the full-load terminal voltage is known as the per-unit or the percentage voltage regulation of the synchronous generator ; thus :

$$\text{Per-Unit regulation} = \frac{\text{Variation of terminal voltage on removal of full-load}}{\text{full-load terminal voltage}}$$

$$= \frac{OB - OD}{OD} = \frac{BD}{OD} \text{ at unity p.f. load}$$

$$= \frac{OC - OD}{OD} = \frac{CD}{OD} \text{ at 0.8 p.f. lagging}$$

$$= \frac{OA - OD}{OD} = -\frac{AD}{OD} \text{ at 0.6 p.f. leading}$$

It is seen that in the last case, i.e., 0.6 p.f. leading, the regulation is negative (capacitive loads), as indicated by the negative sign.

4.23 Review Questions

Q 1. Discuss the main constructional features of cylindrical rotor and salient-pole alternators.

Q 2. Explain the constructional features of a salient-pole alternator. (Aug 82, B.U.; Jan 88, Jan 93, B.U.)

Q 3. With sketches distinguish between salient-pole and non-salient pole alternators. Where are the two types used ? (Oct 85, B.U.)

Q 5. Starting from basic principles, develop an expression for the e.m.f induced in an Alternator. (Dec. 81, Mar 83, Apr 85, Feb 88, July 89, B.U.; Apr/May 87, M.U.; Mar/Apr 88, M.U.)

Q 7. Derive an expression for E.M.F. equation of an alternator, with the usual notations. (Mar 99, VTU)

Q 8. With a neat sketch explain the construction of Salient-Pole Alternators. (Mar 99, VTU)

4.24 Exercises - Problems

1. Calculate the phase voltage of a 3-phase star-connected alternator having 10 poles, running at 1000 rpm. The coils are full-pitched, the Distribution Factor is 0.98. Flux/pole = 0.025 Wb. Turns/phase = 150. (Nov 90, KUD)

   Answer : 1359.75 V

2. Calculate the no-load terminal voltage of a 3-phase, 8-pole, star-connected Alternator running at 750 rpm, having the following data

   Sinusoidally distributed flux/pole = 50 mWb
   Total number of stator slots = 72
   Number of conductors/slot = 10
   Distribution Factor = 0.96
   Assume full-pitched coils. (Nov 87, KUD)

   Answer : 2214.8 Volts

3. A 4-pole 3-phase, star-connected alternator has an armature with 90 slots and 8 conductors/slot, and rotates at 1500 rpm. The flux/pole is 0.05 Wb. Calculate the e.m.f. generated, if the Winding Factor is 0.97, and all the conductors in each phase are connected in series.

   (April 88, KUD)

   Answer : 1292 Volts/Phase, 2237.9 Volts-Line

MODULE

5

(a) Single Phase Transformers

5.1 Introduction

The main advantage of alternating current over direct current is that it can be easily increased or reduced as per requirement during the generation, transmission, distribution and utilization of electric power. This is made possible by means of transformers. For instance, high voltages may be generated and stepped up by means of transformers to still higher voltages for the transmission lines. Other transformers are employed at suitable points to step the voltage down to values suitable for motors, lamps, heaters and other loads. The full-load efficiency of a medium sized transformer is of the order of 97-98 percent, so that the loss at each point of transmission or distribution is small. As the transformer is a static apparatus, there are no moving parts and so the maintenance of a transformer is easy and the amount of supervision is negligible.

Transformers are associated not only with power system applications but with low power applications (such as electronic circuits) as well. However, we shall discuss only the common power-system transformer.

5.2 Construction of Single-Phase Transformer

The two windings (primary and secondary) of the transformer are insulated from each and from the laminated steel core. The assembled core and the windings are enclosed in a suitable container. Appropriate bushings, either of the porcelain or the capacitor type, are used for insulating and bringing out the terminals of the windings from the enclosure.

Fig. 5.1

The core is invariably constructed of transformer sheet laminations fitted together in such a way as to ensure a continuous magnetic path, with a minimum of air gap. The steel used for these laminations has high silicon content, which is sometimes heat-treated to ensure high permeability and low hysteresis loss at the usual operating flux densities.

Lamination of the core minimizes eddy current loss. These laminations are insulated from each other by a thin coating of a suitable varnish. The thickness of laminations ranges from 0.35 mm for a frequency of 25 Hz to 0.5 mm for a frequency of 50 Hz.

The lamination strips are assembled as shown in Fig. 5.1, where the joints are staggered to avoid narrow gaps all through the cross-section of the core.

The two main type of transformers are

i) Core-type

ii) Shell-type

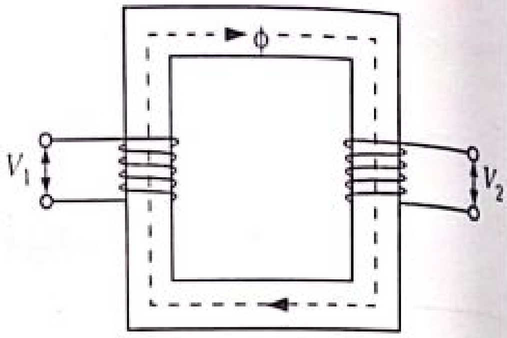


Fig. 5.2

5.2.1 Core-type Transformers

In this type of transformer, a large part of the core is surrounded by the windings. Fig. 5.2 shows the simplified representation of a core-type transformer, where the primary and secondary windings have been shown wound on the opposite limbs. However, in actual practice, half the primary and half the secondary windings are situated side by side on each limb, so as to reduce leakage flux, as shown in Fig. 5.3.

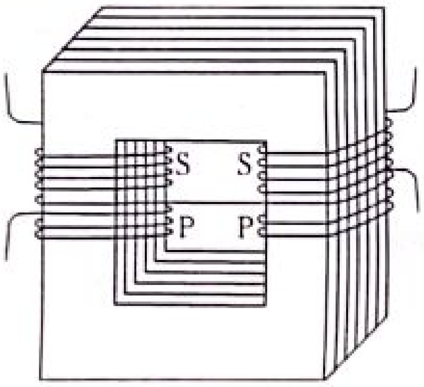


Fig. 5.3

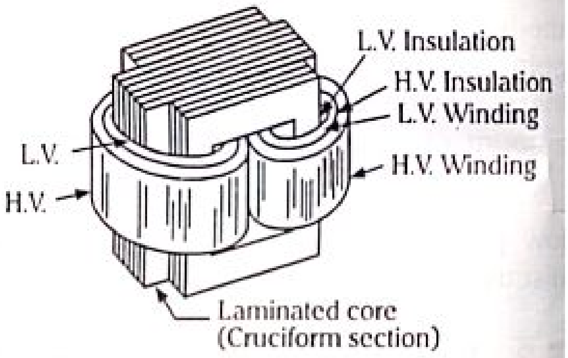


Fig. 5.4

The general form of the coils may be circular or oval rectangular. A rectangular core is used with cylindrical coils for small transformers. However, in the case of large-size core-type transformers, round or circular cylindrical coils are wound over a cruciform core section (as shown in Fig. 5.4), offering considerable mechanical strength. These coils are wound in helical layers, each layer being insulated from the other by using paper, cloth or cooling ducts. The net core cross-sectional area is reduced by about 10 % because of paper and other materials.

5.2.2 Shell-type Transformers

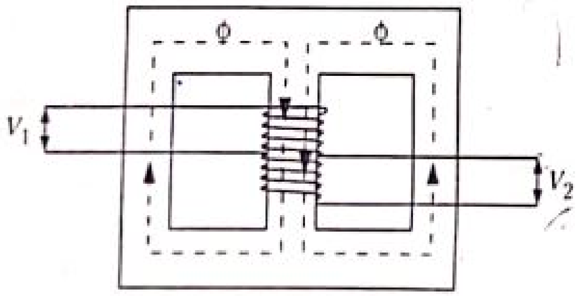


Fig. 5.5

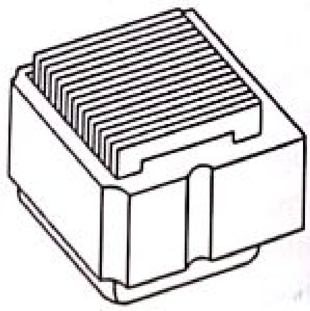


Fig. 5.6

In this type, the windings occupy a smaller portion of the core as shown schematically in Fig. 5.5. The primary and secondary windings are shown located on the central limb.

The coils are form-wound in this case too; they are multilayer disc type, in the shape of pancakes. Each of these multilayer discs is insulated from the other by using paper. The entire winding comprises stacked discs with insulation spaces between the coils, such spaces forming horizontal cooling and insulating ducts. A commonly used shell-type transformer has a simple rectangular form, as shown in Fig. 5.6.

5.3 Basic Principle of Operation

A transformer works on the principle of electromagnetic induction and mutual induction between the two coils. The general arrangement of a transformer is shown in Fig. 5.7.

A steel core consists of laminated sheets, about 0.4-0.7 mm thick, insulated from each other. The core is laminated to reduce eddy current loss. The vertical parts of the core are known as *limbs*, while the top and bottom parts are called *yokes*.

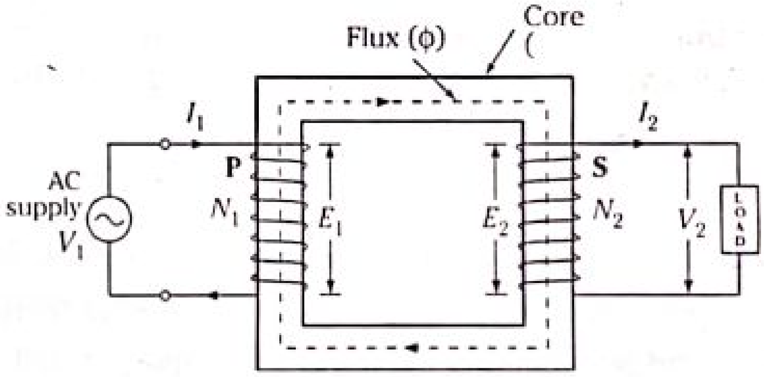


Fig. 5.7 Basic Transformer

There are two separate electrical windings, which are linked through a common magnetic circuit. These windings are isolated from each other electrically.

The coil into which electrical energy is fed is called the **primary winding (*P*)**, while the other coil from which electrical energy is drawn out is called the **secondary winding (*S*)**. The primary winding has N_1 number of turns while the secondary winding has N_2 number of turns.

When the primary winding is connected to an alternating voltage V_1, an alternating current flows through the primary winding *P* and this current produces an alternating flux ϕ in the steel core, the mean path of this flux being indicated by the dotted line. If the entire flux produced by *P* passes through *S*, the e.m.f. induced in each turn is the same for *P* and *S*.

The above mentioned alternating flux ϕ produces self-induced e.m.f. E_1 primary winding P, while due to mutual induction i.e., due to the flux produc the primary linking with the secondary, it produces mutually induced e.m.f. the secondary winding S.

These e.m.f.'s are $E_1 = -N_1 \dfrac{d\phi}{dt}$ and $E_2 = -N_2 \dfrac{d\phi}{dt}$

$$\therefore \quad \frac{E_2}{E_1} = \frac{N_2}{N_1} = K$$

K is known as the voltage transformation ratio

The frequency of the two e.m.f.'s is the same. The voltage transformation may alternatively be obtained as follows :

The e.m.f. per turn is the same for P & S as mentioned earlier. Hence,

$$\frac{\text{Total e.m.f induced in S}}{\text{Total e.m.f induced in P}} = \frac{N_2 \times \text{e.m.f per turn}}{N_1 \times \text{e.m.f. per turn}} = \frac{N_2}{N_1} = K$$

When the secondary is on open circuit, its terminal voltage is the same a induced e.m.f. The primary current is then quite small, so that the applied vo V_1 is practically equal and opposite to the e.m.f. induced in P. Hence

$$\frac{V_2}{V_1} \approx \frac{N_2}{N_1} = K \qquad \text{---(i)}$$

As the full-load efficiency of a transformer is almost 100 percent,

$V_1 I_1 \times$ primary power factor $= V_2 I_2 \times$ secondary power factor

As both the primary and secondary p.f.'s are almost equal on full load,

$$\therefore \quad \frac{I_1}{I_2} \approx \frac{V_2}{V_1} = K \qquad \text{---(ii)}$$

From eqns (i) and (ii) above

$$\frac{V_2}{V_1} \approx \frac{I_1}{I_2} \approx K$$

5.4 E.M.F. Equation of a Transformer

JMP

Let us consider a transformer having :

N_1 = primary turns

N_2 = secondary turns

ϕ_m = maximum value of the flux in the core linking both the windings (webers).

$= B_m A$,

where B_m = Maximum flux density in the core (Wb/m^2)

A = Area of cross-section of the core (m^2)

f_1 = frequency of a.c. input in hertz (Hz).

The flux in the core will vary sinusoidally as shown in Fig. 5.8, so that it increases from zero to maximum value ϕ_m in one quarter of the cycle *i.e.*, in $\frac{1}{4f}$ second.

$\therefore$ Average rate of change of flux $= \dfrac{\phi_m}{\frac{1}{4f}} = 4 f \phi_m$

We know that the rate of change of flux per turn means induced e.m.f. in volts.

$\therefore$ Average e.m.f. induced / turn $= 4 f \phi_m$ volts

Since the flux is varying sinusoidally, the r.m.s. value of induced e.m.f. is obtained by multiplying the average value by the form factor.

$\therefore$ r.m.s. value of e.m.f. induced/turn

$= 1.11 \times 4 f \phi_m$

$= 4.44 f \phi_m$ volts

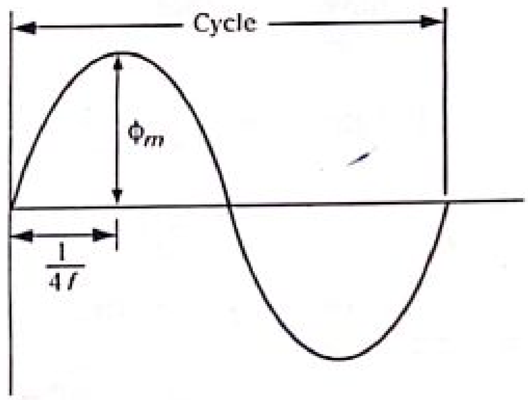


Fig. 5.8

The r.m.s. value of induced e.m.f. in the entire primary winding = (induced e.m.f. / turn) × No. of primary turns, or

$$\mathbf{E_1 = 4.44\, f N_1 \phi_m = 4.44\, f N_1 B_m A} \qquad \text{---(i)}$$

In a similar manner, the r.m.s. value of induced e.m.f in the entire secondary winging is

$$\mathbf{E_2 = 4.44\, f N_2 \phi_m = 4.44\, f N_2 B_m A} \qquad \text{---(ii)}$$

In an ideal transformer on no-load,

Applied voltage, $V_1 = E_1$ and

Secondary terminal voltage $V_2 = E_2$

5.5 Transfer of Transformer Winding Parameters

In the diagram of a transformer in Fig. 5.9, the resistances of the primary and secondary windings are shown as R_1 and R_2 respectively. We shall now proceed to transfer the resistances of the two windings to any one winding. This is done to make calculations simple and easy, as we have to ideal with one winding only.

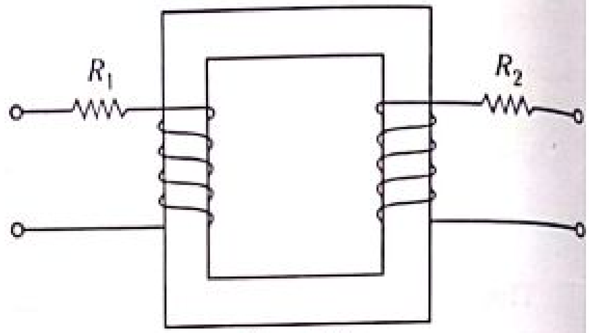


Fig. 5.9

The copper loss in the secondary is $I_2^2 R_2$. This loss is supplied by the primary which takes a current of I_1. Thus R_2' is the equivalent resistance in the primary which would have caused the *same* loss as R_2 in the secondary.

$$\text{Thus} \quad I_1^2 R_2' = I_2^2 R_2 \quad \text{or} \quad R_2' = \left(\frac{I_2}{I_1}\right)^2 R_2$$

We have seen that $\dfrac{I_2}{I_1'} = \dfrac{1}{K}$; however, if I_0 is neglected, $I_1' = I_1$.

$$\text{In that case} \quad \frac{I_2}{I_1} = \frac{1}{K}$$

$$\therefore \quad R_2' = \frac{R_2}{K^2}$$

In a like manner, the equivalent primary resistance as referred to the secondary is

$$R_1' = K^2 R_1$$

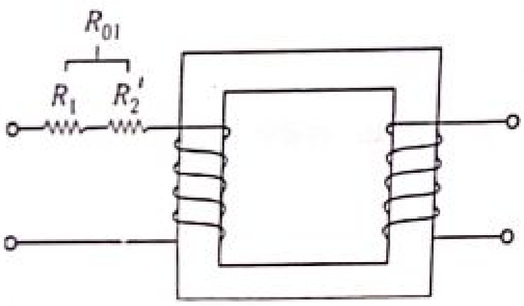


Fig. 5.10

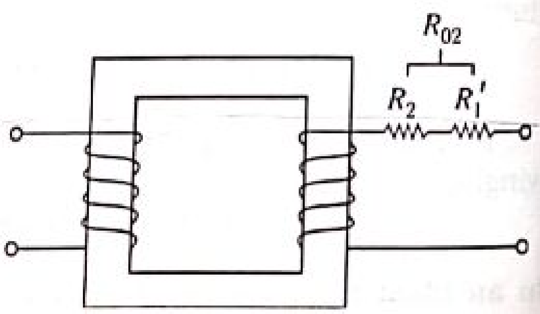


Fig. 5.11

In Fig. 5.10, the secondary resistance has been transferred to the primary. *The resistance* $R_1 + R_2' = R_1 + \dfrac{R_2}{K^2}$ *is called the equivalent resistance of the transformer as referred to the primary and is termed* R_{01}.

$$\therefore R_{01} = R_1 + R_2'$$

or $$R_{01} = R_1 + \frac{R_2}{K^2}$$

In a similar manner, *the equivalent resistance of the transformer as referred to the secondary* is

$$R_{02} = R_2 + K^2 R_1$$

This is depicted in Fig. 5.11.

In a similar way, leakage reactance too can be transferred (Fig. 5.12).

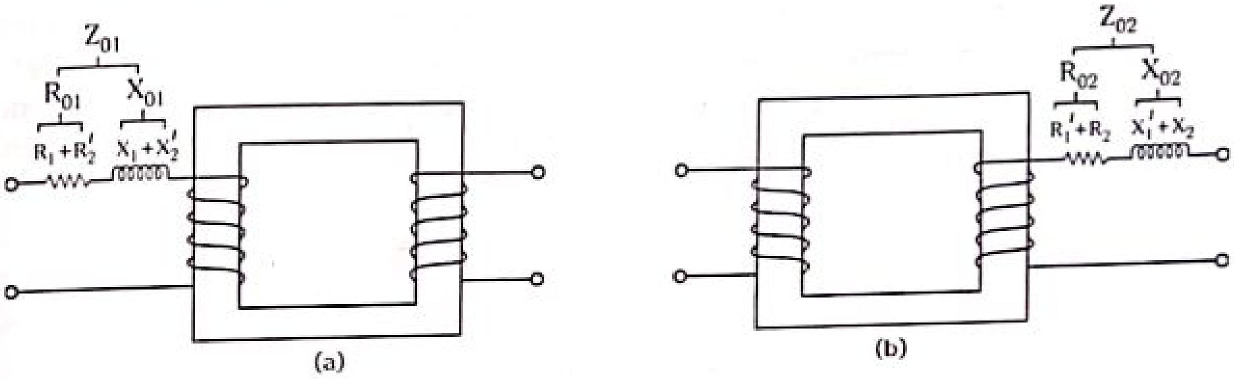


Fig. 5.12

Let X_1' be the primary reactance as referred to the secondary. The voltage drop across it when it is considered to be in the secondary = $I_2\ X_1'$. When it is in the primary, the voltage drop = $I_1\ X_1$. Keeping in mind that the voltages between the two windings are related through K, we have

$$I_2\ X_1' = K\, I_1\, X_1$$

$$\therefore \quad X_1' = K\left(\frac{I_1}{I_2}\right)X_1 = K^2\, X_1$$

Similarly, $$X_2' = \frac{X_2}{K^2}$$

$$\therefore \quad X_{01} = X_1 + X_2' = X_1 + \frac{X_2}{K^2}$$

and $$X_{02} = X_2 + X_1' = X_2 + K^2\, X_1$$

5.6 Power Losses in a Transformer

Being static, the transformer does not have friction or windage losses. However, the only losses occurring are

a) Core or Iron Loss.

b) Copper Losses.

a) Core or Iron Losses : These losses consist of *hysteresis* and *eddy current losses* and occur due to the alternating flux in the transformer core. Because the core flux remains practically constant for all loads (its variation being 1 to 3 % from no - load to full load), the core loss is practically constant at all loads.

i) *Hysteresis Loss* : Since the flux in a transformer core is alternating, therefore, power is required for the continuous reversal of the molecular magnets, which comprise the core. This power is dissipated in the form of heat and is known ashysteresis loss. It depends on the flux density in the core and the supply frequency.

$$\text{Hysteresis loss } W_h = P B_{max}^{1.6} f \text{ watt}$$

where B_{max} : Maximum Flux Density (Wb/m^2)

f : Frequency (Hz)

P is a constant.

ii) *Eddy - Current Loss* : Due to the alternating flux in the core, eddy currents flow in the core. Power is required to maintain these eddy currents. This power is dissipated in the form of heat and is called *eddy current loss*. In order to ensure that these currents are small, a high resistance path is made out for them. This can be done by making the core of thin laminations, the laminations being separated from each other by varnish. The eddy current loss also depends upon flux density in the core and supply frequency.

$$\text{Eddy current loss } W_e = Q B_{max}^{2} f^2 \text{ watt}$$

Q is a constant.

From our above discussion, it is apparent that core losses (hysteresis and eddy current losses) depend upon flux density in the core and supply frequency. As flux density in the core remains practically constant from no-load to full load, and also supply frequency is constant, it follows that **core losses too are constant for a given transformer. These losses are independent of load which is why these are generally termed constant losses.**

Core losses can be minimised by using steel of high silicon content for the core and by using very thin laminations.

b) Copper Losses or I^2R losses : These losses occur due to the ohmic resistance in both the primary and secondary windings. If R_1 and R_2 are the primary

and secondary resistances, and I_1, I_2 are the primary and secondary currents respectively.

Total *Cu* loss $= I_1^2 R_1 + I_2^2 R_2 = I_1^2 R_{01} = I_2^2 R_{02}$

It is obvious that copper loss is proportional to $(\text{current})^2$ or $(\text{kVA})^2$. In other words, *Cu* loss at half the full-load is one fourth of that at full-load.

5.7 Efficiency of a Transformer

As discussed in the previous Section, the power losses in a transformer on load are of two types :

1. *Core* or *Iron losses due to hysteresis and eddy currents* (P_c).
2. I^2R *losses* or *Copper Losses in the primary and secondary windings*, namely $I_1^2 R_1 + I_2^2 R_2$

The efficiency of a transformer at a particular load and power factor is defined as the output power divided by the input power, both being measured in the same units i.e., either in watts or in kilowatts.

$$\therefore \quad \text{Efficiency} = \frac{\text{Output power}}{\text{Input power}} = \frac{\text{Output power}}{\text{Output power} + \text{Core losses} + I^2R \text{ losses}}$$

$$= \frac{I_2 V_2 \times \text{p.f.}}{(I_2 V_2 \times \text{p.f.}) + P_C + I_1^2 R_1 + I_2^2 R_2}$$

Efficiency may be expressed with greater accuracy as follows :

$$\text{Efficiency } \eta = \frac{\text{Output power}}{\text{Input power}} = \frac{\text{Input power} - \text{losses}}{\text{Input power}}$$

$$\text{or} \quad \eta = 1 - \frac{\textbf{losses}}{\textbf{input power}}$$

Here, we may keep in mind that efficiency is based on power output in watts and not on volt-amperes, although losses are proportional to VA. Thus, at any volt-ampere load, the efficiency depends on power factor, being *maximum for unity power factor.*

5.8 Condition for Maximum Efficiency

Copper loss, $W_{cu} = I_1^2 R_{01}$ or $I_2^2 R_{02}$

Iron loss = hysteresis loss + eddy current loss

$$\text{or} \quad W_i = W_h + W_e$$

Taking the primary, primary input = $V_1 I_1 \cos \phi_1$

$$\eta = \frac{V_1 I_1 \cos \phi_1 - \text{losses}}{V_1 I_1 \cos \phi_1}$$

$$= \frac{V_1 I_1 \cos \phi_1 - I_1^2 R_{01} - W_i}{V_1 I_1 \cos \phi_1}$$

$$\text{or} \qquad \eta = 1 - \frac{I_1 R_{01}}{V_1 \cos \phi_1} - \frac{W_i}{V_1 I_1 \cos \phi_1}$$

Differentiating L.H.S. and R.H.S. w.r.t I_1, we have

$$\frac{d\eta}{dI_1} = 0 - \frac{R_{01}}{V_1 \cos \phi_1} + \frac{W_i}{V_1 I_1^2 \cos \phi_1}$$

In order that the efficiency η is maximum

$\dfrac{d\eta}{dI_1}$ should be equated to zero

$$\text{or} \qquad -\frac{R_{01}}{V_1 \cos \phi_1} + \frac{W_i}{V_1 I_1^2 \cos \phi_1} = 0$$

$$\text{or} \qquad \frac{W_i}{V_1 I_1^2 \cos \phi_1} = \frac{R_{01}}{V_1 \cos \phi_1}$$

$$\text{or} \qquad W_i = I_1^2 R_{01} \quad \text{or} \quad I_2^2 R_{02} \qquad \text{(i)}$$

or **Iron Loss = Copper Loss**

The output current in case of maximum efficiency,

$$I_2 = \sqrt{\frac{W_i}{R_{02}}} \qquad \text{from eqn. (i) above}$$

5.9 Load Corresponding to Maximum Efficiency

Once we know the iron loss and copper loss of a transformer, we can determ the load at which the transformer will work at maximum efficiency.

We have found in the previous Section that efficiency will be maximum wh the iron loss W_i is equal to the full-load copper loss W_{Cu}. If X is the load under t condition, W_i becomes the Cu loss for X kVA output. We know that Cu loss proportional to $(\text{kVA})^2$, so

$$W_{Cu} \propto (\text{full-load kVA})^2$$

or $$W_i \propto X^2$$

$$\therefore \quad \left(\frac{X}{\text{F.L. kVA}}\right)^2 = \frac{W_i}{W_{Cu}}$$

or $$X = \text{F.L. kVA} \times \sqrt{\frac{W_i}{W_{Cu}}}$$

or $$\boldsymbol{X = \textbf{F.L. kVA} \times \sqrt{\frac{\textbf{iron loss}}{\textbf{full - load } \textbf{\textit{Cu}} \textbf{ loss}}}}$$

5.10 Definition of Voltage Regulation

Voltage Regulation of a transformer may be defined in various ways, as given below :

A. Constant Primary Voltage : When a transformer is loaded with a *constant primary voltage* the secondary voltage decreases* because of the ohmic resistances and leakage reactances of the windings,

Let V_{02} = secondary terminal voltage at *no load*

$= E_2 = K E_1 = K V_1$, because at no load the impedance drop is negligible

V_2 = secondary terminal voltage on *full-load.*

The change in secondary terminal voltage from no-load to full load is

$$= V_{02} - V_2.$$

This change divided by V_{02} is called 'regulation down'. Thus,

$$\text{Regulation down} = \frac{V_{02} - V_2}{V_{02}}$$

$$\therefore \quad \%\ \text{Regulation down} = \frac{V_{02} - V_2}{V_{02}} \times 100$$

If this change is divided by V_2 *i.e.*, full load secondary terminal voltage, then it is known as 'regulation up'.

* Assuming lagging power factor. It will increase if power factor is leading.

Thus,

$$\text{Regulation up} = \frac{V_{02} - V_2}{V_2}$$

$$\therefore \quad \% \text{ Regulation up} = \frac{V_{02} - V_2}{V_2} \times 100$$

B. Constant Secondary Terminal Voltage V_2 : When the transformer is loaded the secondary terminal voltage falls for lagging power factor or rises for leading power factor loads.

Therefore, to keep the secondary terminal voltage constant, the primary voltage is either increased or decreased depending on the power factor of the load.

Suppose the primary voltage has to be raised from V_1 to V_1', then

$$\% \text{ Regulation} = \frac{V_1' - V_1}{V_1} \times 100$$

5.11 Importance of Voltage Regulation

In the previous Section we have defined voltage regulation under different conditions. The lesser the value of percentage voltage regulation, the better the transformer, because a good transformer should keep its secondary terminal voltage as constant as possible under all conditions of load. Such load could be machinery or equipment which need constant voltage for proper operation. Hence the importance of voltage regulation.

5.12 Illustrative Problems on E.M.F. Equation and Efficiency

Problem 5.1

A 200 KVA, 10000 V/400 V, 50 Hz single-phase transformer has 100 turns on the secondary. Calculate :

(i) the primary and secondary currents

(ii) the number of primary turns

(iii) the maximum value of flux.

Solution :

(i) Full-load primary current $= \dfrac{200 \times 1000}{10000} = \mathbf{20\,A}$

and full-load secondary current $= \dfrac{200 \times 1000}{400} = \mathbf{500\ A}$

(ii) No. of primary turns $= \dfrac{100 \times 10000}{400} = \mathbf{2500}$

(iii) From expression (ii) of Sec. 5.4,

$$E_2 = 4.44 f N_2 \phi_m$$

or $$400 = 4.44 \times 50 \times 100 \times \phi_m$$

$$\phi_m = 0.018 \text{ Wb} = \mathbf{18\ mWb}$$

Problem 5.2

A single-phase transformer with 10 : 1 turns ratio and rated at 25 kVA. 1200/120 V, 50 Hz, is used to step down the voltage of a distribution system. The low tension voltage is to be kept constant at 120 V. Find the value of the load impedance on the low tension side so that the transformer is fully loaded. Find also the value of maximum flux, if the low tension side has 25 turns.

(Apr. 85, B.U.)

Solution :

Full-load current $I_2 = \dfrac{25,000}{120} = 208.33$ A

$$\therefore \quad Z_2 = \frac{V_2}{I_2} = \frac{120}{208.33} = \mathbf{0.576\ \Omega}$$

$$E_2 = 4.44 \times f \times N_2 \times \phi_m$$

or $$120 = 4.44 \times 50 \times 25 \times \phi_m$$

$$\therefore \quad \phi_m = \frac{120}{4.44 \times 50 \times 25} = \mathbf{0.021\ Wb}$$

Problem 5.3

Find the number of turns required on the H.T. side of a 415 / 240 V, 50 Hz single-phase transformer, if the area of cross-section of the core is 25 cm^2 and the maximum flux density is 1.3 Wb/m^2.

(83-84, B.U.)

Solution :

We have already discussed the following e.m.f. equation in Sec. 5.4 :

$$E_1 = 4.44 f N_1 B_m A$$

The HT side e.m.f. $= 415 \text{ V} = E_1$

Also given that f = 50 Hz, B_m = 1.3 Wb/m^2

and A = 25 cm^2 = (25×10^{-4}) m^2

Substituting the above values in the e.m.f. equation.

$$415 = 4.44 \times 50 \times N_1 \times 1.3 \times \left(25 \times 10^{-4}\right)$$

∴ The number of turns required on H.T. side,

$$N_1 = \frac{415}{4.44 \times 50 \times 1.3 \times \left(25 \times 10^{-4}\right)} = \mathbf{575}$$

Problem 5.4

The required no-load ratio in a single phase, 50 Hz, core type transfo[rmer] is 6000 / 250 V. Find the number of turns per limb on the high and low vol[tage] sides if the flux is to be about 0.06 Wb. (Aug 95, [...]

Solution :

Primary induced voltage, E_1 = 6000 V

Secondary induced voltage, E_2 = 250 V

Maximum flux in core, ϕ_m = 0.06 Wb

Supply frequency, f = 50 Hz

Using the relation, $E_1 = 4.44\, f N_1 \phi_m$

∴ Primary winding turns, $N_1 = \dfrac{E_1}{4.44\, f \phi_m}$

$$= \frac{6000}{4.44 \times 50 \times 0.06}$$

$$= \mathbf{450}$$

Similarly, $E_2 = 4.44\, f N_2\, \phi_m$

∴ Secondary winding turns, $N_2 = \dfrac{E_2}{4.44\, f\, \phi_m}$

$$= \frac{250}{4.44 \times 50 \times 0.06}$$

$$= \mathbf{19}$$

Problem 5.5

A 125 kVA transformer has a primary voltage of 2000 V at 60 Hz. Prim[ary] turns are 182 and the secondary turns are 40. Neglecting losses, calculat[e]

i) **no load secondary e.m.f**
ii) **full-load primary and secondary currents**
iii) **flux in the core.** **(Feb/Mar 90, M.U.)**

Solution :

Voltage transformation ratio, $K = \frac{N_2}{N_1} = \frac{40}{182} = \frac{20}{91}$

Now, full-load current $I_1 = \frac{125,000}{2,000} = \mathbf{62.5\ A}$

Full-load current $I_2 = \frac{I_1}{K} = \frac{62.5 \times 91}{20} = \mathbf{284.37\ A}$

No-load secondary e.m.f., $E_2 = KE_1 = \frac{20}{91} \times 2000 = \mathbf{439.6\ V}$

$E_1 = 4.44\ f\ N_1\ \phi_m$

or $2000 = 4.44 \times 60 \times 182 \times \phi_m$

or $\phi_m = 0.0412\,\text{Wb} = \mathbf{41.2\ mWb}$

Problem 5.6

A 25 KVA, single phase transformer has 500 turns on the primary and 40 turns on the secondary winding. The primary is connected to 3000 V, 50 Hz supply. Calculate (i) Primary and Secondary currents on Full-Load (ii) The secondary e.m.f. (iii) the maximum flux in the core.

(May 86; Gulbarga)

Solution :

Given rating = 25 kVA

Primary applied voltage V_1 = 3000 Volts

Number of primary turns, N_1 = 500

Number of secondary turns, N_2 = 40

i) Full-load Primary Current $I_1 = \frac{\text{kVA Rating} \times 1000}{\text{Rated Primary Voltage, } V_1}$

or $I_1 = \frac{25 \times 1000}{3000} = 8.33\ \text{A}$

Turns Ratio, $K = \frac{N_2}{N_1} = \frac{40}{500} = 0.08$

We have, $\frac{I_1}{I_2} = \frac{N_2}{N_1}$ $\left[\text{Current Ratio} = \frac{1}{K}\right]$

or $$I_2 = I_1 \left(\frac{N_1}{N_2}\right)$$

$\therefore$ Full-Load Secondary Current $I_2 = 8.33 \left(\frac{1}{0.08}\right) = \mathbf{104.125\ A}$

ii) Now $\frac{E_2}{E_1} = \frac{N_2}{N_1}$

$\therefore$ Secondary Induced e.m.f. $E_2 = E_1 \left(\frac{N_2}{N_1}\right)$

or $E_2 = 3000 \times 0.08 = \mathbf{240\ Volts}$

iii) The e.m.f equation of a transformer is $E_1 = 4.44\, f\, N_1\, \phi_m$

Substituting the various given values, we get

$$3000 = 4.44 \times 50 \times 500 \times \phi_m$$

$$\therefore \quad \phi_m = \frac{3000}{4.44 \times 50 \times 500} = 0.027 \text{ Wb}$$

Thus, the maximum flux carried in the core = **0.027 Wb.**

Problem 5.7

In a 25 kVA, 2000 / 200 V Transformer, the Iron and Copper Losses are 350 watts and 400 watts respectively. Calculate the efficiency at U.P.F. at Half Full-Load. (Feb 96, B.U.)

Solution :

Power Factor, $\cos \phi = 1$

Output at Half Full-Load

$$= \left(\frac{25}{2}\right) \times \cos \phi \text{ kW} = 12.5 \times 1 = 12.5 \text{ kW}$$

$$\text{Cu. loss at Half Full-Load} = \left(\frac{1}{2}\right)^2 \times \text{F.L. Loss}$$

$$= \frac{1}{4} \times 400 = 100 \text{ kW}$$

Iron Loss = 350 W (constant)

$\therefore$ Total losses = 100 + 350 = 450 W = 0.45 kW

Input = Output + Losses = 12.5 + 0.45

= 12.95 kW.

$$\therefore \quad \text{Efficiency} = \frac{\text{Output}}{\text{Input}}$$

$$= \frac{12.5}{12.95} \times 100 = \mathbf{96.52\%}$$

Problem 5.8

In a 25 kVA, 2000 / 200 V, single phase transformer, the iron and full-load copper losses are 350 and 400 W respectively. Calculate the efficiency at unity power factor on

i) full-load, (ii) half full-load. (June/July 90, B.U.)

Solution :

1. Full-Load

Output at Full-Load = 25 cos ϕ kW

Power Factor, cos ϕ = 1

$\therefore$ Output at Full-Load = 25 × 1 = 25 kW

Iron Losses = 350 W

F.L. Copper Losses = 400 W

Total Losses = 750 W = 0.75 kW

Input = Output + losses

= 25 kW + 0.75 kW = 25.75 kW

$$\therefore \quad \text{Efficiency, } \eta_1 = \frac{25}{25.75} \times 100 = \mathbf{97\%}$$

2. Half Full-Load

$$\text{Output} = \left(\frac{25}{2}\right) \times \cos\phi \text{ kW} = 12.5 \times 1 = 12.5 \text{ kW}$$

$$\text{Cu loss at Half Full-Load} = \left(\frac{1}{2}\right)^2 \times \text{F.L. Loss} = \frac{1}{4} \times 400 = 100 \text{ W}$$

Iron loss = 350 W (constant)

$\therefore$ Total losses = 100 + 350 = 450 W = 0.45 kW

Input = Output + Losses = 12.50 + 0.45 = 12.95 kW

$$\therefore \quad \text{Efficiency, } \eta_2 = \frac{12.5}{12.95} \times 100 = \mathbf{96.52\%}$$

Problem 5.9

A 50 kVA transformer has an efficiency of 98% at full load, 0.8 p.f. and efficiency of 96.9 % at 1/4 full-load, unity p.f. Determine the iron loss and full-load copper loss. (July 93, B

Solution :

Full-Load

Output at Full-Load = 50 cos ϕ = 50 × 0.8 = 40 kW

$$\text{Input} = \text{Output} + \text{Losses}$$
$$= \text{Output} + \text{Full-Load Cu. Losses} + \text{Iron Loss}$$
$$= (40 + FLCL + IL) \text{ kW}$$

$$\text{Efficiency } \eta_1 = \frac{\text{Output}}{\text{Input}} = \frac{\text{Output}}{40 + FLCL + IL}$$

or $$0.98 = \frac{40}{40 + FLCL + IL}$$

or $$40 + FLCL + IL = 40.8 \qquad \text{---(i)}$$

Quarter Full-Load

Iron Loss IL remains unchanged; p.f. = 1

$$\text{Output} = \left(\frac{50}{4}\right) \cos\phi = 12.5 \times 1 = 12.5 \text{ kW}$$

$$\text{Cu. Loss at Quarter Load, } QLCL = \left(\frac{1}{4}\right)^2 \times FLCL$$
$$= 0.0625\ FLCL$$

$$\text{Efficiency } \eta_2 = \frac{12.5}{12.5 + 0.0625\ FLCL + IL}$$

or $$0.969\,(12.5 + 0.0625 \times FLCL + IL) = 12.5$$

or $$12.5 + 0.0625\ FLCL + IL = 12.9 \qquad \text{---(ii)}$$

Solving for eqns (i) and (ii) as follows :

$$\begin{array}{l} 40 + \qquad\quad FLCL + IL = 40.8 \\ \underline{12.5 + 0.0625\ FLCL + IL = 12.9} \\ 27.5 + 0.9375\ \ FLCL \qquad = 27.9 \end{array}$$

FLCL = 0.426 kW = **426 watts**

Substituting this value of FLCL in eqn. (i),

$$40 + 0.426 + IL = 40.8$$

or $IL = 0.374$ kW

$= \mathbf{374\ watts}$

Problem 5.10

A 600 kVA single phase transformer has an efficiency of 92% both at full-load and half-load at unity power factor. Determine its efficiency at 75% of full-load at 0.9 power factor lag. (Aug 94, B.U.)

Solution :-

As the efficiency is the same *i.e.*, 92 % at both full-load and half-load, we will be able to determine the iron and copper losses.

At Full-Load,

Output $= 600 \cos\phi = 600 \times 1 = 600$ KW.

$\therefore$ Input $= \dfrac{600}{0.92} = 652.2$ KW

$\therefore$ Total losses $= 652.2 - 600 = 52.2$ KW

Let x = iron loss – which is constant at all loads

y = F.L. copper loss – which is $\propto (\text{kVA})^2$

$\therefore \quad x + y = 52.2$

At Half-Load,

Output = 300 KW

$\therefore$ Input $= \dfrac{300}{0.92} = 326.1$ KW

$\therefore$ Total losses $= 326.1 - 300 = 26.1$ KW

Since copper loss becomes one-fourth of its full-load value, then

$$x + \frac{y}{4} = 26.1$$

Now, $x + y = 52.2$

Solving, $y = 34.8$ KW and $x = 17.4$ KW.

At 75 % Full-Load,

Copper Losses $= 0.75^2 \times 34.8 = 19.6$ KW.

Iron loss (constant) = 17.4 KW.

$\therefore$ Total loss $= 17.4 + 19.6 = 37$ KW

$$\text{Output} = 600 \cos\phi \times 0.75$$
$$= 600 \times 0.9 \times 0.75$$
$$= \mathbf{405\ KW}$$

$$\text{Efficiency } \eta = \frac{\text{output}}{\text{output + losses}}$$

$$= \frac{405}{405 + 37}$$

$$= \frac{405}{442} = 0.916$$
$$= \mathbf{91.6\ \%}$$

Problem 5.11

Determine the efficiency of a 150 kVA transformer at 50 % full-load of p.f. lag if the copper loss at full-load is 1600 W and the iron loss 1400 W

Solution :

Power factor, $\cos\phi = 0.8$

Iron losses remain constant at all loads. However, copper losses proportional to the square of the load current.

Copper losses at full-load, $P_c = 1600\ \text{W} = 1.6\ \text{kW}$

$\therefore$ Copper losses at 50 % full-load $= \left(\frac{1}{2}\right)^2 P_c$

$$= \frac{1}{4} \times 1.6 = 0.4\ \text{kW}$$

Total losses = Iron losses + *Cu* losses at 50 % F.L.

$$= 1.4 + 0.4 = 1.8\ \text{kW}$$

Output power at 50 % load $= \frac{1}{2} \times 150 \times \cos\phi$

$$= \frac{1}{2} \times 150 \times 0.8 = 60\ \text{kW}$$

Input power $= 60 + 1.8 = 61.8\ \text{kW}$

$\therefore$ Transformer $\eta = \frac{\text{output power}}{\text{input power}} \times 100$

$$= \frac{60}{61.8} \times 100 = \mathbf{97\ \%}$$

Problem 5.12

A 240 / 400 V single phase transformer absorbs 35 Watts when its primary winding is connected to a 240 V, 50 Hz supply, the secondary being on open circuit.

When the primary is short-circuited and a 10 V, 50 Hz supply is connected to the secondary winding the power absorbed is 48 W, when the current has the full-load value of 15 A.

Estimate the efficiency of the transformer at half-load, 0.8 p.f. lagging.

Solution :

The HV side is open and the primary winding has 240 V applied across it

Input power = Iron loss = 35 watts

Full-load secondary current = 15 A

The primary winding is short-circuited and the rated (Full-Load) current passes through the secondary winding.

Power absorbed = 48 W = Full-Load Copper Loss

Now, we have to estimate the efficiency at half full-load, 0.8 p.f.

$$\text{Output} = V_2 I_2 \cos\phi = 400 \times \left(\frac{1}{2} \times 15\right) \times 0.8 = 2400 \text{ Watts}$$

Iron Loss = 35 W

$$\text{C o p p } \frac{1}{2} \text{ Full-load} = \left(\frac{1}{2}\right)^2 \times \text{Full-load Copper Loss}$$

$$= \frac{1}{4} \times 48 = 12 \text{ Watts}$$

$$\therefore \text{ Total Loss} = \text{Iron Loss} + \text{Copper Loss at } \frac{1}{2} \text{ Full-Load}$$

$$= 35 + 12 = 47 \text{ Watts}$$

$$\therefore \text{ Input} = \text{Output} + \text{Total Loss} = 2400 + 47 = 2447 \text{ W}$$

$$\therefore \text{ Efficiency} = \frac{\text{Output}}{\text{Input}} = \frac{2400}{2447} = 0.98 = \mathbf{98\%}$$

Problem 5.13

The primary and secondary windings of a 500 KVA transformer have resistances of 0.5 Ω and 0.002 Ω respectively. The primary and secondary voltages are 10000 V and 400 V respectively and the core loss is 3 KW, the power factor on the load is 0.8. Calculate the efficiency on

(a) Full-load

(b) Half Full-Load.

(a) At Full-Load :

Full-load Primary Current, $I_1 = \dfrac{\text{KVA Rating} \times 1000}{\text{Rated Primary Voltage } V_1}$

or $I_1 = \dfrac{500 \times 1000}{10000} = 50\,\text{A}$

Full-load Secondary Current $I_2 = \dfrac{\text{KVA Rating} \times 1000}{\text{Rated Secondary Voltage } V_2}$

or $I_2 = \dfrac{500 \times 1000}{400} = 1250\,\text{A}$

Primary I^2R loss on Full-load is $(50)^2 \times 0.5 = 1250$ W

and Secondary I^2R loss on Full-Load is $(1250)^2 \times 0.002 = 3125$ W

$\therefore$ Total I^2R loss on Full-Load = 1250 + 3125 = 4375 W = 4.375 KW

$\therefore$ Total Loss on Full-Load = I^2R loss + Core Loss

= 4.375+3 = 7.375 KW

Output power on Full-Load = KVA $\times \cos\phi$

= 500 × 0.8 = 400 KW

$\therefore$ Input power on Full-Load

= Output power on Full-Load + Total loss on Full-load

= 400 + 7.375 = 407.375 KW

In Sec 5.6, we saw that $\eta = 1 - \dfrac{\text{losses}}{\text{input power}}$

$$= \left(1 - \frac{7.375}{407.375}\right)$$

= 0.9819 per unit

= 98.19 percent

(b) At Half Full-Load

Total I^2R loss at Half Full-Load $= \left(\dfrac{1}{2}\right)^2 \times \text{F.L. Loss}$

$$= \frac{1}{4} \times 4.375$$

= 1.09 KW

and Total Loss at Half Full-Load = 1.09 + 3 = 4.09 KW

Now, Output at Half-Full Load $= \left(\frac{500}{2}\right) \times \cos\phi$

$= 250 \times 0.8 = 200$ KW

$\therefore$ Input at Half F.L = Output at Half F.L + Total Loss at Half F.L

$= 200 + 4.09 = 204.09$ KW

As already seen in Sec 5.6

$$\text{Efficiency} = 1 - \frac{\text{losses}}{\text{input power}}$$

$\therefore$ Efficiency at Half F.L $= \left(1 - \frac{4.09}{204.09}\right)$

$= 0.98$ per unit

= 98 percent

5.13 Review Questions

Q 1. Explain with a sketch the construction of a core type or a shell type single phase transformer. (Dec 86, Sep/Oct 87, Jan 93)

Q 2. a) Explain why the core of a transformer is laminated ?
b) State why silicon steel is selected for the core of a transformer ?
(Oct 85, B.U.)

Q 3. Explain the principle of operation of a transformer.
(June/July 89, July 93, B.U.)

Q 4. Derive an expression for the e.m.f. induced in the secondary winding of a transformer. (June/July 90, July 93, Aug 95, Feb 96, B.U.)

Q 5. Derive the EMF equation of a transformer from fundamentals. (Aug 96, B.U.)

Q 6. Define 'Regulation of a Transformer.

Q 7. What are the different power losses that occur in a transformer ?
How do they vary with the load ? How are these losses minimised ?
(Mar 99, V.T.U. Mar 94, B.U.)

Q 8. Enumerate the various losses that occur in a transformer. Explain briefly.
(June 81, Aug 82, Sep 83, B.U.; Mar/Apr 88, M.U.)

Q 9. What do you understand by the efficiency of a transformer ?

(Apr 85, B.U.)

5.14 Exercises - Problems

1. A 10 kVA, 50 Hz single phase transformer has a turns ratio 300 / 23. The primary is connected to 1500 V, 50 Hz supply. Find the secondary voltage on open circuit and maximum flux in the core. [BU - March 89]

Answer : 115 V, 22.52 mWb

2. A 10 kVA, single phase transformer has 400 primary turns and 1000 secondary turns. The net cross-sectional area of the core is 60 cm^2. When the primary winding is connected to a 500 V, 50 Hz supply, calculate (i) maximum value of flux density in the core (ii) the voltage induced in the secondary winding (iii) the secondary full load current. [BU-Dec 84]

Answer : (i) 0.938 Tesla (ii) 1250 V (iii) 8 A.

3. Find the number of turns required on the H.V. and L.V. sides of a 415 / 240 V, 50 Hz, single phase transformer, if the net area of cross-section of the core is 25 cm^2 and the maximum flux density is 1.3 Wb/m^2. [BU - March 84]

Answer : 575, 333.

4. A 125 kVA transformer has a primary voltage of 2000 V at 60 Hz frequency. Primary turns are 182 and secondary turns are 40. Neglecting losses, calculate (i) No-load secondary e.m.f. (ii) full-load primary and secondary currents (iii) flux in the core. [Mysore, March 90]

Answer : i) E_s = 439.56 V (ii) I_p = 62.5 A,

I_s = 284.376 A (iii) ϕ_m = 41.25 mWb,

5. A 1 kVA single phase transformer has a core loss of 15 W and a full-load copper loss of 20 W. Determine its efficiency (i) at full-load, unity p.f. (ii) at 1/3 full-load, 0.9 p.f. and (iii) at 1/4 full-load 0.75 p.f.

Answer : (i) 96.619 % (ii) 94.57 % (iii) 92.0245 %

6. Calculate the efficiency of a 100 kVA, single phase transformer working at full load (i) 0.8 p.f. lag (ii) unity power factor, given copper loss at full-load is 1000 W and iron loss is also 1000 W. [Gulbarga, May 87]

Answer : 97.56 %, 98.04 %.

7. A single phase transformer has 525 turns on primary and 70 turns on secondary. If the primary is connected to a 3300 V supply, find the secondary e.m.f. Also calculate the primary current when the secondary current is 100 A.

[Gulbarga June 92]

Answer : 440 V, 13.33 A.

8. A 440 / 230 V, 50 Hz single-phase transformer is having 525 turns on the L.V. side. Calculate (i) Number of turns on the H.V. side, (ii) maximum flux density in the core, if the effective cross-sectional area of the core is 15 Sq. cm.

[KUD No. 1989]

Answer : 1005, 1.3156 Wb/m^2.

9. Find the number of turns on both H.T. and L.T. side of a 220 / 440 V, 50 Hz single - phase transformer if the area of cross-section of the core is 40 cm^2 and the maximum flux density is 1 Wb/m^2. [KUD May 1990]

Answer 248, 496.

10. A single phase transformer has a turns ratio of 1 : 10 and a secondary winding of 1000 turns. The primary winding is connected to a 25 Volts sinusoidal supply. If the maximum core flux is 2.25 mWb. Find (i) the frequency of the supply (ii) the number of primary turns (iii) the secondary voltage on open circuit.

[KUD. 1984 April.]

Answer : (1) 25 Hz (2) 100 (3) 250 V.

11. A 200 kVA, 3300 V / 240 V, 50 Hz single - phase transformer has 80 turns on the secondary winding. Assuming an ideal transformer, calculate : (i) the primary and secondary currents on full-load; (ii) the maximum value of the flux, (iii) the number of primary turns. [KUD, May 1991]

Answer : 60.60 A, 833.33 A, 0.013 Wb, 1100.

12. A 25 kVA transformer has 500 turns on the primary and 50 turns on the secondary winding. The primary is connected to 3.3 kV, 50 Hz supply. Calculate the full-load primary and secondary currents, the e.m.f. in the secondary and the maximum flux in the core. Neglect leakage and magnetising current.

[KUD, 1988 April]

Answer : 7.576 A; 75.76 A; 330 V; 29.73 mWb.

13. The required no-load ratio in a single phase, 50 Hz, core type transformer is 6000 / 250 V. Find the number of turns in each winding, if the flux in the core is to be about 0.06 Wb.

[KUD, 1988 April]

Answer : Primary - 288 turns, Secondary - 12 turns.

14. A single phase transformer has 300 primary and 750 secondary turns. The net cross-sectional area of the core is 64 sq. cm. If the primary voltage is 440 V at 50 Hz, and rating of the transformer is 10 kVA, find (i) the maximum flux density in the core, and (ii) the e.m.f. induced in the secondary.

[KUD, 1985 Oct.]

Answer : (i) 1.032 Wb/m^2, (ii) 1100 Volts.

15. The following data apply to a single phase transformer : Output : 100 kVA, Secondary Volts : 400 V, Primary turns : 200, Secondary turns : 40. Calculate, neglecting losses, (i) the primary applied p.d. (ii) the normal primary and secondary currents, and (iii) the secondary current when loaded to 30 kW at 0.8 p.f. [KUD 1983]

Answer : (i) 2000 V; (ii) 50 A, 250 A; (iii) 93.75 A.

16. The e.m.f per turn for a single-phase 2200 V / 220 V, 50 Hz transformer is approximately 12 V. Calculate (i) the number of primary and secondary turns and (ii) the net cross-sectional area of core for a maximum flux density of 1.5 T. [KUD, Nov 1991]

Answer : (i) 180.18 (ii) 0.0367 m^2.

17. A single phase transformer has 525 turns on primary and 70 secondary turns. If the primary is connected to a 3300 V supply, find the secondary potential difference. Neglecting losses, what is the primary current, when the secondary current is 250 A ? [KUD, 1986 April]

Answer : 440 Volts ; 33.33 A

18. A 400 / 230 V, 50 Hz single phase transformer is provided with 500 turns on the L.V. side. Determine (i) the number of turns on H.V. side (ii) the effective area of cross-section of core, if the maximum flux density in the core is to be less than 1.4 Wb/m^2. [KUD, 1987 April]

Answer (i) 870 turns ; (ii) 14.8 Sq.cm.

MODULE

5

(b) Three-Phase Induction Motor

5.15 Introduction

The three-phase induction motor is the most widely used a.c. motor. It differs from other types of motors in that there is no connection from the rotor winding to any source of supply. The necessary voltage and current in the rotor circuit are produced by induction from the stator winding, which is why it is called **Induction Motor.**

Advantages :

1. Its cost is low.
2. It has very simple, very robust and rugged, practically unbreakable construction (particularly the squirrel-cage motor).
3. It is very reliable.
4. It is highly efficient.
5. It has a fairly good power factor.
6. Its maintenance requires minimum of attention.
7. It does not need to be synchronised. It has a simple starting arrangement, especially in the case of the squirrel-cage rotor type of induction motor.

Disadvantages :

1. It is essentially a constant speed motor and the speed cannot be varied easily.
2. Its speed reduces to some extent with increase in load, as in the case of d.c. shunt motor.
3. It has a somewhat lesser starting torque, as compared to a d.c. shunt motor.

5.16 Types and Constructional Features

A three-phase induction motor essentially consists of the following two parts:

a) Stator and

b) Rotor

a) **Stator** : It is the stationary part of the motor. The stator core consists of high grade, low electrical loss, thin silicon steel stampings, so that hysteresis and eddy current losses are reduced to a minimum. The thickness of these stampings varies from 0.35 mm to 0.65 mm. The stampings are insulated from each other by a coating of varnish and held together by bolts in insulating sleeves. In order to keep air-gap reluctance minimum the air-gap between the stator and rotor is made as small as practicable (0.3 to 0.35 mm in small machines and 1.0 to 1.5 mm, in high power machines). A 3-phase stator winding is placed in slots on the inner periphery of the core. This winding is fed from a 3-phase supply, and is wound for a definite number of poles, the exact number of poles being determined by the requirements of speed. The greater the number of poles the lesser the speed and vice-versa. When a 3-phase supply is given to the stator winding, a magnetic field or flux is produced, which is of constant magnitude, but which rotates at synchronous speed, given by the expression $N_s = \frac{120f}{P}$. This rotating field induces an e.m.f in the rotor by mutual induction.

b) **Rotor** : It is the rotating part of the motor. The following are the **two types of rotors** employed in 3-phase induction motors :

i) *Squirrel-Cage Rotor :* Motors with this type of rotor are known as Squirrel-Cage Induction Motors.

ii) *Phase-wound or Wound Rotor* : Motors with this type of rotor are known as 'Phase-wound' motors, 'Wound' motors or 'Slip-ring' motors.

i) *Squirrel-cage Rotor*

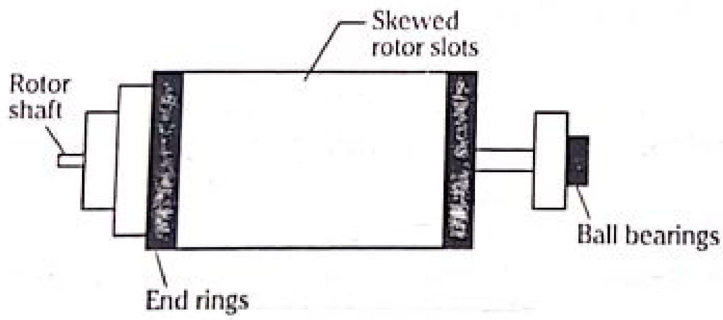


Fig. 5.13

This kind of rotor is very strong and rugged in construction and is practically indestructible, which is why 90% of induction motors are provided with it. This rotor is constructed of a laminated core, with the conductors (heavy bars of copper, aluminium or alloys) placed parallel or approximately parallel (somewhat skewed) to the shaft (Fig. 5.13) and embedded in the surface of the core. The purpose of skewing is three-fold ; *firstly*, the motor runs smoothly as the magnetic hum is reduced; *secondly*, the tendency of the rotor teeth to remain under the stator teeth due to direct magnetic attraction is reduced; *thirdly*, more uniform torque is obtained while running.

The above conductors are not insulated from the core, since the rotor currents naturally follow the path of least resistance, that is, the rotor conductors. At each end of the rotor, these conductors are brazed or bolted to two short-circuiting end-rings of similar material to that of the conductors. The rotor conductors and their end-rings form a completely closed circuit in itself, resembling a squirrel cage, hence the name.

ii) *Phase-wound Rotor : (Slip-Ring Rotor)*

As the name suggests, such a rotor is wound with a 3-phase, double layer, distributed, insulated winding, consisting of coils. The rotor is wound for as many poles as the number of stator poles. The star connection of the three phases is made internally. The winding is made in slots and connected to phosphor-bronze slip-rings at one end of the rotor shaft.

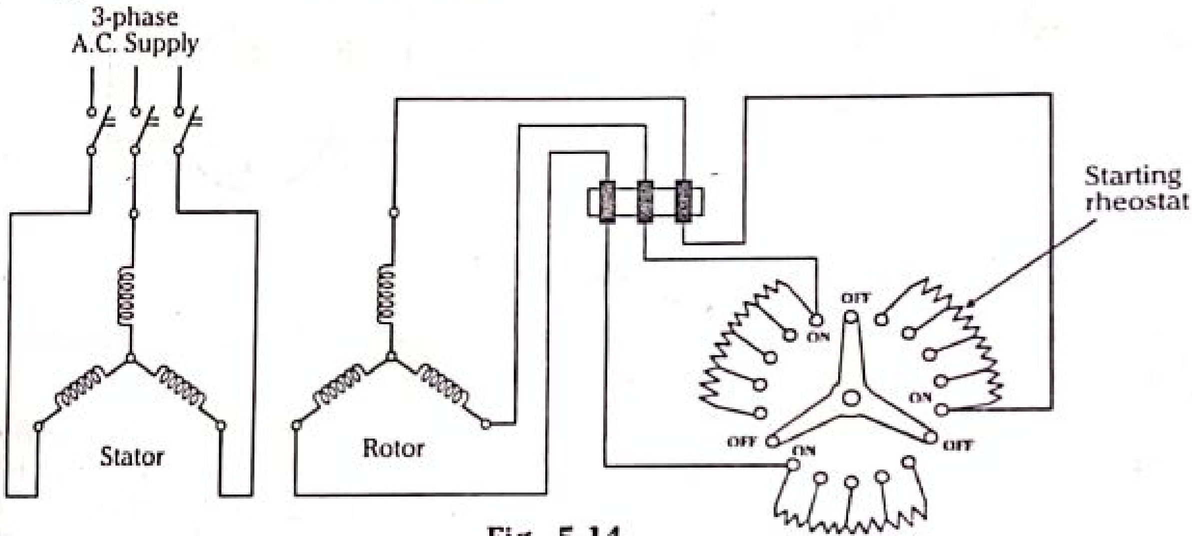


Fig. 5.14

The brushes, which carry the current from and to the rotor windings, are held in box-type holders mounted on insulated steel rods, securely bolted to the end shield. Each brush is suitably tensioned so as to touch each of the slip rings. These brushes are further connected externally to a 3-phase star-connected rheostat for the purpose of starting and speed control (Fig. 5.14). At the time of starting, the entire resistance is included in the rotor circuit and this resistance is gradually cut off as the rotor picks up speed. When running under normal conditions, the entire external resistance is cut out and the rotor windings are automatically short-circuited through the slip-rings by means of a metal collar which is pushed along the shaft and connects rings together. Next, the brushes are automatically lifted from the slip-rings to reduce frictional losses and wear and tear. Thus, we see that, under normal running conditions, the wound rotor is short circuited on itself, just like the squirrel-cage rotor.

The slip-ring induction motors are less extensively used than the squirrel-cage type because of their higher first cost and greater maintenance cost. The slip-ring induction motors are employed only when speed control or high starting torque is required.

5.17 Rotating Magnetic Field

We will show that when three-phase windings, displaced 120 electrical degrees relative to one another, are supplied with three-phase currents, they produce a resultant magnetic flux which rotates in space as if actual magnetic poles were being rotated mechanically.

Let us consider a three-phase induction motor. Let the delta-connected stator windings be supplied from a three-phase a.c. source as shown in Fig. 5.15(a).

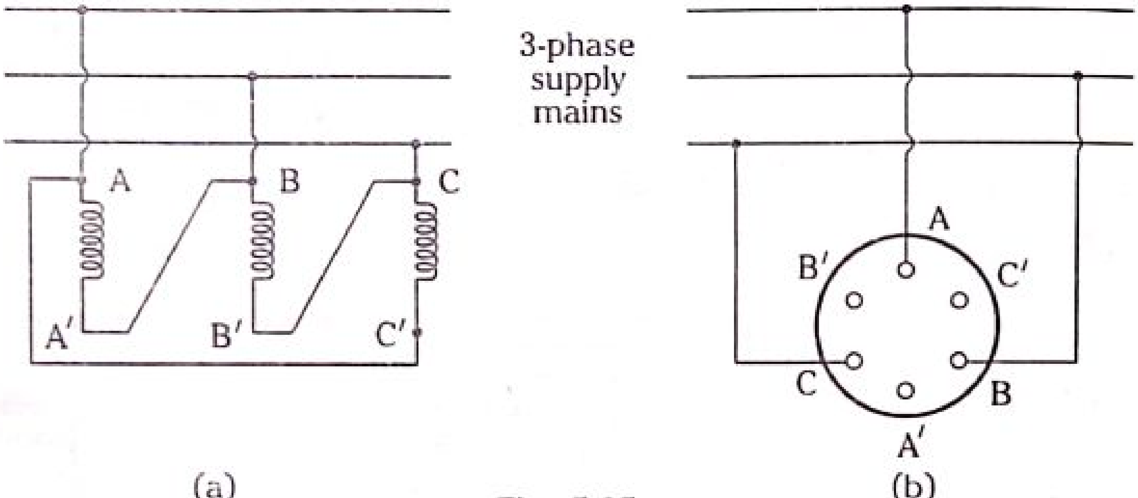


Fig. 5.15

The relative positions of the three phase windings AA', BB' and CC' are shown in Fig. 5.15(b).

As soon as power supply is switched on, alternating currents flow simultaneously in the stator windings. Each current establishes its own magnetic flux. This flux, due to the current in the three phase windings, is shown in Fig. 5.16. The assumed positive directions of the fluxes are shown in Fig. 5.17. Let the maximum value of flux due to any one of the phases be ϕ_m. The resultant flux ϕ_T, at any instant, is given by the vector sum of the individual fluxes, ϕ_R, ϕ_Y and ϕ_B due to the currents in the three phases.

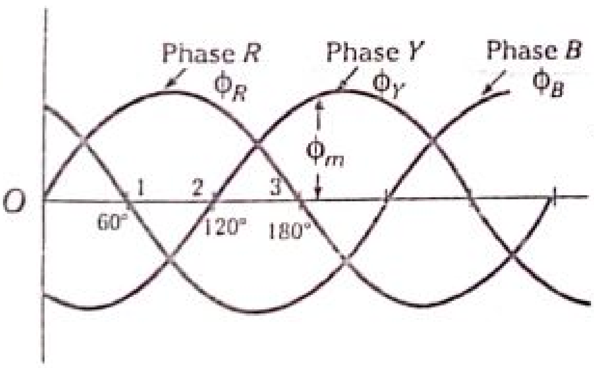


Fig. 5.16

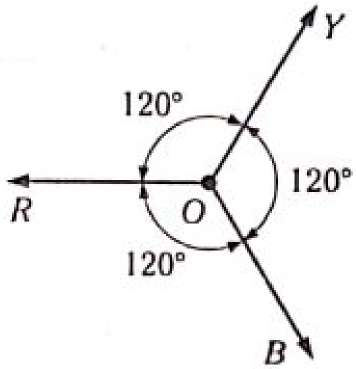


Fig. 5.17

We will consider values of ϕ_T at four instants at intervals of 60° electrical, marked 0, 1, 2 and 3 in Fig. 5.416

(a) At $\theta = 0$

This corresponds to point O in Fig. 5.16. Here $\phi_R = 0$, $\phi_Y = -\frac{\sqrt{3}}{2}\phi_m$, $\phi_B = \frac{\sqrt{3}}{2}\phi_m$. The vector for ϕ_Y in Fig. 5.18(a) is drawn opposite to the direction assumed positive in Fig. 5.17.

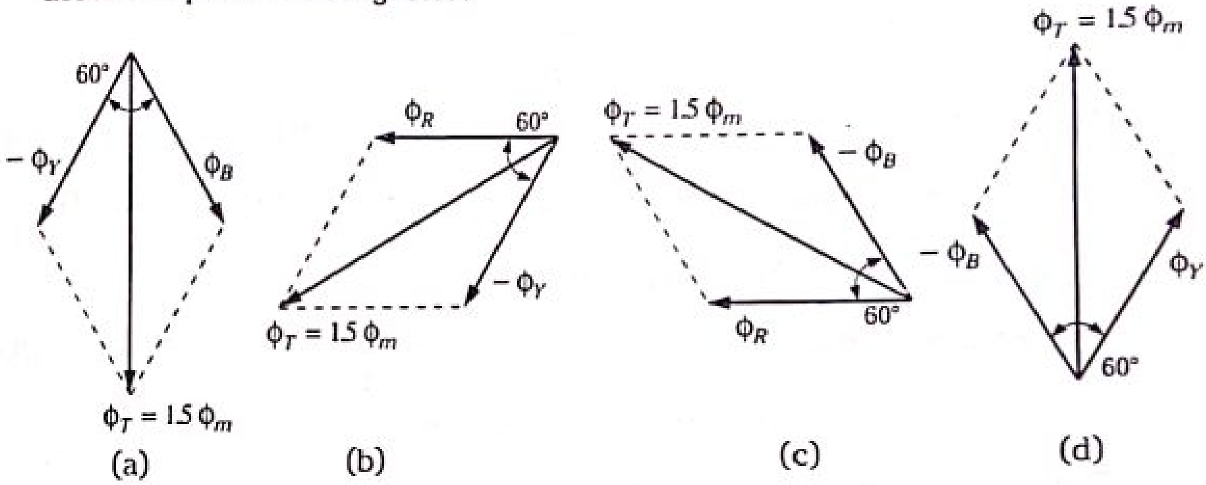


Fig. 5.18

$$\therefore \quad \phi_T = 2 \times \frac{\sqrt{3}}{2}\phi_m \cos\frac{60^\circ}{2} = \sqrt{3} \times \frac{\sqrt{3}}{2}\phi_m = \frac{3}{2}\phi_m \quad [\text{Fig. 5.18(a)}]$$

(b) At $\theta = 60°$, corresponding to point 1 in Fig. 5.16;

Here $\phi_R = \frac{\sqrt{3}}{2}\phi_m$, drawn parallel to OR of Fig. 5.17.

$\phi_Y = -\frac{\sqrt{3}}{2}\phi_m$, drawn in opposition to OY in Fig. 5.17.

$\phi_B = 0$

$$\therefore \quad \phi_T = 2 \times \frac{\sqrt{3}}{2}\phi_m \cos 30^\circ = \frac{3}{2}\phi_m \quad [\text{Fig. 5.18(b)}]$$

We see that the resultant flux is again $\frac{3}{2}\phi_m$, but has rotated clockwise through an angle of 60°.

(c) At $\theta = 120°$, i.e., corresponding to point 2 in Fig. 5.16;

Here $\phi_R = \frac{\sqrt{3}}{2}\phi_m$, $\phi_Y = 0$, $\phi_B = -\frac{\sqrt{3}}{2}\phi_m$.

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We can again prove that $\phi_T = \frac{3}{2}\phi_m$.

Thus, once again the resultant has the same value, but has further rotated clockwise through an angle of 60° [Fig 5.18(c)].

(d) At $\theta = 180°$, *i.e.*, relating to point 3 in Fig. 5.16;

Here $\phi_R = 0$, $\phi_Y = \frac{\sqrt{3}}{2}\phi_m$, $\phi_B = -\frac{\sqrt{3}}{2}\phi_m$.

The resultant is $\frac{3}{2}\phi_m$, and has rotated clockwise through an additional angle of 60°, or through an angle of 180° from the beginning [Fig 5.18(d)].

Thus, we come to the following conclusions :

1. *The resultant flux is of constant value* $= \frac{3}{2}\phi_m$, *i.e.,* 1.5 *times the maximum value of the flux due to any phase.*

2. *The resultant flux rotates around the stator at synchronous speed given by* $N_S = \frac{120\,f}{P}$, *where* P = *number of stator poles and* f = *supply frequency in Hz.*

5.18 Principle of Operation

When the stator of a 3-phase induction motor is connected to a 3-phase a.c. supply, a rotating magnetic field is established which rotates at synchronous speed. The direction of rotation of this field will depend upon the phase sequence of the stator currents, and therefore, will depend upon the order of connection of the stator terminals to the supply. The direction of rotation of the field can be reversed by interchanging the connection to the supply of any two leads of the 3-phase induction motor. The number of magnetic poles of the revolving field will be the same as the number of poles for which each phase of the stator winding is wound.

The magnetic flux of constant amplitude, rotating at synchronous speed, passes through the air-gap and cuts the rotor conductors which as yet are stationary. Due to the relative speed between the rotating flux and the stationary conductors, an e.m.f. is induced in the latter as per Faraday's Laws of Electromagnetic Induction. *The frequency of the induced e.m.f. is the same as the supply frequency.* Its magnitude is proportional to the relative velocity between the flux and the conductors, and its direction is given by Fleming's Right-Hand Rule. Since the rotor conductors form a closed circuit, rotor current is produced, whose direction in terms of Lenz's Law is such as to oppose the very cause producing it. Here, the cause which produces the rotor current is the relative velocity between the rotating field and the stationary

rotor. Hence, to reduce this relative speed, the rotor starts running in the same direction as the stator field in an effort to catch up with it.

The setting up of the torque for causing the rotor to rotate is explained below :

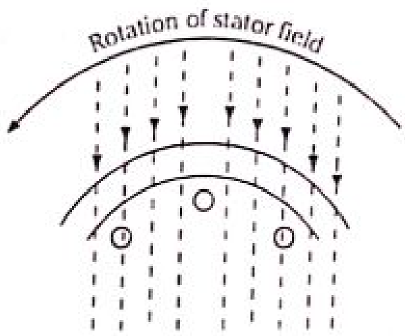

Fig. 5.19(a)

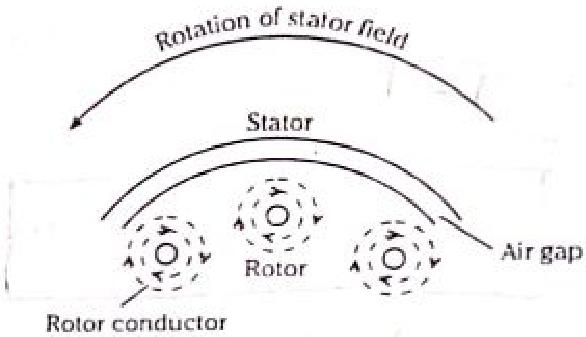

Fig. 5.19(b)

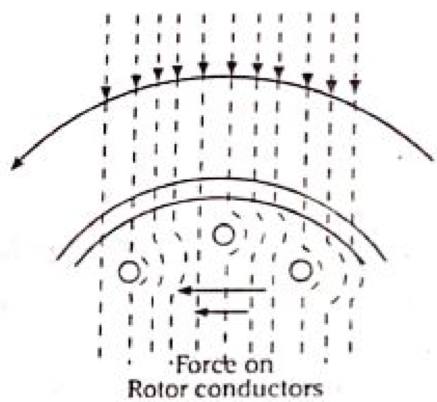

Fig. 5.19(c)

In Fig. 5.19 (a), the stator field is shown as rotating in an anticlockwise direction. The relative motion of the rotor with respect to the stator is clockwise. By applying the Right-Hand Rule, the direction of the induced e.m.f in the rotor is outwards. So, the direction of the flux because of the rotor current alone is as shown in Fig. 5.19(b). Now, considering the effect of both rotor and stator fields, the rotor conductors are subjected to a force tending to rotate them in the anticlockwise direction. Thus, the rotor is made to rotate in the same direction as the stator field (Fig. 5.19c).

5.19 Slip and its Importance

It has been discussed in the previous Section that the rotor follows the stator field. In actual practice, the rotor can never reach the speed of the stator field. If it does so, there would be no relative movement between the stator field and rotor conductors, no induced rotor current, and therefore, no torque to drive the rotor. Hence, the rotor speed is *always* less than the speed of the stator field. The difference in the speed between stator field and rotor depends on the load;

as the load is applied, the natural effect of the load or braking torque is to slow down the motor. Hence, slip increases and with it increases the current and torque till the driving torque of the motor balances the retarding torque of the load. This phenomenon determines the speed at which the motor runs on load.

The difference between the synchronous speed N_s and the actual speed N of the rotor is called slip.

It is usually expressed as a percentage of synchronous speed, *i.e.*,

$$\boxed{\% \text{ age slip, } s = \frac{N_S - N}{N_S} \times 100}$$

The quantity $N_s - N$ is sometimes called the *slip-speed*.

It is apparent that the rotor (or motor) speed is $N = N_s(1 - s)$.

We must remember that the revolving flux rotates synchronously with respect to the stator but at slip-speed relative to the rotor.

In an induction motor, the change in slip from no-load to full-load is hardly 3 – 6 %, so that the induction motor is essentially a constant speed motor.

5.20 Frequency of Rotor Current (or E.M.F.)

When the rotor is at standstill, the frequency of rotor current or e.m.f. is the same as the supply frequency. However, when there is relative speed between the rotor and the stator field, the frequency of the induced e.m.f., and hence the current, in the rotor varies with the rotor speed *i.e.*, slip. Let at any speed N of the rotor, the frequency of the rotor current be f'.

Then, $$N_S - N = \frac{120 f'}{P} \qquad \text{---(i)}$$

Also $$N_S = \frac{120 f}{P} \qquad \text{---(ii)}$$

Dividing (i) by (ii), we have

$$\boxed{\frac{f'}{f} = \frac{N_S - N}{N_S} = s}$$

$6' = s6$

Hence, the frequency of rotor current (or e.m.f.) may be obtained by multiplying the supply frequency by fractional slip.

5.21 Applications of Squirrel-cage and Slip-ring Motors

Three-phase induction motors are used, in practice, for a large number of industrial applications. The choice of the type of motor - whether squirrel-cage or slip-ring-is made on the basis of several factors like starting torque, speed regulation etc.,

1. Squirrel-cage type of motors having moderate starting torque and constant speed characteristics are widely used for driving fans, blowers, water-pumps, grinders, lathe machines, printing machines and drilling machines.
2. Slip-ring induction motors can have high starting torque as high as maximum torque. Hence they are preferred for lifts, hoists, elevators, cranes and compressors.

5.22 Necessity of a Starter

During the normal operation of the motor, it has the full rated voltage applied across it. However, at the time of starting, it draws about 5 to 7 times the full load current and produces only 1.5 to 2.5 times the full load torque, when it is directly connected to the supply. This large initial surge of current is due to the absence of back e.m.f during starting. This large starting current is objectionable, as it is sure to cause damage to the motor; besides it causes large line drop, adversely affecting the operation of other connected apparatus to the line. Hence, in the case of a Squirrel Cage induction motor, where the rotor of the motor is permanently short-circuited, **a reduced voltage is applied at starting, and the voltage is increased to the rated value, when the motor has picked up speed. The reduced voltage is obtained by using a Starter.**

In the case of Slip-ring induction motors, **resistance can be included in the rotor circuit during starting and can be removed when the motor picks up speed.**

The different types of starters are discussed in the following paragraphs.

5.23 Star-delta Starter

Squirrel-cage motors are generally started by operating the changeover switch of a star-delta starter.

The starter connects the three stator windings in star at the instant of starting and as the motor picks up the normal speed, the starter is switched over to the running position, to connect the stator windings in delta (Fig. 5.20).

The voltage of each phase at starting is reduced to $\frac{1}{\sqrt{3}}$ of the line voltage.

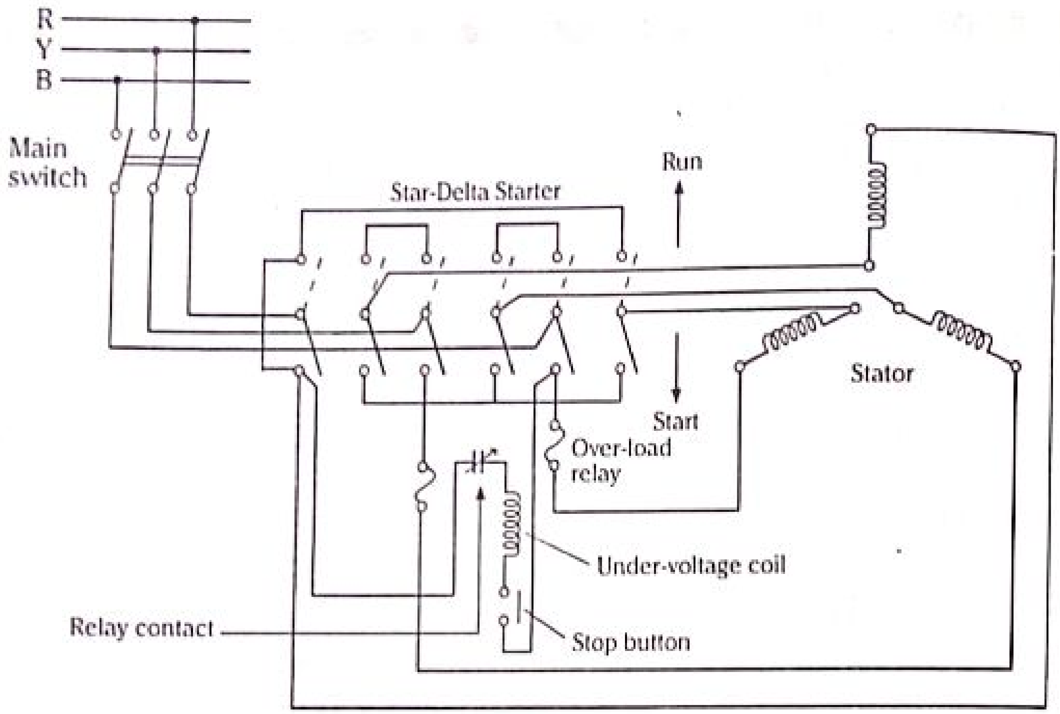

Fig. 5.20

The current in each phase is also reduced by the same factor. Thus, the line current during starting is

$$= \frac{1}{\sqrt{3}} \times \frac{1}{\sqrt{3}} = \frac{1}{3}$$

of the current which the motor would have taken if it had been directly connected across the mains supply.

As $T \propto V^2$, the starting torque is reduced to

$$\left(\frac{1}{\sqrt{3}}\right)^2 = \frac{1}{3} = 33.3\ \% \text{ of the normal torque}$$

The changeover switch is of the double-throw type, with interlocks to prevent the motor from starting when the switch is in the RUN position.

An Overload Relay is provided, whose contact will open in the event of overload, stopping the motor as supply to one of the stator windings is disconnected. An under voltage coil is incorporated, so that if the supply voltage falls below a particular value, the stop button will be operated and supply to a stator phase will be disconnected, again bringing the motor to a halt. The stop button can also be manually operated, in order to stop the machine.

5.24 Illustrative Examples on Speed and Slip Calculations

Problem 5.14

A 3-phase, 4-pole, 400 V, 50 Hz induction motor runs with a speed of 1440 r.p.m. Calculate its slip.

Solution :

Synchronous speed, $N_S = \dfrac{120\,f}{P}$

$$= \frac{120 \times 50}{4} = 1500 \text{ r.p.m}$$

$$\left[S = \frac{N_S - N}{N_S} \right]$$

$$\cancel{S = \frac{N - N_S}{N_S}}$$

Rotor speed, $N = N_S(1 - s)$

or $1440 = 1500(1 - s)$

$1500\, s = 60$ or Slip 's' $= 0.04$ or **4 %**

Problem 5.15

An 8-pole alternator runs at 750 r.p.m. and supplies power to a 6-pole induction motor which runs at 970 r.p.m. What is the slip of the induction motor ?

Solution :

For the alternator, given $P_A = 8$ and $N_A = 750$ r.p.m.

$\therefore$ Frequency of the generated e.m.f $= \dfrac{N_A\, P_A}{120}$

$$= \frac{750 \times 8}{120}$$

$$= 50 \text{ Hz}$$

$\therefore$ Supply frequency for the induction motor = 50 Hz

Given : No. of poles of the induction motor, $P = 6$

$\therefore$ Synchronous speed of the induction motor, $N_S = \dfrac{120\,f}{P}$

$$= \frac{120 \times 50}{6}$$

$$= 1000 \text{ r.p.m.}$$

Now $N = N_S(1 - s)$

or $970 = 1000(1 - s)$

$\therefore$ $1000\, s = 30$

$\therefore$ $s = 0.03$ or **3 %**

Problem 5.16

A 3-phase, 6 pole, 60 Hz Induction Motor has frequency of rotor current at full-load of 1.8Hz. Find the synchronous speed and slip at full-load.

Solution : Synchronous speed, $N_S = \frac{120f}{P} = \frac{120 \times 60}{6} = 1200$ r.p.m.

Frequency of rotor current at full-load

$$f' = sf \quad \text{or} \quad s = \frac{f'}{f} = \frac{1.8}{60} = 0.03 \text{ or } \mathbf{3\%}$$

Problem 5.17

The frequency of the e.m.f. in the stator of a 4-pole induction motor is 50 Hz and in the rotor is 1.5 Hz. What is the slip and at what speed is the motor running ?

Solution :

$$f' = sf \quad \text{or} \quad 1.5 = s \times 50$$

$$\therefore \quad s = 0.03 \text{ or } \mathbf{3\%}$$

$$\text{Synchronous speed, } N_S = \frac{120f}{P} = \frac{120 \times 50}{4} = 1500 \text{ r.p.m.}$$

$$\text{Rotor speed, } N = N_S(1 - s) = 1500(1 - 0.03) = \mathbf{1455 \text{ r.p.m.}}$$

Problem 5.18

If the electromotive force in the stator of an 8-pole induction ...

$$\text{Synchronous speed, } N_S = \frac{120f}{P} = \frac{120 \times 50}{4}$$

$$= \mathbf{750\ r.p.m.}$$

$$\text{Actual speed, } N = N_S(1 - s)$$

$$= 750\ (1 - 0.03)$$

$$= \mathbf{728\ r.p.m.}$$

5.25 Review Questions

Q 1. Explain the construction of Squirrel-cage and Phase wound (Slip-Ring) Induction Motors. (July 93, Mar 95, Feb 96, B.U.)

Q 2. Explain the difference in construction between Squirrel-cage and Wound-rotor (Slip-Ring Rotor) types of rotor of an Induction Motor. (July 88, B.U.)

Q 3. Write a brief note on different types of Induction Motors, their constructional features and specific applications. (June/July 89, B.U.)

Q 4. Briefly explain the construction and principle of operation of a 3-phase Squirrel-cage Induction Motor. (83-84, Nov/Dec 84, Sep/Oct 87, B.U.; Aug/Sep 89, M.U.)

Q 5. Explain the principle of operation of an Induction Motor. Why does it require a starter ? Can it be run at synchronous speed ? Why ? (Mar/Apr 88, M.U.)

Q 6. What is "slip" in an Induction Motor ? Explain why slip is never zero in an Induction Motor. (Dec 86, B.U.)

5.26 Exercises - Problems

1. An 8-pole alternator runs at 750 rpm and supplies power to a 6-pole induction motor which has a full-load slip of 3%. Find the full-load speed of the motor and the frequency of the rotor e.m.f.

   Answer : 970 rpm; 1.50 Hz.

2. A 12-pole, 3-phase alternator is coupled to an engine running at 500 rpm. It supplies an Induction motor which has a full-load speed of 1440 rpm. Find the percentage slip and the number of poles of the motor.

   Answer : 4 % ; 4

3. A 3-phase Induction Motor has 6 poles and runs at 960 r.p.m. on full-load. It is supplied from an alternator having 4 poles and running at 1500 r.p.m. Calculate (i) the Full-load slip of the motor, (ii) frequency of the rotor current.

(May 91, K.U.D)

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Answer : (i) 4 % (ii) 2 Hz.

4. A 4-pole, 50 Hz, Induction Motor has a rotor frequency of 2 Hz. Determine (i) the synchronous speed (ii) the speed of the motor (iii) the slip.

(June 90, Gulbarga)

Answer : (i) 1500 rpm (ii) 1440 rpm (iii) 4 %

5. A 6-pole Squirrel-cage Induction Motor is connected to a supply of 400 V, 50 Hz. It runs at 960 r.p.m. Calculate (i) Synchronous Speed (ii) Slip.

(Sep 86, Gulbarga)

Answer : (i) 1000 r.p.m. (ii) 4 %.

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